The volume and the number of lattice points of the permutohedron are given by multivariate polynomials with remarkable combinatorial properties. We give several formulas for these polynomials. We extend our results to a more general class of polytopes that includes the permutohedron, the associahedron, the cyclohedron, the Pitman-Stanley polytope, various generalized associahedra related to wonderful compactifications of De Concini-Procesi, matroid polytopes, and graph zonotopes. We study the mixed Eulerian numbers, which are coeffiecients in our polynomials. They are equal to the mixed volumes of hypersimplices. Various specializations of these numbers give the usual Eulerian numbers, the Catalan numbers, the numbers of trees, the binomial coefficients, etc. Many results are extended to an arbitrary Weyl group. (Received August 15, 2005)

