A Partnership in Mathematics Education

School and University Faculty Working Together

Anne Roberts

Introduction

Have you ever wished that the mathematics program at your local elementary school could be improved—and wondered what it would take? If you've had a child in elementary school recently, chances are pretty good that you've at least considered this question. Even if you don't have such a connection to K–12 mathematics programs, articles in recent issues of AMS and MAA journals [3,4,6,8] and the publication of the National Council of Teachers of Mathematics, *Curriculum and Evaluation Standards* [5], have probably alerted you to the fact that reforms in teaching mathematics are being considered not only at the university level but in precollege education as well. Educational reform is certainly not new, but there is something a bit different and promising about this current reform in K–12 mathematics education. That is the fact that, unlike the "new math" of the 1960s, the changes in the precollege mathematics curriculum being considered today are growing out of a much needed dialogue among classroom teachers, mathematics educators, and mathematicians, both about the topics to be studied and about ways to teach mathematics.

Developing a plan for educational reform is one thing. Implementing the intended changes and determining whether they produce the desired effect is another. Cohen and Ball address this issue in their article, *Policy and practice: An overview* [2], which looks at the early stages of implementing the California Mathematics Curriculum Framework of 1985. At the elementary school level, broadening the primary focus of the mathematics curriculum on arithmetic skill and learning through memorization to develop mathematical thinking and understanding of concepts as well will require a major shift in emphasis for many teachers. Unless teachers themselves are given an opportunity to reconsider their own images of mathematics and to take part in the kinds of learning experiences they are asked to provide for their students, the reforms envisioned, whether those proposed in the *NCTM Standards* or elsewhere, will fail. One lesson learned from the "new math" movement is that professional development for teachers is a critical factor in reform.

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But what kind of professional development is necessary to help teachers make significant change in their mathematics teaching? Should mathematicians be involved in these efforts? What role can they play? This article describes one attempt to answer these questions, a project in mathematics education that involved a two-year partnership between the mathematics department at the University of Utah and Washington Elementary School, a K–6,

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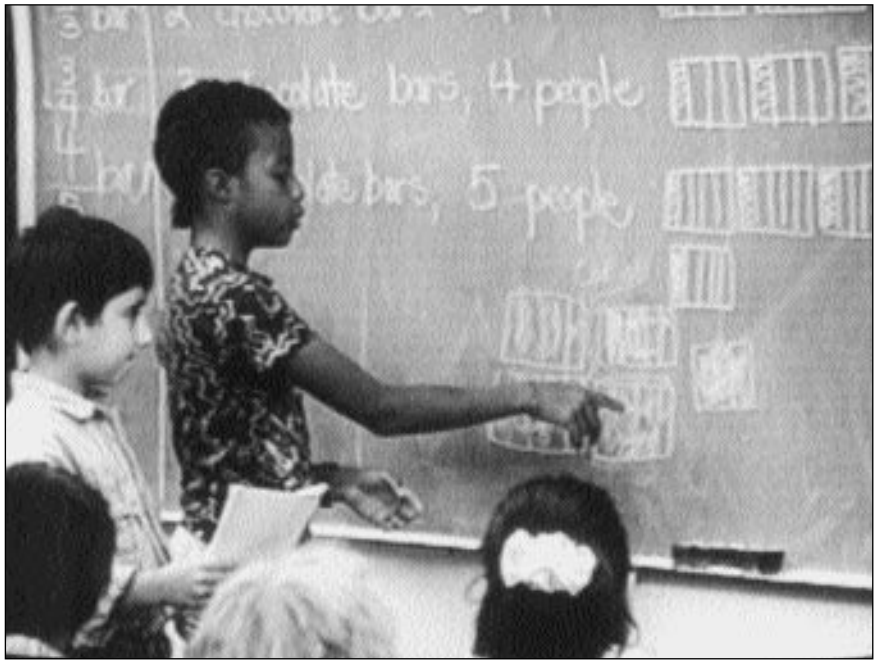
inner-city school in the Salt Lake City School District. The project, called Elementary Mathematics Through Teacher Partnerships (EMTTP), provided an opportunity for mathematicians, mathematics educators, and elementary school teachers to share their understandings about mathematics content and pedagogy. We are writing this article for several reasons. Members of the AMS are being called on to play a greater role in mathematics education efforts at the precollege level [1]. The work in the EMTTP project is an example of how mathematicians and school teachers can work together to improve the mathematics culture in an elementary school. We hope our experiences will provide useful information for those members of the AMS who are involved in or are contemplating similar activities. Finally, we believe that it is in partnerships that foster an exchange of ideas across the continuum of mathematics teaching that we see the hope of improving mathematics education at all levels.

The Project: How We Began, What We Hoped To Do, and Why

The EMTTP project, funded by the National Science Foundation, began in August 1992 and ended in June 1994. Herb Clemens of the mathematics department directed the project, and I led the school-year work with the teachers. We had each worked with the principal of Washington Elementary, Dr. Carol Lubomudrov, prior to the project by teaching classes of students at schools where Lubomudrov was the principal. For several years, the three of us had shared our thinking on how to help elementary school teachers, those now in education programs and those already in schools, become effective teachers of mathematics. During the year before the project began, Clemens, who was then teaching a class at Washington Elementary, worked together with the school faculty and Lubomudrov to develop the plan that became the EMTTP project.

What kind of considerations go into such a plan? What does each group hope to achieve or see as potential benefits? First, we need a little background on the school. Washington Elementary, as an inner-city, Chapter One school, had a large minority population, nineteen different languages being represented among the students, and a high student turnover rate during the school year. The school faculty was a very interactive group of teachers, used to dealing with the many social problems of their students. Although very flexible in handling nonacademic issues, the teachers had been encouraged to adopt a direct instruction, or didactic, style of teaching by a former principal. In mathematics they focused primarily on arithmetic presented as a series of rules for computation, and students were grouped in math classes by their ability in computation. When Lubomudrov became principal two years prior to the project, she initiated a dialogue with the faculty on different approaches to student learning; and the faculty, discouraged about their students' performance in mathematics, was willing to consider how this might take place in the context of their mathematics program. They were also willing as a group to take part in an extended professional development effort.

Rather than simply drilling students on material, one teacher began using "concrete objects and diagrams" to help them to understand concepts.



From our own work in elementary schools, we believed that improving a school mathematics program is not simply a question of a change in the topics taught or the way these topics are taught. Nor is new mathematics learning for teachers sufficient. While each of these changes may have some benefit and all are necessary to make a substantive impact on a school mathematics culture, there is another crucial consideration. Since it is teachers who bring about change in the classroom, teachers need to understand the rationale for change, take responsibility for determining the implications for their teaching practice, and, together with their colleagues, evaluate the effect of changes made. There are many examples which demonstrate that teachers' uncritical acceptance of reform ideas proposed by experts in content and/or pedagogy can lead to mathematics instruction that runs counter to the intent of the proposed reforms. Consulting with outside experts on content and pedagogy is important,

but ultimately how one incorporates this information into one's teaching practice is what will determine growth in teaching mathematics. It is this individual, reflective growth that, combined with a schoolwide sense of direction, seems essential for lasting improvement in the mathematics learning environment at a school. To support this kind of professional development, we believed a partnership between school and university faculty was required, one that recognizes the expertise each group brings and is based on the willingness of those involved to consider and learn from each other's perspective on mathematics and the process of teaching and learning mathematics. Mathematicians can play an important role in this process if they can work together with teachers, sharing their insight into the main questions or goals in mathematics.

The goal of the EMTTP project was to develop an extended conversation between school and university faculty. This conversation would be based on the mathematical experiences in elementary school classrooms and hopefully would lead the school faculty themselves to find ways to improve the mathematics program at Washington Elementary. Both groups stood to gain from this conversation: the teachers would have the opportunity to consider, in some depth, the mathematics they taught and what and how their students were learning; from actual experience in the elementary school classroom, university faculty could learn to deal with complex pedagogical situations and to determine ways to improve their courses in mathematics for teachers.

Project Structure: Setting the Stage for a Conversation

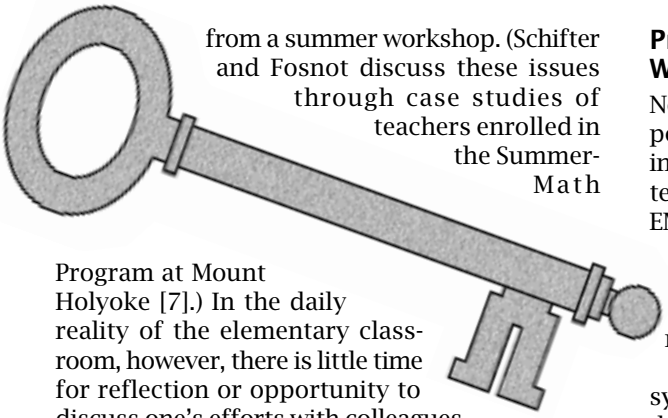
During each year of the EMTTP project, there was a three- to four-week summer workshop, which all but a few of the faculty at Washington Elementary attended, and a system of support for teachers during the school year, both in the classroom and through regular meetings at each grade level. In this section we describe each of these components and discuss how they fit into the goal of developing a partnership in mathematics education.

Summer Workshop. Each year the summer workshops began towards the end of July, just prior to the start of the school year. The workshops were held at Washington Elementary, and they focused on mathematics content and pedagogy. In the first two weeks of the workshop, teachers selected one theme per week that they would follow in morning math sessions. During the first summer, Clemens and I led sessions on: fractions, number systems, geometry, estimation. In the second summer another theme on math and science was added and taught by members of the mathematics department. The morning math sessions began with problems and/or questions presented on the theme that would challenge the teachers mathematically, could be explored at several levels of expertise, and would relate directly to classroom work at the K–6 level. The experience of working on these questions led to pedagogical issues as teachers and project staff shared ideas of how one learns mathematics, which models work best, and what the difficulties are in mastering certain concepts. Each day included teacher presentations on solutions to the problems and connections to the classroom. The main focus of this two-week period was to give teachers an opportunity to learn more about elementary mathematics in a way that encouraged them to ask questions about concepts, think about their understanding, justify their answers, and discuss their work with their colleagues. Then, using their own experience as learners in this context, they could consider their students' learning and the role of the teacher in that learning process.

One teacher noted, "I find myself saying, 'See what methods you can devise to solve this problem.'"

The second half of the workshop focused on demonstration lessons given by Deborah Ball, a mathematics educator from Michigan State University. For three days Ball taught one primary (grades 1–3) and one intermediate (grades 4–6) class, using students who attended Washington Elementary with the teachers as observers. Briefing sessions were held before and after the classes so that the teachers could discuss with Ball what worked, what didn't work, and what direction the next session might take. The goal in this activity was to set the stage for a collegial discussion of the lessons which would include an analysis of instructional decisions and their effectiveness, possible alternatives, and the children's thinking and evidence of understanding of concepts. This group discussion was to be a model for faculty exchange during the year on the process of evaluating lessons and modifying instruction based on mathematics goals and student response.

School-Year Support. Making significant change in teaching requires the time to think seriously about one's efforts and their impact in the classroom. Support and feedback during the school year are critical as teachers try out ideas



from a summer workshop. (Schifter and Fosnot discuss these issues through case studies of teachers enrolled in the Summer-Math Program at Mount Holyoke [7].) In the daily reality of the elementary classroom, however, there is little time for reflection or opportunity to discuss one's efforts with colleagues. Many teachers will attest to the fact that summer workshop plans are often placed on the top shelf and not used when the immediate concerns of the classroom arise. This is even more true in an inner-city school where the many nonacademic problems of one's students can be so demanding. Moreover, when teachers begin to change their mathematics practice, it is natural to encounter problems and concerns. Here opportunities to continue learning about mathematics and to discuss one's teaching with colleagues can be especially important in supporting a teacher's efforts and helping the teacher to evaluate the changes being made.

Most professional development programs consist of teachers drawn from a number of different schools, so that when the teachers return to their classes for the school year, there may be only one or two colleagues at their school with whom they can share their work and discuss their goals. This is in contrast to the EMTTP project which involved virtually an entire school faculty and thus provided a natural setting for the kind of faculty interaction necessary to improve a school mathematics program. To continue the conversation about mathematics begun in the summer, during the school year I met biweekly with the teachers at each grade level and worked with individual teachers in their classrooms. A graduate student in mathematics education, Lorel Preston, assisted me in this work. The regular forty-five minute, grade-level meetings gave teachers an opportunity to share their math lessons, to refine their thinking on both mathematics content and pedagogical issues, and to consider long-range goals for their mathematics curriculum. By working in the classrooms with the teachers, giving lessons or leading groups of students, and then debriefing afterwards, we were able to learn about an individual teacher's goals and concerns. By sharing the elementary teaching experience in this way, we were better able to offer support that recognized each teacher's potential for growth and that could relate the actual classroom work to the grade-level discussions.

Project Results: Developments at Washington and What We Are Learning

Now, after two years of project work, what is happening in the mathematics program at Washington Elementary? We will look first at what two teachers have to say about their work during the EMTTP project.

At the end of the first summer workshop, Diane (not her real name), a second/third-grade teacher, wrote the following in her journal.

"My understanding of the subject [number systems] has changed— there is more to understand than I realized! I have been caught up in the computing of a problem rather than reasoning it through—exactly the way I was taught. There is a much greater thinking process involved instead of just memorizing steps to come up with an answer....the thinking process which is most important has many times been overlooked or passed over quickly. Hopefully I can find a good balance of this in my own teaching."

During the summer, Diane had felt a sense of power when she began to understand why some of the arithmetic rules worked. Wanting this advantage for her students, she listened carefully in the grade-level meetings as her colleagues discussed ways to represent a number such as 345 with concrete objects or diagrams that would illustrate the relative amounts represented by the digits 3, 4, 5. In this way, the teachers could better examine the concepts of "carrying and borrowing" in addition and subtraction with their class. Gradually Diane, who was used to simply drilling the children in the steps of the subtraction algorithm, began to experiment with concrete objects and diagrams to discuss the reasons behind these steps with her students. A few weeks later when I visited Diane's class, she and her students explained their work to me. Considering the effect of this work, Diane was especially delighted to see how the students' *understanding* of borrowing improved their ability to subtract. In the past when she moved from problems such as (345 minus 157) to (305 minus 157), it had seemed like a whole different process to her students. Now "moving across the zero to borrow from the hundreds seemed like no big deal to the students."

Diane continued to use diagrams throughout the year to examine various concepts with her students. In her grade-level meetings, discussions focused on the concepts of multiplication, division, fractions, their properties, connections, and what understandings students can develop from different approaches to these concepts. The teachers were especially interested in looking at the area model for multiplication on graph paper and considered how this model

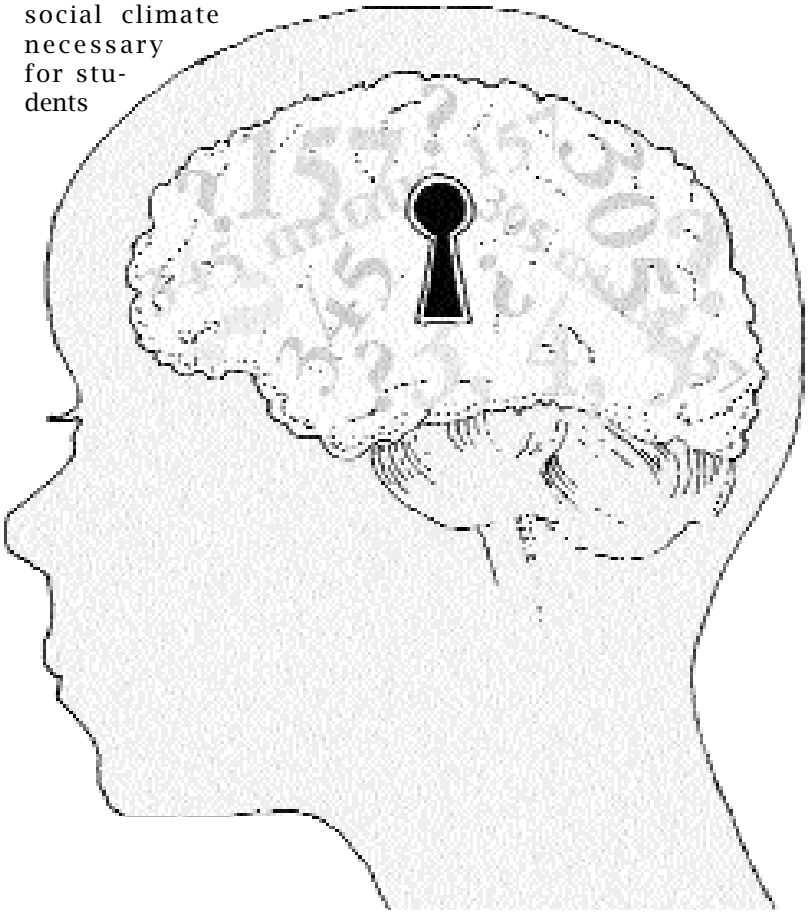
could be used to examine fractions and division as well. Towards the end of the first year when I visited Diane's class, I found the students having a lively discussion about products and factors using overhead graph paper. As Diane asked her students various questions and demonstrated their answers on the overhead by drawing rectangles, she brought in very naturally related questions about fractions and division. Afterward Diane said that these questions on connections between topics "just popped out" now after the grade-level discussions. Towards the end of the second year of the project, Diane wrote the following words about her work.

"I find that the 'textbook approach' is a thing of the past. Rather than just opening the book in order to teach the next lesson and then proceeding to do a page of problems, I now have many resources I draw from, including several books, ideas from other teachers, and perhaps more importantly, the students themselves....I have learned perhaps to think of mathematical concepts at a higher level as we participated in classwork ourselves....Because I have given greater thought to the reasoning of some things (for example, borrowing in subtraction), I feel I now understand some of these concepts much better myself....I find myself asking different kinds of questions...And, perhaps most importantly, I find math much more fun!"

Diane's growth in mathematical understanding seemed to be the impetus for many of the changes she made in her teaching practice. There were other motivations for change, however, and Bob, a fourth/fifth-grade teacher, had a different one. Bob had more mathematical training than Diane and had pursued mathematics through calculus. Dealing with the upper grades in elementary school, a main issue for Bob was how to achieve the technical skills that his students would need in middle school and yet at the same time develop students' conceptual understanding. Bob was intrigued by the style of teaching that Deborah Ball modelled in her demonstration classes and how this approach affected the students' learning. Ball generally began the class by posing a question that led to groups working on an investigation. About midway through, she asked the groups to report their results and consider the original question. Throughout the class the emphasis was on the students' thinking and their communication about the mathematics. Ball accepted all responses and expected the students themselves to examine these responses seriously as they shared their reasoning with their classmates. By neither praising nor correcting the responses given herself but instead focusing the discussion on key points with questions to the groups, Ball seemed to encourage the students to assume responsibility for determining the validity of the work presented. This interaction pleased Bob. He could see that the students were more involved in the class and that this approach would contribute to students becoming independent learners, a goal he valued. At the end of the second summer workshop, he described his thinking in the following way.

"I am again impressed (and a little bit confused) concerning Debra's [Deborah Ball's] low-key style of responding to the students. The behaviour modification techniques of praising correct response is not seen in her teaching (although I observed her saying good-bye to the students as they left the room and taking that opportunity to praise them)....is this style of interaction with students superior to Skinner's in that it enables students to take more responsibility for their learning and develop intrinsic gratification? I am leaning toward the latter as I watch Debra and the students. The learning—the content—seems to come more from the students than the teacher."

Bob had a new style of classroom work in mind as he began the second year with the topic of geometry. During the first few weeks he worked to establish the social climate necessary for students



to discuss each other's work. This was a very important step for the kind of discussions he aimed for, and he also felt this work in itself was valuable for the students' learning. About a month into the school year he asked me to visit his class, because he was very excited about the changes he saw in the students. As I watched, the students worked in groups on problems from Clemens book, *The graph paper geometry book*. Bob had arranged an overhead at the front of the room, and after a few minutes he invited students to tell what their group had found for the area of a particular triangle. At this point the students did not know a formula for the area of a triangle, so there were several different answers, and students were eager to show their solutions. Bob moderated the discussion that followed, while students compared their methods and used class comments to revise their answers. Bob was delighted to see the students more excited about their work. He reported that they had a better attitude towards mathematics as a discipline and that this new approach allowed more students to take part in the classroom work. Students were encouraged to take initiative in examining mathematical concepts, and Bob felt his class had a deeper understanding of area now than in years past. He continued this approach in his math class throughout the year and thought seriously about the effect of this approach on his students' learning, both in mathematics and in other areas. At the end of that year Bob described the benefits he saw in the following way.

To help implement new ideas, teachers met bi-weekly to refine their thinking and lesson plans as well as share goals and concerns.

"Students seem to regard themselves as more competent mathematically...Emphasis has shifted away from a right or wrong, black or white, 'all or nothing' perspective. Students have more opportunity to regard themselves as contributors to a growth process...to view their partial understanding as normal. Relative (or casual) to this shift in student attitude is a reformation of teacher presentation. Instead of 'This is the way you do it, class', I find myself saying, 'See what methods you can devise to solve this problem.' The latter instruction allows considerably more opportunity for challenge, comparison, esteem, fun, success, and original thinking."

The work of these two teachers in the EMTTP project illustrates an especially important aspect of what is happening at Washington Elementary. That is, the school faculty is actively engaged in thinking about the mathematics they teach and in analyzing the impact of their teaching on student learning. Lubomudrov, the principal during the project, summarizes this by saying that the teachers have moved from teaching as direct instruction to a reflective, critical practice. This process of reflection places additional demands on a teacher's own content knowledge. By examining a mathematical concept through examples of their own students' work, then sharing their thinking with colleagues and project staff, many teachers' mathematical knowledge has grown as well as their interest in the subject. Project staff observe this increased knowledge affecting the teachers' decisions about classroom work and goals, and there is a greater emphasis on mathematical thinking as teachers and students examine topics in more depth. The project evaluator, Deborah Schifter from the Educational Development Center in Boston, visited the project twice each year, and in her final report, she offered the following comments on the changes she observed in the faculty.

"In the last round of debriefing sessions (April 1994) [held after a demonstration lesson given by Deborah Ball], I was struck by the attention teachers gave to the mathematics of the lesson. They listened carefully to what the children said and considered what these comments reflected about their understanding. Furthermore, the feeling that Deborah was a model who demonstrated strategies they should judge seemed to have disappeared. Rather, Deborah's teaching gave them a context for reflecting on issues of learning and teaching mathematics. The fact that some teachers...chose to co-plan with Deborah, do the same lesson in their different classrooms, and then debrief together illustrates a dramatic shift in the teachers' relationship to Deborah, to each other, and to their work."

This sense of a community involved in the process of reflecting on teaching mathematics seems to us to be a critical first step in improving a school mathematics program. Can it be maintained over time, and will it eventually

translate into greater mathematical learning for students? These are questions we need to consider, but at this point we are cautiously optimistic. The teachers see many benefits from their project work, both for themselves and for their students. As a faculty they believe in the value of their grade-level meetings and during this first year after the project, they are continuing these meetings through the use of school funds to pay for release time. In addition, they are organizing three half-day math workshops led by project staff. The teachers report that student attitudes towards mathematics have improved, that the students are more willing to take risks and to tackle complex problems, and that they are better able to explain their thinking both in mathematics and in other areas. Overall, student SAT scores have remained stable even though the teachers have stopped "teaching to the test" and have emphasized conceptual understanding and mathematical dialogue.

Conclusions

So, what does it take to improve the mathematics program at an elementary school, and should mathematicians be involved in such efforts? Not all will agree on what constitutes an improvement. We have only to look at current efforts at reform in undergraduate mathematics programs to see an appropriate parallel. However, we believe that real improvement depends on a school faculty's ability and opportunities to develop a broad, cohesive vision for their school mathematics goals, one that is based on increased understanding of both mathematics content and pedagogy. The extended conversation about mathematics and teaching that was a central component of the EMTTP project provided one such opportunity. Mathematicians have both a lot to offer and a lot to gain from this kind of exchange.

The mathematical experiences of many school faculty are often limited in scope and the source of painful memories. New mathematical knowledge is important, but what may be even more critical to teachers is the opportunity to gain new insight into what doing mathematics can mean. Through the EMTTP project we have found that when mathematicians take part in classroom-based discussions, relating their own knowledge to the elementary classroom experience and to a teacher's pedagogical concerns, they can have a powerful influence on what teachers see as the process of doing mathematics and the general goals of the subject. This kind of collaboration, however, must develop over time and requires a sense of mutual respect and trust.

Mathematicians can learn a lot from school faculty about what it means to teach and to support mathematical growth. In the EMTTP project we've learned that teaching elementary school is harder than we'd imagined and that the issues elementary teachers face on a daily basis go far beyond issues of mathematical content. The issues of content are certainly central, but the classroom culture and student and teacher interaction play a much greater role in elementary schools where students are in the early stages of finding out what learning means. Along with our differences, however, there are issues that we have in common. For example, the balance between technical skill and understanding that the upper elementary grade teachers are concerned about is also of concern in the calculus reform efforts, and as teachers at all levels reconsider their mathematics curriculum, we must address the issue of assessment jointly. Through our work in this project we've learned new ways to approach our preservice mathematics classes, and we believe even more strongly in the need to provide preservice teachers with a program that is classroom-based and integrates both content and pedagogical knowledge in mathematics. For mathematicians, there is simply no substitute for regular work in schools to prepare one for working with preservice teachers. While not all members of mathematics departments will be involved in this work, it seems important that as a community we become more aware of the issues of teaching mathematics at all levels and of the need for on-going connections between undergraduate and precollege mathematics programs.

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