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with the assistance of AMS staff

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Israel Moiseevich Gelfand
(1913–2009)

REMEMBRANCES OF ISRAEL MOISEEVICH GELFAND

Israel Moiseevich Gelfand passed away at the age of 96 on 5 October 2009 in Piscataway Township, New Jersey, in the U.S.

Gelfand signified a whole epoch in mathematics, characterized by a perception of this science as a unified body of knowledge where the traditional division into separate areas is determined only by this or that topic of study, but on a deep level all the branches are conceptually and systematically connected with one another. He always persistently encouraged such an attitude toward mathematics, and confirmed it in his own creative works. He left his mark in many areas of mathematics: Banach algebras, the spectral theory of operators, the theory of representations of Lie groups, integral geometry, the theory of generalized functions, linear topological spaces, differential equations, dynamical systems, the theory of hypergeometric functions, numerical methods, mathematical physics — this is far from a complete list of directions with which he was directly concerned. Moreover, to varying degrees he influenced the development of an even greater number of mathematical topics in the sphere of his interests. He possessed a subtle intuition, and his thinking process was richly associative. He could sense new and substantive problems and was able to find a suitable key, a "master key", that was adequate for the most diverse problems. All who worked with him at one time or another well know and recall these characteristics, but they can also be seen in his published works, where final refinements often cover up the intricate strategems of the probing thought processes.

Gelfand's role was significant in the construction of the bridge between modern theoretical physics and mathematics, a bridge which led to the emergence of new and unexpected directions in the development of fundamental investigations in this area. He always had esteem for applied research, in which he was able to use deep mathematical ideas and find new formulations of mathematical problems. His many years of scientific activity in the area of biology and medicine should especially be mentioned. There he also studied a diversity of topics: neurophysiology, classification of nucleotides, algorithmetization of medical practice, and so on.

Besides his creativity in pure research, an important side of his scientific life was his pedagogical and — on a broader plane — general educational activity. Here one has only to mention the famous "Gelfand seminar", which met in the Faculty of Mechanics and Mathematics at Moscow State University for almost half a century! Several generations of now well-known mathematicians passed through this seminar. After his move to the U.S., Gelfand led a similar seminar at Rutgers University. For decades he lectured brilliantly in required courses and optional courses at Moscow State University. These courses and seminars were always attended by many listeners and participants. Furthermore, he was able to attract talented young people and to captivate them by new and interesting problems, drawing them into his scientific orbit and acting as their patron over time. His seminar on biology, which he led in Moscow for about 30 years (and later continued

in a similar form at Rutgers University) was highly valued by biologists and medical specialists.

Gelfand was the author of more than 30 books on mathematics and biology, monographs and textbooks. Among them a special place is occupied by the series of monographs under the general title of "Generalized Functions", written together with other mathematicians. Several generations of mathematicians, both in Russia and abroad, have been brought up on these books.

Many remember the time when Gelfand was president of the Moscow Mathematical Society. He succeeded in reorganizing the operation of the Society so that its weekly meetings, which had previously been somewhat dull, became very lively and attracted a large audience.

Of the scientific projects undertaken by Gelfand one should mention the two mathematical journals he founded: *Trudy Moskovskogo matematicheskogo obshchestva* (translated as *Transactions of the Moscow Mathematical Society*) and *Funktsional′nyĭ analiz i ego prilozheniya* (translated as *Functional Analysis and its Applications*). In each of them he remained the editor-in-chief for many years.

Another important creation of Gelfand was the Mathematics Correspondence School, which he founded and supervised for many years. Thousands of Soviet schoolchildren from the most remote towns and villages were able to join in serious mathematics thanks to this project. Gelfand also created a similar program in the U.S.

Last but not least in importance is the Interdisciplinary Interfaculty Laboratory created at Moscow State University by Gelfand together with other scholars. This laboratory was connected in part with his biology seminar, and for a long time he headed its division on mathematical methods in biology.

This has been a brief account of landmarks in the scientific biography of Israel Moiseevich Gelfand — a remarkable human being and an outstanding mathematician.

The members of the editorial board of this journal express their deep condolences to the family, colleagues, and close friends of Israel Moiseevich.

Translated by H. H. McFADEN

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