

Meeting: 999, Nashville, Tennessee, SS 14A, Special Session on Graph Theory and Matroid Theory

999-05-273 **Neil Robertson*** (robertso@math.ohio-state.edu), Department of Mathematics, Ohio State University, 231 West 18th Avenue, Columbus, OH 43210. *Some impediments to extending the four-color theorem.*

The four-color theorem for planar graphs continues to stimulate the development of chromatic graph theory. Most mathematicians would like to see the reliance on the computer in the proof reduced. A direct approach would involve developments in the theory of Kempe chains. Indirectly, the dual forms of the theorem involving vertex colorations or facial colorations generalize in different ways. These extensions should lead to new techniques, as they are longstanding open problems, and some of these may apply to the theorem itself. The Hadwiger conjecture, that graphs with chromatic number k contract onto complete graphs on k vertices dominates the vertex coloration side. Facial colorations led Tutte to his famous nowhere-zero 4-flow conjecture. Seymour made conjectures about edge-colorations extending the four-color theorem and close to conjectures about Vizing's theorem. In general, these open problems have been solved in special cases going well beyond the four-color theorem, but have impediments to carrying out the natural inductions in their proofs, many having stood the test of time. This talk will describe these impediments, which must be removed or shown to contain counterexamples to the associated conjectures if the theory is to progress significantly. (Received August 25, 2004)