A Geometer's View of Space-Time

Poetry of the Universe:
A Mathematical Exploration of the Cosmos
Robert Osserman
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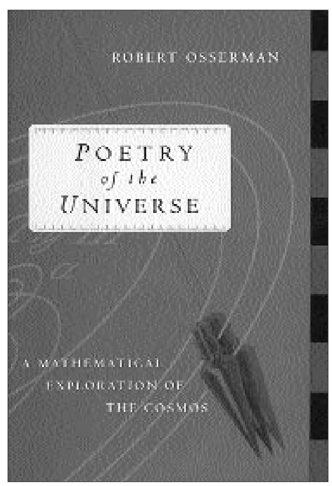
Mathematicians have always struggled with the difficulty of describing to nonmathematicians what they do. This difficulty is said to account for all sorts of ills, such as meager federal funding for mathematics research, low student interest in mathematics, and lack of appreciation for mathematics as a force in human culture. The easiest way to deal with the difficulty is to describe applications of mathematics. The hardest way is to address mathematics head-on, in all its abstractions.

In his book *Poetry of the Universe: A Mathematical Exploration of the Cosmos*, Robert Osserman deftly combines these two approaches in a serious yet lively work. Most people have wondered from time to time about what the universe is like outside of what we can observe from Earth, though fewer have wondered why euclidean space is flat and a sphere is curved. Osserman exploits readers' natural curiosity about the universe to take them on a remarkable journey through mathematical lands unfamiliar to most.

Along the way, Osserman weaves together diverse strands of cultural, scientific, and mathe-



matical developments, each of which makes an important contribution to the exposition. The danger in this tightly organized approach is that the reader is expected to remember too many different things in order to understand the overall picture. By some sleight of hand of which the reader remains unaware, Osserman devotes to each strand just the right emphasis and just the right amount of revisiting so that the strands illuminate the story that emerges, rather than obscuring it. A number of times, when reading about a particular topic in the book, I felt as if I were following a path through a densely packed forest that, while interesting, was unrelated to the main thrust of the book. It was delightful to emerge from the path to an open vista and see that the topic had strong and important connections to ones discussed earlier.

The writing style is concise and elegant, unadorned with flashy tricks that a writer less sensitive to the beauty of the subject might be tempted to throw in. Osserman does not beat readers over the head with technicalities, nor does he apologize for technical aspects when he needs to introduce them. He takes it on faith that readers will be just as enraptured by the subject as he is, rather than moralizing about why they should be enraptured. The simplicity of this approach works.

The book begins in antiquity, with Greek and Egyptian attempts to measure the size of the earth. At that time, it was commonly accepted that the earth is a ball, not a flat surface; it was only later, when much of the knowledge of the Egyptians and Greeks was lost after the fall of ancient civilizations, that the idea of a flat earth took hold in the West. Osserman points out that by the time of Columbus, the notion of a round earth was once again in favor. Columbus did not set out on his voyage assuming that he would reach the "edge" of the earth (though he and his comtemporaries did wonder if his ships would fall off the earth when they sailed around the "bottom").

Throughout the ages people wrestled with the problem of making accurate maps of the earth. Osserman describes Mercator's projection and Gauss's work on geodesy, and along the way describes why a completely accurate two-dimensional map of the earth is impossible. The notion of curvature is explained at first with down-to-earth pictures of what orchards of trees would look like on differently curved surfaces. These pictures are gradually replaced by more abstract ones of surfaces with positive, negative, and zero curvature.

In the early 1800s, Lobachevsky and Bolyai simultaneously introduced the intriguing notion of noneuclidean geometry, asking what would happen if one replaces Euclid's parallel postulate by different notions. (Actually, Osserman points out that Johann Lambert, a contemporary of Euler's, had explored these ideas earlier but convinced himself that they would lead to a contradiction and did not pursue the work further.) Osserman's ingenious explanations of the oddities of hyperbolic space would be enormously useful to a student who is exploring the subject for the first time.

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Once one admits the possibility that the universe might be a three-sphere, the problem of how to make a useful "map" arises. At this point Osserman draws on his earlier discussions of the analogous problem of making maps of the two-sphere of the earth. And it is at this point that one sees clearly how the strands of the book come together. There is nothing superfluous here; every point made is related to the central theme of exploring the shape of the universe. Given the richness of the subject matter and the historical twists and turns, Osserman must have been tempted by many interesting digressions, but he does not pursue them. Because the book carefully develops the important ideas while bypassing peripheral ones, it manages to be concise yet unhurried.

Taking up where Lobachevsky and Bolyai left off, Riemann further developed the ideas of noneuclidean geometry. He constructed a model in which the universe is seen as a three-sphere: the way to visualize it is as two separate three-balls in three-space, with their boundaries identified. Looking at this as an "egocentric map" of the universe, one imagines Earth at the center of one of the balls and its antipodal point at the center of the other. If one crosses at any point the boundary of the first ball, one enters the second. A geodesic path starting at Earth passes through the antipodal point in the other ball and closes up back at Earth again. Even though such constructions are commonplace in mathematics today, the power of Riemann's ideas remains awe inspiring. "[E]ven today—a century and a half later—it stretches our imaginations to the limit to encompass Riemann's vision," writes Osserman.

Osserman notes Dante's description of the universe in the *Divine Comedy*, which bears striking resemblance to Riemann's three-sphere. Dante pictured the universe as two balls, one called the "Primum Mobile", which has the earth at its center, and the other called the "Empyrean". The surface of the Empyrean is inhabited by angels, and the center is a light of blinding intensity. "[W]e are to think of the Empyrean as somehow both surrounding the visible universe and adjacent to it," Osserman writes. "If that is the case, then the universe according to Dante would coincide exactly with the universe according to Riemann; they would differ only in the labels."

There are infinitely many possibilities for the geometry of the universe, and in order to understand which ones might be correct, one must uncover clues hidden in astronomical observations. One of the central tools is Hubble's Law, which says that all stars and galaxies are moving away from Earth at a speed proportional to their distance from Earth. The notion that as we examine distant galaxies we are actually looking back in time is one that, for most readers who are not scientists, will take some getting used to. The emphasis Osserman devotes to this important point pays off later on, when he describes Einstein's work and adds time as the fourth dimension.

Having explained the model of the universe as a three-sphere, Osserman goes on to examine other manifolds that might be possible candidates for representing the shape of the universe. The three-sphere yields a model of the universe with constant positive curvature. By contrast, one could also consider a toroidal universe with a flat metric. In a mathematically precise yet nontechnical way, Osserman describes these objects and manages to convey not only their utility as models for the universe but also their innate fascination. Even if one cannot prove that the universe has this or that particular shape, there is great satisfaction to be had in contemplating the possibilities. In addition, these mind-expanding explorations illustrate the power of mathematics to explain subtle and important aspects of the physical world and to provide clues about where to look for new information.

The denouement of the book comes not on the last page, but in the seventh of the nine chapters. There Osserman discusses the 3-degree microwave background radiation that emanates from every point in the sky. This radiation is thought to be the oldest that can be detected, originating about 300,000 years after the Big Bang. This is as far back in time as human beings can see. As Osserman puts it, "A curtain suddenly drops, permanently blocking from view what lies beyond." This was foreshadowed in the prelude to the book, which includes a "picture" of this radiation, assembled from observations made by the Cosmic Background Explorer that was launched in 1989. Just as Mercator wrestled with the question of how best to draw a flat map of the earth, astronomers had to come up with a way of representing in a two-dimensional picture this background radiation. Essentially, the book is devoted to explaining just what that picture means.

Although it is aimed at those with little background in science and mathematics, mathematicians are sure to love this book. With no equations, few symbols, and no proofs, the book nevertheless is scrupulously faithful to the mathematics it describes. Those who work outside geometry and topology will gain new appreciation and intuition, and those who do work in these areas will value Osserman's insights into familiar problems. The book would be an excellent antidote for the mathematics student ensnared in thickets of equations and definitions. Detailed notes at the end of the book point to sources for further reading.

But will this book appeal to the general public, to those with no background in science or mathematics? It seems unlikely that someone with no previous interest in the subject would pick up this book and read it from start to finish; but then, that is true of just about any book, particularly nonfiction works. The mind-stretching visualizations required to really understand the ideas in the book would tax the patience of many readers, especially those accustomed to instant visual recognition so common in today's world of television and video games. The book is engaging, lively, and well paced, but (to its credit) not what might be called "entertaining". The book's overarching organization is a model of clarity, and it nimbly skirts technical jargon. However, some readers could be confused by small details, such as the interchangeable use of terms like "the parallel axiom" and "the parallel postulate", or "circumference" of a circle and "length" of a circle.

Such minor quibbles aside, this book offers many mathematical riches to readers of any background. The "poetry" referred to in the title is a genre that too few are familiar with. This mathematically honest and highly readable account explores realms of beauty that deserve more attention.

—Allyn Jackson