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A. T. Szankowski*, Institute of Mathematics, Hebrew University, Jerusalem, Israel. *Hereditary Approximation Property.*

We say that a Banach space has the hereditary approximation property (or is a HAPpy space) if all of its subspaces have the approximation property. In 1976 Johnson constructed the first examples of HAPpy spaces not isomorphic to a Hilbert space. Later Pisier developed the theory of weak Hilbert spaces and proved that they all have the HAP. The spaces constructed by Johnson as well as all weak Hilbert spaces are asymptotically Hilbertian. An asymptotically Hilbertian space cannot have a symmetric (or even subsymmetric) basis unless it is isomorphic to ℓ_2 . This immediately prompts the following problem: “Is there a HAPpy space which has a symmetric basis but is not isomorphic to ℓ_2 ?” . We give an affirmative answer to this question. The crucial quantity is $d_n(X) = \sup\{d(E, \ell_2^n) : E \subset X, \dim E = n\}$. Our main result is that if $d_n(X)$ goes sufficiently slowly to ∞ , then X is HAPpy. The proof is rather tricky. Then it remains, given any sequence α_n which goes to ∞ , to construct a space X with symmetric basis such that $d_n(X) \leq \alpha_n$ but $d_n(X)$ goes to ∞ . We construct a corresponding Tsirelson/Schlumprecht type space and an Orlicz space.

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