Nominations for President-elect

Nomination for Frederick W. Gehring

Clifford J. Earle and Franklin P. Peterson

It is an honor and a great pleasure to place Frederick W. Gehring's name in nomination for the presidency of the American Mathe-matical Society.

Fred is very well qualified for this important post. He has done ground-breaking mathematical research, some of which we shall describe in the following paragraphs, and has been an inspiring mentor of numerous graduate students and young Ph.D.s. He has been an invited speaker at three International Congresses of Mathematicians, a sign of widespread respect both for his research and for his lecturing skills. Fred's good judgment is also widely respected, and his advice and leadership are often in demand. For example, he has served three terms as chair of the Mathematics Department of the University of Michigan in Ann Arbor and has served on fifteen external visiting committees for other mathematics departments. His past service to the Society includes ten years on the Board of Trustees, two of them as its chair.

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Fred received his Ph.D. from the University of Cambridge in 1952, under the supervision of J. C. Burkill. His earliest papers deal with the theory of measure and integration. His first paper about complex analytic functions appeared in 1958. Since then, complex analysis has been one of Fred's major interests, and his papers in that subject make him arguably the premier geometric function theorist of his generation. However, even more of Fred's impressive stock of creative energy has gone into his pioneering work on the theory of quasiconformal mappings in higher-dimensional Euclidean spaces.

Let $\overline{\mathsf{R}^n} = \mathsf{R}^n \cup \{\infty\}$ be the one-point compactification of R^n, $n \geq 2$, and let V be a subregion of $\overline{\mathsf{R}^n}$. By definition, a homeomorphism f of V into $\overline{\mathsf{R}^n}$ is a *quasiconformal map* (a *qc map* for short) if there is a finite number M such that

$$\begin{aligned} H_f(x) &= \limsup_{r\to 0^+} \frac{\max\{|f(y)-f(x)| : |y-x| = r\}}{\min\{|f(y)-f(x)| : |y-x| = r\}} \\ &\leq M \quad \text{for all } x \in V \setminus \{\infty, f^{-1}(\infty)\}. \end{aligned}$$

The qc map f is K-*quasiconformal* (K-*qc*) if and only if $H_f \leq K$ almost everywhere in $V \setminus \{\infty, f^{-1}(\infty)\}$. (Warning: When $n > 2$, there are several different definitions of K-quasiconformality. This is not the most common one, but it gives the usual set of 1-qc maps.)

Complex analysis provides a powerful tool for the study of qc maps in two dimensions because the orientation-preserving 1-qc maps from a subregion V of C ($= \mathsf{R}^2$) into C are precisely the injective complex analytic functions on V. The theory in higher dimensions is harder and requires tools from geometric measure theory and partial differential equations. Two fundamental papers by Fred Gehring in 1961 and 1962 developed a comprehensive theory of qc maps in dimension three and laid the foundation for the theory in higher dimensions. The 1961 paper studies the conformal capacity of ring domains in R^3. Gehring uses spherical symmetrization and the then-new co-area formula of H. Federer and L. C. Young to determine certain extremal ring domains. The second paper uses these extremal rings to establish the properties of qc maps in R^3.

Two theorems in this 1962 paper deserve special attention. A Möbius transformation in $\overline{\mathsf{R}^n}$ is a member of the group of homeomorphisms of $\overline{\mathsf{R}^n}$ generated by the translations $x \mapsto x + a$, $a \in \mathsf{R}^n$, and the inversion $x \mapsto x/|x|^2$. Every Möbius transformation is a 1-qc map. In a remarkable generalization of a classical theorem of Liouville, Gehring proved that if $V \subset \mathsf{R}^3$ then every 1-qc map of V into R^3 is the restriction to V of some Möbius transformation. Gehring's proof of this beautiful result is based on the fact that a 1-qc mapping preserves the conformal capacity of ring domains. It carries over to subregions of $\overline{\mathsf{R}^n}$ in every dimension $n \geq 3$. Note the striking contrast with $n = 2$, where every injective complex analytic function is 1-qc.

A second important result concerns boundary values of qc maps. Let f be a qc map of the half space $H^+ = \{(x, y, z) \in \mathsf{R}^3 : z > 0\}$ onto itself. Gehring proves that f extends to a qc map of $\overline{\mathsf{R}^3}$ onto itself. Furthermore the restriction of f to the boundary of H^+ in $\overline{\mathsf{R}^3}$ is a qc map of $\overline{\mathsf{R}^2}$ onto itself. This is a strong regularity statement about the boundary values of f. G. D. Mostow discovered that it can be used to help prove that any qc map of one finite-volume hyperbolic 3-manifold onto another is homotopic to an isometry. Mostow then extended Gehring's theory to all $n \geq 3$ as part of the proof of his famous rigidity theorem.

Gehring's theorem about the boundary values of a qc self-map of H^+ raises a converse question. Can every qc self-map of $\overline{\mathsf{R}^2}$ be extended to a qc self-map of $\overline{\mathsf{R}^3}$? Ahlfors gave a positive answer to this question in 1963. Using Ahlfors's result Gehring proved in a remarkable 1965 paper that a topological 2-sphere S in $\overline{\mathsf{R}^3}$ is the image of the standard sphere S^2 under a qc self-map of $\overline{\mathsf{R}^3}$ if and only if each component of $\overline{\mathsf{R}^3} \setminus S$ can be mapped quasiconformally onto the 3-ball. The problem of determining exactly which regions in $\overline{\mathsf{R}^n}$ can be mapped quasiconformally onto the n-ball is still unsolved. Gehring has the best partial results.

Gehring's criterion for S to be a qc image of S^2 in $\overline{\mathsf{R}^3}$ (which has since been extended to all $n \geq 3$) reveals another striking contrast between two dimensions and higher dimensions. By the Riemann mapping theorem and the Jordan curve theorem, every Jordan curve C in $\overline{\mathsf{R}^2}$ has the property that both components of $\overline{\mathsf{R}^2} \setminus C$ are conformally equivalent to the 2-ball, but only very special Jordan curves are the images of S^1 under a qc self-map of $\overline{\mathsf{R}^2}$. These curves are called quasicircles and are characterized by a number of equivalent good properties. One of the most remarkable characterizations is due to Gehring who proved in a 1977 paper that a simply connected region $D \subsetneq \mathsf{C}$ is bounded by a quasicircle (in $\overline{\mathsf{R}^2}$) if and only if there is a positive number ε such that every holomorphic function f on D that satisfies the inequality

$$|S_f(z)d(z)^2| < \varepsilon \qquad \text{for all } z \in D$$

is injective. (Here $S_f = (f''/f')' - \frac{1}{2}(f''/f')^2$ and $d(z)$ is the distance from z to the boundary of D.) This result is only one of many indications that qc maps are intimately related to very classical ideas in complex function theory.

The geometric properties of qc maps in R^n lead rather directly to the conclusion that their first partial derivatives in the sense of distributions are measurable functions locally of the Lebesgue class L^p for $p = n$. The properties of the Beltrami equation and the two-dimensional Hilbert transform imply that the first partial derivatives of a qc map in R^2 are locally in L^p for some $p > 2$. That is a significant conclusion since it makes Sobolev's regularity theorems available. In an important 1973 paper Fred Gehring solved the difficult problem of proving for all n that the first partial derivatives of a qc map in R^n are locally in L^p for some $p > n$. New methods are required. Gehring based his proof on a lemma involving reversed Hölder inequalities, a result that has useful applications to partial differential equations.

In recent years Fred and his former student Gaven Martin have jointly studied discrete groups of Möbius transformations acting on the 3-ball. They have had impressive success in finding sharp geometric inequalities for such groups. In addition, they have introduced the important new concept of a convergence group, which captures many of the crucial properties of Möbius groups. Let $Homeo(S^n)$ be the homeomorphism group of S^n. By definition a subgroup G of $Homeo(S^n)$ is a convergence group if every sequence of distinct elements of G has a subsequence $\{g_n\}$ such that either (a) there is a g in $Homeo(S^n)$ such that $g_n \to g$ in the compact-open topology, or (b) there are points x and y on S^n such that $g_n|(S^n \setminus \{x\}) \to y$ and $g_n^{-1}|(S^n \setminus \{y\}) \to x$ in the compact-open topology. The convergence group G is called discrete if possibility (a) never occurs. D. Gabai and, independently, A. Casson and D. Jungreis have proved that every discrete convergence group on S^1 is conjugate in $Homeo(S^1)$ to a group of Möbius transformations of $\overline{\mathsf{R}^2}$ (restricted to S^1). That fact has important applications to the topology of surfaces and 3-manifolds.

Fred Gehring's contributions to the theory of quasiconformal maps and geometric function theory go far beyond his own research. He has been a vigorous and enthusiastic leader in shaping these theories, working closely with senior mathematicians and with young mathematicians as a teacher and mentor. He recently won the prestigious Sokol Prize awarded by the University of Michigan for excellence in graduate teaching. He has had twenty-six Ph.D. students, now spread around the world, and has been the sponsor of forty foreign visitors who have come to work with him in Ann Arbor.

Fred has served the AMS in a number of important capacities. He was on the *Mathematical Reviews* Editorial Committee from 1969–1975, on the Executive Committee of the Council from 1973–1975 and from 1981–1983, and was a Trustee from 1983–1992. During this period, the Society had many difficult problems to resolve and Fred's advice on scientific and financial matters was always well thought out and very much appreciated by his colleagues. He was chair of some very sensitive committees such as the Search Committee for the Executive Director in 1987 and the Committee on Governance in 1993. He did a wonderful job of organizing the work of these committees and contributed substantially to their reports.

In addition to his service to the Society and on external visiting committees for mathematics departments, Fred has helped many other organizations govern themselves, for example, the Institute for Mathematics and Applications and the Geometry Center in Minneapolis.

Having seen the high quality of advice he gives, we can easily understand why so many organizations seek to benefit from Fred Gehring's advice and judgment. He is an excellent person to lead the AMS in the upcoming years as its president.

Nomination for Arthur Jaffe

Barry Mazur and Vaughan Jones

Happily, the enterprise of mathematics is enriched by the presence of a number of people who have the talent, the energy, and the generosity of spirit to be at the same time deeply devoted mathematicians and equally gifted and effective organizers. Arthur Jaffe is one of these people, and we enthusiastically nominate him for the presidency of the American Mathematical Society.

Arthur Jaffe was born in 1937, and he received his Ph.D. from Princeton University in 1966. He is currently the Landon T. Clay Professor of Mathematics and Theoretical Science at Harvard University.

Arthur Jaffe has played an important role in bringing the mathematics and physics communities together. For his work in mathematical physics, Arthur Jaffe received the Dannie Heinemann Prize in 1980 for mathematically "demonstrating the compatibility of relativistic invariance, quantum mechanics, and local field theory.

The field of mathematical physics enjoyed a period of rapid development in the late sixties and early seventies, attracting the attention of many mathematicians and physicists including E. Nelson, I. Segal, K. Symanzik, A. Wightman, and others. In a series of ground-breaking papers and continued over a span of 14 years, Jaffe and Glimm established the discipline of Constructive Field Theory, which remains the most significant approach to answering the question: "Can quantization as predicted by renormalized perturbation theory of physics be implemented mathematically?" Even if Jaffe had done nothing else, his and Glimm's heroic and successful construction of a nontrivial quantum field in 3-dimensional space-time would have assured him an honored place in the mathematical physics literature. But Jaffe's contributions are hardly limited to one field, and his general attitude towards research is marked by an openness to diverse ideas and interests: He has made important contributions, for example, to classical gauge theory and cyclic cohomology. With Jaffe's enthusiasm, his congeniality, and his mathematical ideas, it is no wonder that he is always surrounded by young bright students. Many of Jaffe's students are now important contributors to mathematical physics; but some have become leading figures in quite different branches of pure mathematics; some have made their mark in applied mathematics.

Jaffe's manifold activities and organizational innovations show him to be keenly interested in the well-being of the mathematical community. Besides serving on numerous AMS committees, Jaffe has recently completed terms on the Council and on the Executive Committee of the Council. He has served as president of the International Association of Mathematical Physics (IAMP) for the period 1991–1996. He has done much to secure a future for that fledgling organization, including chairing the scientific committee for its successful 1994 Congress at UNESCO in Paris. He helped organize numerous conferences, and, in particular, was the co-founder of a series of successful Cargèse Summer Schools. Jaffe remains the moving force in America for these Summer Schools, which are widely known for the role that they play in facilitating communication between mathematicians and physicists.

Jaffe was an active member of the David Committee and was the author of the fine appendix to that report *Ordering the Universe: the Role of Mathematics*, which had the mission of explaining to a broader public the unique character of mathematics. This article played an influential role, both in this country and abroad, in justifying increased government support for mathematics.

Jaffe's wise appointment of editors, his own editorial talents, and his energetic devotion to the *Journal for Communications in Mathematical Physics* (CMP) have been key in making that journal play the inspirational role that it does play in the subject. Jaffe's enthusiasm and scientific openmindedness is a great factor in maintaining the high level of CMP. He is always on the lookout for exciting new papers; he vigorously solicits articles which contain advances in a range of subjects, keeping that journal fresh, and keeping its scope broad.

His controversial piece *Theoretical Mathematics*, coauthored with Frank Quinn, displays the lively concern that Jaffe has for the status of our subject, and the desire he

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has to understand the trends within it, and to stimulate meaningful discussion about it. Indeed the responses of Atiyah, et al. in the *Bulletin of the AMS* (April 1994) show the sensitivity and importance of the issue raised by Jaffe and Quinn. No matter where one stands on the issues of rigor, one must admire Jaffe and Quinn for taking a position and publicly defending it.

In a more parochial arena, Jaffe served as chair of the Harvard Mathematics Department (1987–1990), during which period he did much to win from Harvard's administration a level of respect for his energy and for his innovative ideas, which included a significant expansion of department space and the establishment of an extensive visitor's program.

Let us now discuss the main work of Jaffe's career—Constructive Field Theory. The physicists' perturbative approach to Quantum Field Theory is to begin with the Lagrangian of a classical field theory, which in a very simple case might look like

$$\mathcal{L}(\phi) = \partial_\mu \phi \partial_\mu \phi + m^2 \phi^2 + \lambda \phi^4 \ .$$

One then calculates observable quantities using the Feynman path integral

$$\int [D\phi] e^{i \int \mathcal{L}(\phi) d^4 x} \ .$$

One does "calculate" despite the fact that the integral is mathematically ill-defined (for starters, one has not defined a measure on the path space over which the integration is to be performed). The integral can nevertheless be approached by expanding it as a perturbation series (formal power series) in the parameter λ. Alternatively in "non-perturbative" quantum field theory one would seek a Hilbert space of "states", where "observables" are given by self-adjoint linear operators on that space. Both approaches, perturbative and non-perturbative, are fraught with problems due to the infinite dimensionality of the path space and due to an incompatibility of ordinary (several-particle) quantum mechanics with special relativity. In the perturbative approach, even the coefficients of the expansion may diverge. The physicists deal with divergences that occur by methods which go under the name of "renormalization."Though apparently ad hoc, techniques of renormalization have been extraordinarily successful and have given the best agreements between theory and experiment in all of science. Quantum field theory has also led to profound insight into purely mathematical questions (witness operator algebras, analysis of infinite-dimensional spaces, the work of Atiyah and Singer, and Witten's many recent successes in geometry). There must be a broad and deep structure at work here and its rigorous elucidation is one of the big mathematical challenges of our time.

The Constructive Field Theory program tackles the problem head-on by constructing path space measures, i.e., measures on distributions in space-time (and as a consequence, the program constructs Hilbert spaces and observables) for specific interesting Lagrangians. This theory is tremendously difficult, and highly original, mathematics. We recommend the book *Quantum Physics*, by Glimm and Jaffe themselves. The central difficulties lie in the initial construction of the measure, and (in the course of studying properties of the constructed measure) in understanding the spectrum of the corresponding Laplacian. This is equivalent to proving existence and establishing properties of the solutions to certain nonlinear, infinite-dimensional, and highly singular PDE's.

To begin, Jaffe and Glimm established the first existence theorem for a 2-dimensional nonlinear quantum field obeying the Wightman axioms. They also made a profound study of its properties. Later, in a true analytic *tour de force*, they went on to lay the foundations for the much more difficult 3-dimensional problem. In a series of papers written with Spencer, they analyzed the spectrum of the infinite-dimensional Laplacian in question, proving conjectures of physicists about phase transitions. The technique used in proving these results (cluster expansions) became a standard tool of mathematical physics.

But Jaffe's contribution is by no means limited to constructive field theory. His book with Taubes, *Vortices and Monopoles*, became a standard reference in the theory of classical, nonlinear field equations. In the late eighties, Jaffe turned to the new subject of noncommutative differential geometry, originally introduced by Connes. In collaboration with Lesniewski and Osterwalder, he constructed a cocycle in entire cyclic cohomology. While motivated by his earlier work on quantum field theory, the cocycle condition arises from the interplay between algebraic and analytic aspects of supersymmetry. Now known as the JLO cocycle, it is a non-commutative generalization of the Chern character and has become a standard tool in index theory.

Arthur Jaffe has deep scientific accomplishments, has a firm commitment to the well-being of the mathematical community, and has the gift of being able to follow through successfully on his organizational ideas. We feel that Arthur Jaffe, with his energy, resourcefulness, and dedication, would make an excellent and highly successful president of the American Mathematical Society.