1995 Frank Nelson Cole Prize in Algebra

The Frank Nelson Cole Prize in Algebra is awarded every five years for a notable research memoir in algebra which has appeared in the previous five years. This prize and the Frank Nelson Cole Prize in Number Theory were established in honor of Professor Frank Nelson Cole, who served as secretary of the AMS from 1896 to 1920 and also served as editor-in-chief of the *Bulletin* for twenty-one years. The original fund was donated by Professor Cole out of monies presented to him on his retirement. The fund has been augmented by contributions from members of the Society, including a gift made in 1929 by Charles A. Cole, Professor Cole's son, which more than doubled the size of the fund. In recent years, the Cole Prizes have been augmented by awards from the Leroy P. Steele Fund; currently they amount to $4,000.

The Twenty-fifth Cole Prize is shared by DAVID HARBATER of the University of Pennsylvania and MICHEL RAYNAUD of Université de Paris-Sud, Orsay. The prize was presented at the Society's 101st Annual Meeting in San Francisco in January 1995. The prize is awarded by the AMS Council, acting through a selection committee consisting of Barry Mazur, Shigefumi Mori, and Jean-Pierre Serre (chair).

The text below includes the citations from the selection committee, the recipients' responses upon receiving the prize, and a brief biographical sketch of each recipient.

Citation

The Frank Nelson Cole Prize in Algebra for 1995 is awarded to Michel Raynaud and David Harbater for their solution of Abhyankar's conjecture. This work appeared in the papers *Revêtements de la droite affine en caractéristique* $p > 0$, Invent. Math. **116** (1994), 425–462 (Raynaud), and *Abhyankar's conjecture on Galois groups over curves*, Invent. Math. **117** (1994), 1–25 (Harbater).

As a first application of his reworking of algebraic geometry, A. Grothendieck constructed a theory of the fundamental group of an algebraic curve over a field of arbitrary characteristic. He could prove that when the curve had a usual fundamental group, the algebraic one was the profinite completion of the topological one (case of characteristic 0). In characteristic $p > 0$, the same statement is true if one lifts the curve to characteristic 0 and restricts attention to the prime-to-p part of the group. However, the theory is powerless for the p part of the fundamental group in characteristic p. Already in 1957, S. Abhyankar had seen that the situation was bound to be more complicated for the p part than for the usual (prime-to-p) part (Amer. J. Math. **79**, 825–856). For example, the affine line in characteristic p is not simply connected, because the Artin-Schreier coverings preclude this. Abhyankar made the following conjecture in his cited paper: For a finite group G, write $p(G)$ for the subgroup generated by all the p-Sylow subgroups of G. If X is a projective curve in characteristic $p > 0$, and if $x_0, \dots, x_t$, are points of $X (t > 0)$, then a necessary and sufficient condition that G occur as the Galois group of a finite covering, Y, of X—branched only at the points $x_0, \dots, x_t$—is that $G/p(G)$ have $2g + t$ generators. (Of course, g is the genus of X, and the generator condition is merely the statement that $G/p(G)$ appear as a quotient of the usual fundamental group of the open curve $X - \{x_0, \dots, x_t\}$.)

Michel Raynaud is among the best active specialists in algebraic geometry and its applications to number theory. A thread common to most of his work is its great generality that still provides the means to attack concrete problems very effectively. Thus, several of his results, notably those concerning finite group schemes of type $(p, p, \ldots, p)$, rigid analytic geometry, Neron models, and Picard functors, have become the tools of choice in algebraic geometry and arithmetic.

David Harbater has made pioneering contributions to formal algebraic geometry. A thread common to most of his work is the use of power series methods. He has made significant advances in such areas as approximation theory and formal geometric methods, which include his theory of mock coverings and patching methods.

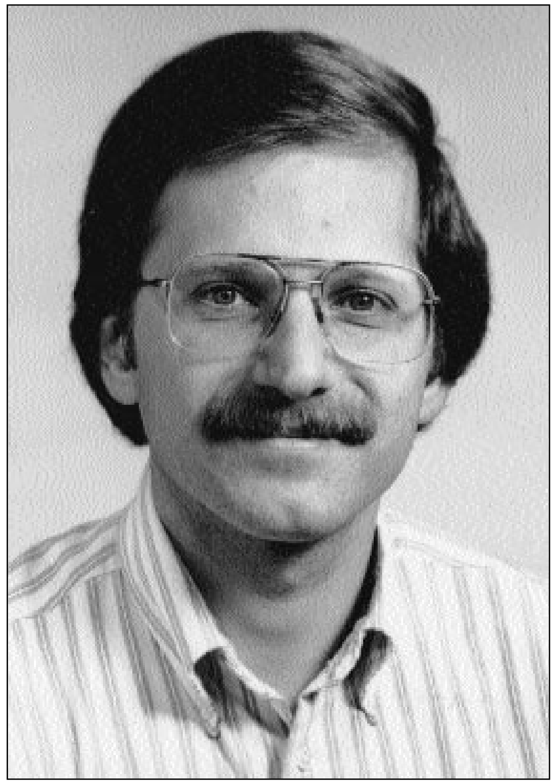
In the prize winning works, Raynaud solved the Abhyankar problem in the crucial case of the affine line (the projective line with a point deleted) by using rigid analytic methods (à la Tate), combined with a very interesting study of the action of the Galois group on the graph of components, in the case of bad reduction. Harbater then proves the full Abhyankar conjecture by building upon the solution of the conjecture for the case of the affine line and by using the powerful methods mentioned above that he has developed.

David Harbater

Response

I am very happy to accept this award and would like to thank the AMS and the selection committee. I also would like to express my thanks to my thesis advisor, Michael Artin, for having initially suggested to me the problem of studying fundamental groups in characteristic p. In addition, I would like to take this occasion to express my appreciation for the continuing support over the years that I have received at the Penn Mathematics Department from Steve Shatz and that I have received at home from Judy Axler.

The papers that have been cited prove a conjecture that was posed in 1957 by S. S. Abhyankar as an outgrowth of his work on resolution of singularities. In that work he considered varieties over fields of characteristic p as well as arithmetic varieties. By taking linear slices of surfaces that he wished to resolve, he found interesting examples of covers of curves in characteristic p and especially unramified covers of the affine line. Based on these examples, he conjectured which finite groups can arise as the Galois groups (i.e., deck transformation groups) of covers of a curve of genus g with r points removed over an algebraically closed field k of characteristic p. A few years later, Grothendieck showed that the conjecture was correct in the case of groups of order prime to p. Grothendieck's method did not generalize to arbitrary groups, however, because it involved comparison between coverings of curves over k and of curves over C (i.e., Riemann surfaces).



David Harbater

Little further progress was made until the 1980s, when Nori and Abhyankar each found examples of infinite classes of finite groups that, as conjectured, do occur over the affine line over k. In 1990, Serre proved the conjecture for solvable groups over the affine line, using a cohomological approach. The following year, I was able to obtain partial results toward Abhyankar's conjecture over general affine curves, using formal patching. Upon sending a copy of this manuscript to Serre, I learned from him that Raynaud had just obtained related results over the affine line using rigid patching and that Raynaud was hopeful that he could prove the full conjecture over the affine line by combining these results with an argument using semistable reduction. In 1992, Raynaud succeeded in doing just that. The year after, I was able to combine Raynaud's result with my formal patching methods in order to prove the conjecture over arbitrary affine curves. The key step involved going from the affine line to the once-punctured affine line, using a formal patching construction with a ramified closed fibre. From there, another formal patching construction yielded the general case.

Michel Raynaud

Both patching methods—formal and rigid—allow for the possibility of treating curves in characteristic p much as though they were Riemann surfaces. In particular, these approaches permit "cut-and-paste" constructions, which would not make sense if one worked only within the Zariski topology. In my case, the formal approach was one that I had previously used in the 1980s to realize all finite groups as Galois groups of branched covers of the line over p-adic and algebraically closed fields. Raynaud had previously shown that the formal and rigid approaches are in many situations essentially equivalent, in the sense of almost having a dictionary between them. Intuitively, however, they are quite different, and the intuition behind the rigid approach is probably more accessible. On the other hand, the formal approach enables one to draw on the edifice constructed by Grothendieck in EGA.

I am convinced that patching methods, in whichever guise, will permit much further progress to be made toward many of the open problems concerning Galois groups and fundamental groups in characteristic p and in arithmetic situations. And I am very pleased to be sharing this award with Michel Raynaud.

Biographical Sketch

David Harbater was born on December 19, 1952, in New York City. While attending Stuyvesant High School, he participated in summer mathematics programs sponsored by the National Science Foundation at the University of New Hampshire and at Ohio State University. He received degrees from Harvard University (A.B., 1974), Brandeis University (M.S., 1975), and the Massachusetts Institute of Technology (Ph.D., 1978). His thesis "Deformation theory and the fundamental group in algebraic geometry" was written under the direction of Michael Artin.

Since 1978 Professor Harbater has been on the faculty of the University of Pennsylvania, beginning as an assistant professor. He became an associate professor in 1983 and has been a professor since 1991. He has been a visitor to Harvard, MIT, and the Mathematical Sciences Research Institute for one-year periods, as well as visiting the University of Heidelberg and the University of Bordeaux for shorter stays.

Professor Harbater has held an AMS Postdoctoral Fellowship (1978–1979), an NSF Postdoctoral Fellowship (1982–1983), and a Sloan Fellowship (1984–1987). He delivered addresses at the International Symposium on the Inverse Galois Problem (Oxford, 1990), the Arbeitstagung (Bonn, 1993), and the International Congress of Mathematicians (Zürich, 1994). His mathematical interests lie at the intersection of algebraic geometry, number theory, and topology, with a focus on curves and their coverings.

Michel Raynaud

Response

It is a great honor and a great pleasure for me, as a foreign algebraist, to receive the Frank Nelson Cole Prize. I hope that the topologists will not be too much troubled by the fact that the affine line has a very large fundamental group. In fact, they knew already that nothing reasonable can be expected from this fancy characteristic, p.

I thank heartily the AMS and want to congratulate my corecipient and good friend, David Harbater.

Biographical Sketch

Michel Raynaud was born on June 16, 1938, in Riom, France, and worked on his thesis at the Centre National de la Recherche Scientifique under the supervision of A. Grothendieck (1962–1967). Since 1967 he has been a professor at the Université de Paris-Sud, Orsay. He received the Prize Ampère from the Académie des Sciences in 1987.

Raynaud's interests lie in algebraic geometry and its relationship with arithmetic. He has been guided by A. Grothendieck and J.P. Serre, and he admires and is still impressed by the way they think and do mathematics, even after thirty years. Grothendieck introduced him to the techniques of representability; he has been fond of group schemes and Picard functors for a long time. Later, invited by John Tate, he focused on rigid geometry. For many years he has concentrated on curves of p-adic discrete valuation rings. He has used them to prove that a curve of genus > 2 over C contains only finitely many torsion points of its Jacobian. The semistable reduction of curves and its combinatoric aspect have led to quite amusing problems. For instance, they give the key of the proof of Abhyankar's conjecture for the covers of the affine line in characteristic p.