We present examples of singular measures on the 2-torus such that each measure imposes rigid constraints on the group of
measure preserving diffeomorphisms. All our measures are constructed as invariant measures for an auxiliary non-linear
Anosov diffeomorphism $f: \mathbb{T}^2 \to \mathbb{T}^2$. In the first example, we construct equilibrium states for $f$ such that the group of
measure preserving diffeomorphisms is virtually infinite cyclic. In the second example, we show that, for a larger class
of measures, the set of entropies for all measure preserving diffeomorphisms has a semi-group structure isomorphic to
$\mathbb{N}$. (Received September 02, 2010)