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Donald Nelson, Michael Plummer, Neil Robertson and Xiaoya Zha* (xzha@mtsu.edu),
Department of Mathematical Sciences, Middle Tennessee State University, Murfreesboro, TN
37132. *On a conjecture concerning the Petersen Graph.* Preliminary report.

Robertson has conjectured that the only 3-connected internally-4-connected graph of girth 5 in which every odd cycle of length greater than 5 has a chord is the Petersen graph. We prove this conjecture in the special case where the graphs involved are also cubic. Moreover, this proof does not require the internal-4-connectivity assumption. An example is then presented to show that the assumption of internal-4-connectivity cannot be dropped as an hypothesis in the original conjecture.

We then summarize our results aimed toward the solution of the conjecture in its original form. In particular, let G be any 3-connected internally-4-connected graph of girth 5 in which every odd cycle of length greater than 5 has a chord. If C is any girth cycle in G then $N(C) - V(C)$ cannot be an independent set, and if $N(C) - V(C)$ contains a path of length at least 2, then the conjecture is true. If the conjecture fails and H is a counterexample, then for any girth cycle C in H , $N(C) - V(C)$ consists of a matching M together with an independent set of vertices. Moreover, M can be partitioned into (at most) two disjoint non-empty sets where we can precisely describe how these sets are attached to cycle C . (Received September 14, 2010)