Meeting: 999, Nashville, Tennessee, SS 4A, Special Session on Universal Algebra and Lattice Theory

999-06-108 Nikolaos Galatos* (ngalatos@math.vanderbilt.edu), Vanderbilt University, Dept. of Mathematics, 1326 Stevenson Center, Nashville, TN 37240. Non-associative residuated lattices.

Residuated lattices are ordered algebraic structures that constitute algebraic semantics for substructural logics extending the Full Lambek calculus. Examples of residuated lattices include lattice-ordered groups, Brouwerian algebras and MV-algebras. The underlying multiplication in a residuated lattice is assumed to be associative. We extend the structure theory and the algebraization for residuated lattices to their non-associative generalizations, residuated lattice-ordered groupoids with unit (rℓu-groupoids).

In this general setting we provide two logical systems, one formulated in Hilbert and one in Gentzen style, whose algebraic semantics are rℓu-groupoids and show, in an algebraic way, that the first enjoys the strong separation property and the second the cut elimination property. As corollaries we obtain the decidability of the equational theory of rℓu-groupoids and axiomatizations for all classes of subreducts of rℓu-groupoids. Both properties are consequences of a quasi-embedding construction for logical matrices, which also yields the embedding that establishes the finite embeddability property for integral residuated lattices. Finally, we discuss the importance of the parameterized local deduction theorem for substructural logics. (Received August 17, 2004)