Eugene Paul Wigner 1902–1995

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Eugene Wigner died in Princeton, NJ, on January 1, 1995, at the age of 92. He had been one of the last survivors of the generation that witnessed the creation of quantum mechanics and participated in the exciting initial years of its development. He spent most of six active decades on the faculty of Princeton University. Although he was best known for his physical and mathematical analyses of symmetry in quantum mechanics, he also made important contributions to solid-state physics, physical chemistry, nuclear engineering, and epistemology. In his later years, he found himself in the unusual position of being highly esteemed by physicists, mathematicians, chemists, engineers, and philosophers.

Eugene Wigner was born Jenö Pál Wigner in Budapest, Hungary, on November 17, 1902. Since he was a somewhat sickly child, his parents arranged for his early education to occur at home. However, later on he spent four years at the famous Lutheran gymnasium (high school) of Budapest, where he had the good fortune to have as friend and classmate (one class behind him) Jancsi (=Johann=John) von Neumann. Wigner was attracted by mathematics and physics, but, following his father's wish that he study something that could be useful in the leather tannery where his father was a foreman, Wigner got a degree in chemical engineering from the Technische Hochschule in Berlin. His thesis (1925), written under the supervision of Michael Polanyi, was on the theory of chemical reactions. Wigner's acumen so impressed Polanyi that he recommended him for his first position as a physicist, assistant to the physicist Richard Becker.

During the next decade and a half Wigner continued his study of the theory of chemical reactions but used the then new quantum mechanics. He did related work with Victor Weisskopf on the theory of line breadth in atomic spectra as well as a study of nuclear reaction rates with Gregory Breit. However, the main focus of his effort was in the application of group theory to the study of the symmetry properties of stationary states of atoms, molecules, atomic nuclei, and crystals.

It was also during this period that Wigner made a transition from Germany to the United States. From 1930 to 1933 Wigner and von Neumann had a common arrangement: they spent one term each year at their jobs in Berlin and one at Princeton University. In the spring of 1933 the National Socialists came to power in Germany, and the Berlin positions of von Neumann and Wigner vanished. Von Neumann then joined the faculty of the new Institute for Advanced Study. Wigner spent three years full-time in Princeton and then went to Wisconsin for two years. In the fall of 1938 he was back in Princeton in an endowed professorship, just in time to hear the news of the discovery of nuclear fission, a phenomenon whose consequences dominated the next decade of his life.

Wigner and his friend Leo Szilard foresaw as clearly as anyone the disastrous consequences of the Third Reich's acquiring nuclear weapons before the Allies. They persuaded Albert Einstein to write a letter alerting President Roosevelt. The result was the Manhattan Project, a large-scale effort to separate U^{235} from U^{238} in uranium ore and to create nuclear reactors to pro-

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duce plutonium as well as to design bombs which used these products as explosives. Wigner was heavily involved in design studies for the nuclear reactors. Most of the work took place in the mathematics department of the University of Chicago, code-named the Metallurgical Laboratory. It was here that Wigner acquired his reputation as a formidable nuts-and-bolts engineer. When his design work for the plutonium production reactors was done, Wigner turned to the design of power reactors. This continued after the war; he spent 1946–47 as director of research at Oak Ridge on leave from Princeton. Wigner and his coworker Alvin Weinberg collected their knowledge and experience in the definitive treatise, *The Physical Theory of Neutron Chain Reactors* (1958).

He returned to academic life in 1947 but, over the next three decades, often served as a consultant to the federal government in nuclear matters. Both his scientific achievements and his service were richly recognized with prizes and awards, of which only a few will be mentioned: Medal for Merit, 1946; Fermi Award, 1958; Max Planck Medal, 1961; Nobel Prize for Physics, 1963; National Medal of Science, 1969. After his retirement in 1971, he remained active until the late 1980s.

In the remainder of this obituary I have singled out for discussion a few points from Wigner's work on the application of group theory to the study of symmetry in quantum mechanics. More details and more complete coverage are to be found in Volume I of Wigner's *Collected Works* in the magistral annotations of Brian Judd, Part II "Applied Group Theory 1926–1935", and George W. Mackey, Part III "Mathematical Papers".

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Wigner began his study of symmetry in quantum mechanics with the problem of classifying the behavior of eigenfunctions of the Schrödinger Hamiltonians for atoms under permutations of the electrons. Inspired by a paper of Heisenberg on helium (two electrons), he first treated the case of three electrons, without recourse to the representation theory of the permutation group. However, he found the case of $n \geq 4$ electrons too complicated to do by hand. On the advice of von Neumann, he studied the pre-World War I papers of Frobenius, Schur, and Burnside on the representation theory of finite groups, as well as the later papers of Weyl and of Schur on continuous groups. The latter enabled him to enlarge his study to the consideration of the action on eigenfunctions, of rotations R of the coordinates of n electrons $\vec{x}_1, \ldots, \vec{x}_n$:

$$\vec{x}_1, \ldots \vec{x}_n \to R\vec{x}_1, \ldots R\vec{x}_n.$$

He recognized that if the Hamiltonian commutes with the action on wave functions of permutations of coordinates or with the action on wave functions of rotations of coordinates, then the linear subspace spanned by the eigenfunctions of a fixed eigenvalue is left invariant by these actions and, in the subspace, yields a unitary representation of the permutation and the rotation group. Such a representation is a direct sum of irreducibles, so the dimension of the linear space spanned by the eigenfunctions must be a sum with possible multiplicities of the dimensions of the irreducible representations of the groups. This is the elementary group theoretical explanation for the ubiquitous appearance in atomic physics of degenerate multiplets of multiplicity $2j+1$ where j is a positive integer or half-odd integer. Later on, in the context of nuclear physics, this argument led to a theory of super-multiplets in which the group $SU(2)$ is replaced by the group $SU(4)$.

Up to this point, Wigner had treated the unphysical case of spinless electrons. However, in the very same issue of the *Zeitschrift für Physik* in which he had published these considerations, there appeared Pauli's paper on nonrelativistic electrons with spin 1/2. Within a few months von Neumann and Wigner published the first of three papers generalizing everything that Wigner had done to the case of spin 1/2 Pauli electrons. These papers were not easy to read, and it seems plausible that they, together with Hermann

Weyl's classic book *Gruppentheorie und Quantenmechanik* (1928) were the origin of the use by physicists of the phrase "die Gruppenpest" to describe this approach to spectroscopy. (The German word is often translated "group pest", but the alternative "group plague" is probably better, if you take into account some of the passionate animadversions on group theory by physicists in those days.) In any case, it is instructive to compare Weyl's book with Wigner's *Gruppentheorie und Ihre Anwendung auf die Quantenmechanik der Atomspektren* (1931).

Both books have introductory chapters on linear transformations, groups, and quantum mechanics. Here Weyl puts more emphasis on vector spaces; Wigner on calculations with matrices. Wigner confines his attention to the permutation group and rotation group or its covering group $SU(2)$. However, he goes into much more detail on the representation theory of these groups. For example, he derives the Wigner Eckart formula for the matrix elements of tensor operators. This permits him to derive the intensity relations for spectral lines that follow from rotation invariance. He also gives a quantitative and general analysis of the splitting of spectral lines in the presence of external symmetry-breaking interactions. Weyl, on the other hand, discusses the Lorentz group, its covering group $SL(2, C)$, and the relation of their finite-dimensional representations to quantum field theory. Physicists interested in spectroscopy naturally preferred Wigner's book to Weyl's, but, of late, there has also been mathematical interest in the kind of detailed formulae for the Clebsch-Gordon coefficients that Wigner's book contains.

The work of Wigner best known among mathematicians is undoubtedly his construction of a class of irreducible unitary representations of the inhomogeneous Lorentz group. This group is not compact, and all its irreducible unitary representations except the trivial one are infinite dimensional. The representation theory of such groups was still unknown territory when Wigner published his fundamental paper in 1938. Of course, later, as a result of the work of Gelfand, Naimark, Bargmann, and others on such groups as $SL(2, C)$ and $SL(2, R)$, this theory became highly developed. Wigner limited his considerations to those irreducible representations in which the spectrum of the representation of the translation subgroup lies in or on the future cone:

$$(k^0)^2 - (k^1)^2 - (k^2)^2 - (k^3)^2 \geq 0.$$

The irreducibles turned out to be characterized by the squared mass (= the left-hand side of the inequality) and the representation of the so-called little group, the group of transformations

leaving a vector of mass$^2 = m^2$ fixed. When $m^2 > 0$, the little group is isomorphic to $SO(3)$ or $SU(2)$ and so is determined by a positive integer or half-odd integer, the *spin*. For $m^2 = 0$, the little group is isomorphic to the euclidean group of a two-dimensional plane or to the two-sheeted covering of such a group; the physically interesting irreducible representations are determined by a *helicity* which is an integer or half-odd integer.

Wigner came to the problem of the determination of the unitary ray representations of the inhomogeneous Lorentz group by adopting a space-time point of view in a discussion of symmetry in quantum mechanics. By an analogue of the argument he had presented in his book for the case of symmetry in space at a fixed time, he showed that a quantum mechanical theory invariant under inhomogeneous Lorentz transformations has an associated unitary ray representation of the inhomogeneous Lorentz group. It is a remarkable fact that the law of evolution of states in the most general quantum mechanical theory can be characterized by a measure class and multiplicity function on the masses spins and helicities.

In his later years Wigner devoted most of his scientific effort to sharpening what he saw as the paradoxes in the standard interpretations of the quantum theory of measurement. He became convinced that an essential extension of physical theory to include consciousness was necessary.