On Attracting and Retaining Mathematics Majors—Don't Cancel the Human Factor

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Potsdam College of the State University of New York, a traditional four-year liberal arts college with an enrollment of 3,500 and a mathematics faculty of thirteen, has for the past several years granted more B.A.s in mathematics than all but a few institutions in the U.S. While nationally approximately 1 percent of the graduating seniors are mathematics majors, at Potsdam approximately 10 percent are. The natural questions are: How come? and Can—and should—this be duplicated elsewhere? First, a little about the program.

In our liberal arts tradition, the requirement for the mathematics major is minimal: 33 semester hours of mathematics, consisting of 12 semester hours in calculus, and seven one-semester courses beyond calculus, these being: set theory and logic, linear algebra, modern algebra, advanced calculus, problem seminar, and two electives. Given this requirement, many students choose to "double major" in mathematics and x, as x ranges through all majors on campus. Roughly half of the students are seeking teacher certification. It should be noted that all students, including the certification students, must satisfy the same requirements; there are no special tracks. Almost all of the students at Potsdam are graduates of New York high schools.

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The mean high school average of incoming students has stabilized in the past few years at about 85, and the mean SAT at about 980. For mathematics majors, the means are a little higher (of course?), approximately 87 and 1040. It is noteworthy that these students are high-achievers in college. In 1993–1994, sixty-four mathematics majors were on the President's List with a GPA of 3.5 or higher on a scale of 4. Among the graduates in 1994, seven math majors graduated Summa Cum Laude, fifteen Magma Cum Laude, and sixteen Cum Laude. The graduates who do not take teaching jobs take jobs in many facets of industry. Although these students seem to fare well in entry-level positions, most claim to have chosen to study mathematics because they like it rather than as a career-planning move. Roughly 15 percent continue into graduate school in several different fields, including law, medicine, business, computer science, as well as mathematics.

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All of the above says something about the school and the program in mathematics but does not answer the questions this article started with. The reader who expects some brilliant and innovative answers will probably be disappointed by the proposed answers. These are summed up in a Departmental Position Paper which includes the following: "The major program in mathematics is based on the premise that the study of pure mathematics can be undertaken successfully by a large number of students if they are provided with a supportive environment including: careful and considerate teaching by a well-trained and dedicated faculty, continual encouragement, successful (student) role models, enough success to develop self-esteem, enough time to develop intellectually, recognition of their achievement, and the belief that the study is a worthwhile endeavor. We are dedicated to providing this supportive environment." Providing this environment means making working with students not only the primary activity of the faculty but, as the Position Paper states, "Indeed, we consider for any activity how it will affect our ability to work with our students, and generally eschew those that have a significant negative impact on this activity."

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Those readers who expect a discussion of "The Potsdam Teaching Method" are also in for a disappointment. The thirteen faculty members use a diverse collection of methods. But given the diversity, there are some things that are shared. All have put the welfare of their students first and have subjugated personal academic achievement to concentrate on having their students achieve. All have put developing the students' abilities above filling the students' minds; all have agreed that it is far more important to consider how much information the students can deal with than how much the professor can "cover". This seems to be the appropriate place to take up the issue of "standards".

It is natural to ask, Have they simply lowered standards to keep more students? The answer is yes and no. If by "lowering standards" we mean dealing with less material in order to have the students actually be able to do something beyond merely repeating what they've been told, the answer is usually yes. If by "lowering standards" we mean accepting students at the level we find them and having them grow from that point, the answer is almost always yes. If by "lowering standards" we mean working with students for whom studying mathematics is a challenge and waiting for those students to develop rather than dealing only with the naturally precocious students, the answer is definitely yes. If, however, lowering standards means replacing proof with example, replacing understanding with rote memorization, replacing analysis with algorithm, the answer is no. If by "lowering standards" we mean sending students into the world ill prepared to compete, the answer is no. While it is true that some students have been denied admission to some graduate schools based on the amount of information the students don't have, that is rare, and in those cases such students have gone to other graduate schools and done well. One student in particular, when he didn't recognize some "well-known theorems" was told by an interviewer that he had half an education and had no chance to get a degree at XYZ University. The student was accepted into a more prestigious program, graduated with honors, and joined our profession. The change from undergraduate mathematics to graduate mathematics is, for many, a real shock, especially in the amount of information covered and the amount of information their classmates seem to possess. One student in particular said he was very intimidated and almost quit, but noticed that when it came time to do some problems, the skills he had developed as an undergraduate were much more important than a volume of information. He also said that learning from a lecture was very difficult, but given that he had learned to read mathematics texts, he could, and did, learn in spite of his teacher.

The proper environment for learning is critical and includes much more than classrooms, libraries, and laboratories. It also requires a sense of belonging. We like our students to regard themselves as part of the department. Some students are hired to work in a tutoring lab. There is a large and active Pi Mu Epsilon chapter, which focuses on inclusion of students based on eligibility, not their exclusion. The highly accelerated students who are working for their B.A. and M.A. degrees simultaneously represent role models of outstanding achievement and, rather than generating envy, generate enthusiasm. Bulletin board displays of student achievement and honors serve to motivate younger stu-

dents to high achievement. More than once students have been heard to say, "Gee, look at that. My name will be up there some day." The department keeps in touch with former students, and they often visit and tell the current students about the value of their education. Our students seem to draw inspiration from the visits and colloquium talks given by former students.

As far as facilities go, ours are adequate but not lavish. Very little "hi-tech" teaching goes on. Mostly it is basic paper and pencil, blackboard and chalk work. Indeed the facilities look like facilities at any college. Private offices are provided and, given the amount of individualized student contact, are essential. Office doors are almost always open, and most often there will be a student or two in the office with the professor. We have kept the enrollment in classes moderate: forty in the lower-division courses, twenty-five to thirty in the upper division, and fifteen in the Problem Seminar. We have resisted going to large lecture sections. We believe that the single most important thing that keeps students encouraged and persisting in mathematics courses is not a career objective nor a technological experience, but careful teaching, especially at the freshman and sophomore level. All members of the faculty teach at all levels, and there are no teaching assistants. Students typically take Calculus I and II as freshmen. There is no special service course for nonmajors; indeed the only special calculus course is an honors course open to all qualified students independent of intended major. This practice reemphasizes our contention that the study of mathematics should be available to a large set of students and not to the select few. In the sophomore year, prospective majors take multivariate calculus, set theory and logic, and perhaps linear algebra. In the latter two courses, special emphasis is placed on having the student learn to create and write correct proofs of mathematical propositions. This emphasis continues throughout the major program.

If there is anything unique about the curriculum at Potsdam, other than being rather old-fashioned, it would be the careful attention paid to helping students make the transformation from the mathematics of "find" to the mathematics of "prove". This change is not terribly difficult for the "natural mathematicians", a set that probably includes all of the persons reading this. However, this can be quite difficult for some students, who without significant help would never make the transformation successfully, but who with some careful teaching can make the change and become good mathematics students. Not all students reach the level we might hope, but it is our belief that the traditional education in mathematics, even if limited in its information base, is the best foundation for a whole array of careers; and the success of our students seems to validate this belief.

Following the question about standards, the second most frequently asked question about our program is, How about research? The position of the department is that mathematical research as usually perceived by mathematicians is an activity complementary to undergraduate teaching. We believe that it is complementary in the sense that it is separate and distinct and that the very nature of the activities means that the single-minded attention, time, and energy needed to perform one well will necessarily detract from the other. At Potsdam we have made working with students our absolutely most important activity. Eleven members of the faculty have Ph.D.s, and all have engaged in significant research. We do try to provide, as best we can, favorable conditions for those faculty members who have research projects going, knowing full well that we cannot hope to duplicate the conditions at a major research university. We believe that there is a whole array of intellectually respectable activities that can serve as alternatives to the pursuit of publishable research. Given the standard refrain that research and teaching go hand in hand and given the usual criteria for reappointment, promotion, and tenure, the department's position would not weather too well in some institutions. In fact there has been some tension from time to time at Potsdam about this position vis-à-vis the position held by the administration of the institution. Having been on the faculty of a "regular" mathematics department, where we considered ourselves roughly half-time teacher–half-time researcher and where performance was judged primarily on research output, I am convinced that a program like ours cannot succeed in such a setting.

A discussion of the mathematics program at Potsdam is not complete without mentioning the man primarily responsible for it. As is the case with most innovative ideas, the program is not the work of a committee but the fruition of one man's dream. Dr. Clarence Stephens chaired and guided the department with a firm but gentle hand for twenty years. His concept of teaching permeates department philosophy and activities. Some of his ideas are summed up in what I'll call Clarence Stephens's aphorisms: Believe in your students—everyone CAN do mathematics, philosophy, art, literature, etc.… Know your students—their names, what they know, their hopes and fears… Don't say this is easy, or this is trivial… Go fast slowly (most teachers go slow fast)… You cannot push students from the bottom; you must raise them up from above… It is very difficult to learn how to solve problems by watching someone else do it or by

reading a book; the best method to learn to solve problems is by solving a lot of problems yourself… High standards do not mean having unrealistic expectations so students feel that they have failed… Teach students how to learn, reason, and think… Write tests carefully—know what your average student can do and what your best student can do. Give a balanced test…build students' confidence by giving them problems they can do… And most of all—teach the students you have, not the ones you wish you had.

What then is necessary to have a program like the one at Potsdam? (1) A caring and dedicated faculty, chosen for their commitment to teaching and sensitivity to student needs, who see their personal achievement through the achievement of their students. (2) A commitment to the principle that the study of pure mathematics is worthwhile in its own right and that it can be done by many people, not just the precocious few. (3) The belief that the true measure of a student is how well the student fares in post-college experiences and that the true measure of a program is how well it prepares students for these experiences. (4) The freedom for a department to define its own mission within a college mission and to have its members' performance judged by how well they carry out that mission.

As our program has received some publicity, several persons have visited our department to see first-hand what is going on. Two people who have visited and studied our program and who have written about their experiences merit special mention: Dr. Pat Rogers [2] of York University in Toronto, who became interested when she found that a majority of our students are women, and Dr. Dilip Datta [1] of the University of Rhode Island. Each has a special perspective on the program. I hope interested persons will contact them.

Some mathematicians seem to take perverse pleasure in the ignorance the general public has about the discipline. However, we suffer from this ignorance, for example, in financial support and more generally in the wide misuse of statistics that would not be possible in a more mathematically literate society. There seems to be general agreement that there is a need to encourage more young people to undertake the study of mathematics. Recently, in part to address this need, there has been a great deal of concern expressed about curriculum, technology, and career opportunities. Certainly, attention to the curriculum is important. Certainly, attention to the proper role of technology is important. And certainly, attention to career opportunities is important. However, none of these begins to approach in importance the attention we give the people we teach and the environment in which they learn.

References

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[3] *Transforming mathematics pedagogy*, On Teaching and Learning **4** (1992), 78–98.

[4] *Student sensitive teaching at the tertiary level: A case study*, Psych. of Math. Ed. **2** (1998), 536–543.