

New GRE Mathematical Reasoning Test

by Alan Tucker

The General Test portion of the Graduate Record Examinations will be reorganized in fall 1997 to consist of the following five parts:

1. A revised version of the current quantitative test;
2. A revised verbal test;
3. A revised analytical reasoning test;
4. A new mathematical reasoning test; and
5. A new writing test.

These five General Test measures will be offered in two packages: (1) verbal, analytical, writing, and revised quantitative, and (2) verbal, analytical, writing, and mathematical reasoning.

Graduate departments will select the package of measures most appropriate for graduate study in their disciplines. Fall 1994 freshmen will be the first graduating class to take this new GRE exam. The new mathematical reasoning test will include problems of the type associated with 'reform calculus'.

The new GRE mathematical reasoning test is expected to replace the current quantitative test for students planning graduate study in engineering, the physical sciences, the mathematical sciences, computer science, economics and some areas of biology. Unlike most widely used tests for post-baccalaureate study, such as the LSAT, MCAT, GMAT, and current GRE exams, the new GRE mathematical reasoning test will draw on knowledge acquired in a first-year course in cal-

The image shows a computer screen with a window titled "GRE test". The window contains a coordinate system on the left with a grid and a curve labeled $y = f(x)$. To the right of the coordinate system are five separate curves. Below the curves, the text reads: "Given the graph of $y = f(x)$, produce the graph of $y = f'(2x) - 3$ by selecting one of the 5 curves and positioning it in the coordinate system." On the right side of the window, there are several control buttons: "Previous Item", "Next Item", "Back", and "Quit".

Sample new GRE test question

culus. This writer was a member of the advisory committee that recently set the specifications for this new mathematical reasoning test.

It is important for mathematics departments to be aware of this direct link between learning in calculus and performance on the new GRE mathematical reasoning test. The new test has drawn high praise from faculty in the sciences and engineering, and they expect the test to carry considerable weight in graduate admission decisions in their disciplines. Thus, mathematics departments should become familiar with the general types of mathematical prob-

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lem-solving on this new test. In particular, the new test will include problems of the sort found in calculus reform texts such as the Harvard Consortium text.

Readers might be interested to know that all the GRE testing program is likely to be fully computerized in 1997. That is, examinees will be required to take the test on computers. Currently, the GRE General Test is available on computer only at selected locations. The current computerized General Test is an adaptive test, as each of the new General Test measures will be (with the exception of writing). In an adaptive test, each question an examinee receives will depend to some extent on the answers to previous questions. Examinees receive fewer questions, because an adaptive test allows the test to estimate the examinee's approximate ability level and focus questions around this level. There is one serious drawback, in the minds of many faculty, to the adaptive testing scheme—examinees may not skip a question and return to it later (or change an answer after it has been confirmed).

The current GRE quantitative test assumes only two years of high school algebra, to accommodate humanities students who have had no mathematics since high school, and produces uniformly high scores among college seniors in mathematically oriented disciplines. These high scores limit the value of the test for admission decisions in mathematically oriented graduate programs. At the same time, non mathematical graduate programs are also concerned because they feel these high quantitative scores bias the GRE General Test to favor mathematically oriented disciplines. Such a bias may influence decisions about university-wide fellowships at the many institutions that use the GRE General Test in selecting fellowship recipients.

Following is the proposed content of the new text in terms of both reasoning skills tested and the mathematical background assumed. Many questions may involve more than one category of reasoning skill and also possibly more than one content category.

Mathematical Reasoning

Mathematical Modeling

- Recognize whether a mathematical model applies to a given situation (e.g., linear function)
- Develop a mathematical model (e.g., graph, equation, schematic, geometric figure) from a description of a system
- Recognize what assumptions underlie a particular mathematical model and how those assumptions can affect the validity of the model

(e.g., employing a continuous model for a discrete situation, neglecting aspects of a physical situation in developing a mathematical model of it)

- Reason with symbols rather than numbers (e.g., determine the influence of a parameter on a mathematical system)
- Express in everyday language relationships that are given mathematically

Logical Reasoning

- Formulate a conjecture or draw conclusions from a given set of results or observations
- Construct a valid argument to support or refute a conjecture or hypothesis
- Determine the validity of an argument or identify the flaw in an invalid argument

Patterns and Similarities

- Recognize patterns, trends, or symmetries; continue a pattern
- Investigate relationships between various mathematical ideas and processes (e.g., relationship between slope and rate of change)

Problem-solving Strategies

- Reduce a problem to a simpler case
- Determine when a certain procedure is appropriate for solving a problem (e.g., differentiation, integration, averaging)

Estimation and Approximation

- Determine when estimation techniques are appropriate and determine the degree of accuracy needed in an estimate
- Recognize the reasonableness of results (e.g., orders of magnitude, signs of physical quantities)

Mathematical Content

Arithmetic and Algebra

- Basic properties of integers (e.g., divisors and multiples, prime and composite numbers)
- Solving equations and inequalities; factoring
- Laws of exponents
- Ratio, proportion, and percent

Geometry and Analytic Geometry

- Properties of geometric figures (e.g., perimeter, area, volume, Pythagorean Theorem, similarity)
- Coordinate geometry (e.g., equations of lines, planes, and conic sections, distance between points, relationship between rectangular and polar coordinates)

Functions and Their Graphs

- Relationship between algebraic and graphic representations of functional relations (continuous or discrete)

- Properties of functions and their graphs (e.g., domain, range, continuity, symmetry, composites, inverses, intercepts, asymptotes)
- Elementary functions, their properties, and their graphs (e.g., polynomials, rational functions, trigonometric functions, exponential functions, logarithmic functions)

Calculus

- The limit concept, properties of limits, computation of limits
- Rates of change (e.g., speed, average rate of change)
- The derivative of a function and its relationship to the behavior of the function and to the slope of its graph
- The definite integral and its relationship to the area of a region in the plane
- Relationship between integration and differentiation
- Differentiation and integration of elementary functions (e.g., polynomial, trigonometric, exponential, and logarithmic functions)
- Standard applications of the differential and integral calculus (e.g., velocity and acceleration, exponential growth, work, compound interest, optimization problems)

Probability and Statistics

- Measures of central tendency (mean, median, mode) and dispersion (range, standard deviation, variance)
- Interpretation of tables, graphs, histograms, etc.

The new mathematical reasoning test is designed to test mathematical reasoning skills and not mathematical content knowledge, and so its calculus prerequisite will not result in questions directed at knowledge of calculus techniques. The types of questions to appear on this test were developed somewhat independently of the calculus reform movement but reflect the same concern for an understanding of concepts and their interpretation in graphical and applications settings.

For further information about the new GRE mathematical reasoning test, readers should

Click on an answer box and enter or edit your answer using the on-screen keyboard.

Figure I

Figure II

The area of the shaded region in Figure I is 6 times the area of the shaded region in Figure II.

What is c in terms of a ?

$c =$

Click on an answer box and enter or edit your answer using the on-screen keyboard.

$$\begin{aligned} x + Ay &= d \\ ax + y &= e \end{aligned}$$

$A, a, d,$ and e are nonzero constants.

If the system of equations has infinitely many solutions, what is c in terms of A ?

$c =$

Sample GRE test questions supplied by J. Levin of the Educational Testing Service and generated by ETS staff.

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Editor's Note

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