We prove that if $G$ is a 5-connected graph embedded on a surface $\Sigma$ (other than the sphere) with face-width at least 5, then $G$ contains a subdivision of $K_5$. This is a special case of a conjecture of P. Seymour, that every 5-connected non-planar graph contains a subdivision of $K_5$. Moreover, we prove that if $G$ is 6-connected and embedded with face-width at least 5, then for every $v \in V(G)$, $G$ contains a subdivision of $K_5$ whose branch vertices are $v$ and four neighbors of $v$. (Received September 14, 2010)