Upstairs, Downstairs

Mark Saul

The Mathematician and Pre-College Education

Humans are rarely in control of their own institutions. However much we prize freedom of action, we are often constrained to behave in ways which are dictated by our institutions' needs rather than our own.

The role of mathematicians in education illustrates this clearly. While many mathematicians are interested in pre-college education, there is a scarcity of opportunities for mathematicians to contribute to the national effort in education. When these opportunities do arise, the rewards structure attached to such work often makes it difficult to take advantage of them.

John Kenelly illustrates this with a striking anecdote. He had worked for several years to increase participation in his state in the Advanced Placement program in mathematics. When his efforts resulted in a major increase in enrollment in the program, he reported this to the chair of his department. The reaction? "Yes, such volunteer work is good. Your colleague in the next office is a scoutmaster. But we don't credit community service in our evaluations." Happily, the president of the university, hearing this, disagreed. And, happily, the growth in mathematical knowledge of the students and teachers in the state continued, bolstered by the efforts of several mathematicians.

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Our institutions do not dictate our individual actions, but they often do set our priorities. To achieve the rewards of tenure and promotion, the college professor must get her or his name out there in publication. The university must become a center of research, a producer of new knowledge, as measured by research articles, and graduate programs. The issue of individual mortality—of nurturing new mathematicians to take over from the current generation—is one that The University largely ignores.

Ignored, too, is the issue of seeking out mathematical ability in those who don't know they possess it. The gifted graduate student was once the insecure undergraduate, and before that the uncertain high school student, and before that the child, with a mind awaiting inspiration. It's a severe series of filters to pass through. We can only guess at how much mathematical ability remains untapped. We need this ability not only to create mathematicians, but also across the spectrum of a growing number of fields which use mathematics. The narrow view of university mathematics does not contribute to this effort.

Priorities in pre-college education differ vastly from those on the university level. The classroom teacher must maintain order and show student achievement. The administrator must show that The District is progressing, and that this progress is due to his or her own actions. The teacher who publishes, who "produces new knowledge", is an exception. Indeed, there are cases in which a teacher's publications have roused the jealousy of colleagues, and threatened the pride of unenlightened administrators. The District is often unaware that the transfer of expertise among teachers benefits everyone. And so our institutions perpetuate the caricature of the otherworldly professor and the circumscribed teacher. They attempt to mold research mathematicians into elitist strivers, and pre-college teachers into docile workers. The situation is not unlike that of the Edwardian household in the television series whose title I have borrowed: two social systems in proximity, struggling to reach a personal working relationship against the grain of the roles cast them by society.

Faculty members of schools of education find themselves pinched especially painfully in this vise. While they are charged with investigating the processes of pre-college education, and by extension doing something about it, the institutions they work with keep them out of the classroom and away from the most fertile source of knowledge. When they do interact with students and teachers, they are often coming "down" from upstairs to do so, and report their results in journals read mainly by other researchers. The unfortunate result is that much of this research is of little interest either to mathematicians or to mathematics teachers.

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It is also true that the field of education is changing rapidly, so that mathematicians who want to help find themselves aiming at a moving target. We are more and more aware of our crisis situation in pre-college mathematics education and more willing to try new ideas. It is inevitable that some of these ideas will not succeed and that the system will experience a certain amount of "thrashing". The message this sends to some mathematicians is that education is a mess with which they should not dirty themselves.

Of course, none of these descriptions is fair to those individuals of good will who reach across institutional barriers, who leave the campus for the classroom or the classroom for the library. The point is that the work of these individuals to bind us together runs counter to the institutional forces that tear us apart.

Translated to the national level, these forces produce several anomalies. Most conspicuously, our graduate and research programs in mathematics, often deemed the finest in the world, attract the best students from all over the globe. Victims of our own success, American-born graduates students are outnumbered by gifted immigrants. Yet our pre-college programs, and particularly our public schools, suffer with perhaps the lowest level of mathematical achievement of any of the industrialized countries.

Another related anomaly: we have a surfeit of Ph.D.s in mathematics, but a major shortage of teachers on the pre-college level with strong mathematical backgrounds. There is an obvious solution, but most people who have recently earned the Ph.D. in mathematics would rather perform respectable manual labor—or study law—than take a job in public education. Their mentors and advisors, anxious for their departments to achieve where it counts, often discourage such a step.

There are alternatives. For example, the end of the Cold War has fortuitously presented us with a different model, in the mathematical culture of Eastern Europe and the former Soviet Union. Readers of this journal are perhaps the only group in the country who can appreciate the successes of Russian mathematicians in the Soviet era. These extraordinary achievements were the product of a society in which rewards and status were assigned in very different ways from those we are used to. A second outstanding program was the "correspondence school" started by I. M. Gelfand, and which he is continuing in the United States (for more information on this, see Mark Saul: "Love Among the Ruins: The Education of High-Ability Mathematics Students in the USSR," *Focus*, Vol. 12, No. 1, February, 1992). His program ultimately reached tens of thousands of students in small towns and cities across the expanse of the Soviet Union. College admissions programs recognized participation in the Gelfand school as indicating both a deep interest and careful training in mathematics. Of the many remarkable features of this program, we may note first that the tremendous labor required to assess and respond to student assignments was undertaken by graduate students of mathematics, by faculty members and by high school teachers, each of whom played closely coordinated roles. Also noteworthy was the personal interest Gelfand took in the program. One of the great mathematicians of the century routinely took time out of his research career to write books and construct exercises for high school students. The care and thought that Gelfand put into this effort was entirely commensurate with the efforts he put into his mathematical (and biological) research. While we must not copy the Soviet system, with its many faults (and probably cannot copy any system at all from abroad), we can learn and borrow from their practices.

The Russian "mathematical circles" are an example of a practice from which we can learn. It was fairly common for the faculty of Russian universities, from well-established professors to younger faculty and graduate students, to go into the high schools and organize study groups for students who were interested in mathematics. Often these were the high schools that the mathematicians had graduated from, or which their own children attended. Out of these circles grew a body of popular mathematical literature on a very high level, written by some of the most accomplished Soviet mathematicians. Kolmogorov, Yaglom, Markushevitch, Khinchin, Dinkin, and many others spent significant parts of their careers considering mathematics curricula or writing books that bring rather advanced materials within reach of high school students. This body of literature constitutes a Russian national treasure, only some of which has been made accessible in English.

I.M. Gelfand

The organization of mathematical circles was closely connected with the Russian system of mathematical competitions. Russian contests involved almost exclusively long-answer questions requiring experimentation and proof (American contests are usually based on short-answer or multiple choice questions). The expertise of university faculty was routinely tapped, both to construct questions and to grade papers. This led to a healthy interaction among professors, graduate and undergraduate students, and high school staff and students. In turn, this massive effort forged new links among members of the mathematics community and led to still more cooperative activity, including mathematical summer camps and evening schools. High schools for students specializing in mathematics and the sciences were opened across the Soviet Union, often motivated by the actions of a well-known physicist or mathematician.

Amid this ferment of educational and mathematical activity, two particular programs stand out. The first is the journal *Kvant*, founded in 1970 by a group of mathematicians and physicists, including teachers on the secondary level. This remarkable journal presented articles in these two areas written by researchers or graduate students, and accessible to high school students. Its style was interactive, with exercises and examples posed throughout the expositions.

Russian mathematics has thus given us a rich set of models on which to build, and there have been several fine beginnings. In 1988, the editors of *Kvant* reached an agreement with the National Science Teachers' Association to publish a parallel journal in America, called *Quantum*. A marvelous but underutilized resource, *Quantum* presents monthly translations of Russian articles, problems and brainteasers. Some American mathematicians and teachers have contributed articles (indeed, some American authors had already published in *Kvant*). But it has been difficult to find suitable contributions from American authors. Perhaps in the future, publication in *Quantum* will become an accepted and perhaps rewarded academic activity.

American mathematical contests have also been influenced by foreign models, from Russia and elsewhere. To the older short-answer contests, some of whose histories reach back to the early years of the century, we have added the USA Mathematical Olympiad, administered to about 150 of the country's most talented high school mathematics students. The Olympiad consists of five long-answer questions, and is essentially modeled after numerous Eastern European competitions (the International Mathematical Olympiad originated in Eastern Europe when it was the Soviet Bloc). From Hungary has come the model for the American Mathematical Talent Search, run by George Berzsenyi out of Rose-Hulman Institute of Technology. This contest, with an associated NSF-funded summer program, offers sets of long-answer questions to almost 1,000 students. The American Regions Mathematics League, composed mainly of high school teachers, is about to launch a Power Contest, in which groups of high school students will cooperate in solving olympiad-style questions. In each case, the struggle has been to find mathematicians who would be willing to write questions and grade solutions. Support for these enterprises has come from government agencies, from research laboratories, and from mathematicians employed in the high schools, but it has been difficult to find university faculty who are free to engage in this kind of work.

I. M. Gelfand, now at Rutgers University, has been working to establish a form of his correspondence school in the United States. This effort, called the Gelfand Outreach Program in Mathematics, has been remarkably successful on a local level. Gelfand has published three texts on elementary mathematics, and plans a full English-language series. As with the Russian materials, he takes the same care in preparing these as in preparing his research articles for publication. Using resources offered by Rutgers University, personal connections, and private funding, Gelfand has set up a program serving 400 students, a remarkable number of whom are female or come from minority backgrounds. The effort to duplicate this model in other localities has proven difficult: American mathematicians are rarely supported in this by their institutions.

Not all the involvement of American mathematicians in education has its roots in foreign influences. There are a number of striking American originals. David Kelly has been running a summer program at Hampshire College which attracts interested mathematicians and creates a support network of peers and teachers for students interested in mathematics. Arnold Ross has been doing much the same thing since before Sputnik at Ohio State University. Ross's work is being continued by Glenn Stevens at Boston University. Other instances of mathematicians working with high school teachers or students include the Westinghouse contest, which is judged partly by mathematicians, and the Research Science Institute at MIT. Started by Admiral Rickover, this last program arranges for summer internships in many fields, including mathematics, for interested high-ability students. The University of Minnesota has a well-developed program serving high school students who are qualified to take college-level courses. As a result, more and more such students are being identified in Minnesota. And numerous mathematicians, like John Kennelly, have taken an interest in the Advanced Placement program of Educational Testing Service. (See also the article, "The Math Circle", by Robert Kaplan, this issue of the *Notices*).

The National Science Foundation contributes to some of these programs through a relatively small division which funds student summer programs. Most of the programs funded target specific underrepresented populations. However, in general, the NSF has not yet taken an interest in applying the American expertise in mathematics to America's educational difficulties. For example, while there is a thriving program supporting postdoctoral students in industry, there is not at present any corresponding effort to involve them in education. And there is no program at all which connects elementary school children and their teachers, or children without a special interest or ability in mathematics, with the mathematical community.

We are in desperate need of institutional commitments, on the part of NSF, on the part of our professional organizations, and on the part of our schools and universities, to eliminate the barriers between the various constituencies of the mathematical community. Maintaining an "upstairs, downstairs" relationship between pre-college and higher education is proving much too costly to the national effort in the field. The very institutions charged with furthering that effort find their progress slowed to a crawl by an unconscious desire to maintain this relationship.

Perhaps studying the model of the former Soviet Union and extending the work of the individual efforts described above can help us remove the institutional barriers which hamper our movements. And perhaps our institutions can begin to take positive steps to encourage mathematicians and educators to act in concert for our common good.

Teachers can learn from mathematicians about the dynamism of the subject they teach. Mathematicians can learn from teachers an expertise in transmission of knowledge which has been proven in the field. Each can enrich the other.