Waterloo, Waterloo, N2L 3G1, Canada. Product Representations of Polynomials.
In this talk I will discuss the algorithmic problem of efficiently determining the existence of a linear dependence amongst a set of vectors in a finite dimensional vector space over $F_{q}$. To do so, a more general framework is introduced, where we look for integer factorizations of points in the value set of a polynomials.

For a polynomial $f \in Z[X]$ and positive integers $k$ and $N$, let $\rho_{k}(N ; f)$ denote the maximum size of a set $A \subset$ $\{1,2, \ldots, N\}$ such that no product of $k$ distinct elements of $A$ is in the value set of $f$.

Using a little algebraic geometry, the probabilistic method and some extremal combinatorics, we prove that for every polynomial $f$ of prime degree $d$, either $\rho_{k}(N ; f)$ is linear in $N$, or $|f|$ is the $d^{\text {th }}$ power of a monic linear polynomial and $\rho_{k}(N ; f) \sim c \pi(N)+O\left(N^{1-1 / 2 d}\right)$ and $c$ is completely determined. This generalizes earlier results of Erdős (1963), Erdős, Sós and A. Sárközy (1995), Györi and G. Sárközy (1997). We conclude with some open questions. (Received August 07, 2005)

