Undergraduate Mathematics Education Needs Your Critical Concern

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Steve Krantz's article ("Math for Sale", *Notices* **42**, p. 1116) and the responses it has received on various networks and in these pages raise a number of questions about mathematics, mathematics education, and the relationship between them. Krantz expresses concerns; others disagree with what he says and criticize several aspects of his style. I would like to try to change the direction of the discussion.

What we have here is the emergence of a new field—undergraduate mathematics education. It has to do with research into learning and teaching, which may be thought of as a pure component, and with postsecondary curriculum development, which may be considered as an applied component. This emergence parallels and is inspired by a flowering of mathematics education research at the school level [3], which has attracted a number of mathematicians, e.g., Freudenthal [2]. However, because of the particular importance of mathematical content in collegiate mathematics education, there is no way that such a field could develop fully in an environment that is not very close to the mathematics community. This development within the world of mathematics is actually happening [5, 6], and mathematicians have only the choice of helping or hindering. The very worst alternative would be to tell this field to find some other home for its infancy and childhood, if not its adolescence and adulthood.

We have been here before. There is a time-honored tradition in mathematics of giving birth to, and nurturing, new fields: statistics, applied mathematics, computer science, to name a few. I have been an ordinary observer in one of these three and a little more involved in the other two. I can tell you that much of today's discourse also occurred when these fields were developing. I am experiencing an uncanny déjà vu of a new field arising at a time when mathematics is in some trouble with respect to falling enrollments, job shortages, and diminishing external support. There is a familiar ring to current suggestions that only weak mathematicians go into education, that the field is siphoning off graduate students, that it is taking funding away from mathematics, that the quality of work in the field is very poor, and that people only go into it for the money.

I would not have much credibility here if I did not acknowledge that many of these concerns are, to a certain extent, as well founded as they may have been in the early days of statistics, applied mathematics, and computer science. In fact, many of the complaints could be, and have been, applied to emerging fields of mathematical research, as witness the derisive "generalized nonsense" phrase that was used to describe much of the functional analysis of the 1950s and 1960s. Nevertheless, the new fields *were* established, and I think it can be agreed that they are more than respectable. Indeed, the mathematical community has every right to be proud of its contributions to their emergence and development. But more than feeling good, in the case of each of these fields, the current relationship between it and mathematics is enriching for both. I believe this can happen, indeed is happening, with collegiate mathematics education.

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Indeed, it is possible to argue that the unfortunate characteristics are unavoidable features of the emergence of a new field within mathematics. One way of analyzing the growth of quantity and quality in a field of research is to postulate that the percentage of all research that is high quality will be fairly constant (assuming a fixed amount of special training for gifted individuals) and that one way of increasing the amount of high-quality researchers is to increase the total number at the same time that efforts are made to establish standards of quality. Thus, it would be in the interest of all to stop dwelling on what are certainly unsatisfactory realities and, acknowledging their (possibly necessary) existence, to focus on looking for ways to move on to the next, more satisfactory stage. For people in postsecondary mathematics education, that means to improve quality, to develop and maintain the highest standards of scholarly endeavor, to stop being defensive about the criticisms of mathematicians, and to work closely with as many of our mathematician colleagues whom we can interest in our work. For people in mathematics it means to stop looking at research and development in collegiate mathematics education as a threat and to work closely with the practitioners of the new field so as to be a positive influence on its development.

I try very hard to make these points to my colleagues and students in mathematics education. For mathematicians, I would like to make the following specific suggestions for ways in which you can be of immense help.

- Judge our work in terms of our own paradigms. That is, consider what we are trying to do, what may be possible, and what is out of the question (no matter how desirable it may be). Then evaluate us in terms of how well we do with what we are actually trying to accomplish.

  What we are trying to do is very different from the mathematician's work of proving theorems and finding counterexamples or solving problems in the physical universe. Basically we are trying to build models of what might be in students' minds—what they understand, how they think and learn. These models cannot be derived formally from theoretical analyses or completely tested empirically, i.e., we can never really know what is going on in someone's mind. However, we do develop theoretical frameworks and conduct empirical studies using questionnaires, exams, interviews, and/or teaching experiments—and these data are analyzed in great detail. Sometimes, plausible implications can be drawn for teaching and curriculum development. For example, freshman have ideas of proof quite different from ours. Some accept proofs while feeling their conviction would be strengthened by additional empirical evidence [1], while others simultaneously accept both inductive *and* correct deductive arguments as valid [4]. Such results would not tell one how to teach proofs, but they might help one avoid some pitfalls.

  In general, the connections between theoretical analyses, empirical investigations, and education applications are complex, and various practitioners operate according to different paradigms. For detailed discussions of some of these paradigms and how they are used in postsecondary mathematics education, the reader might consult the expository papers by Schoenfeld [7] and Asiola et al. [8].

- Don't let our insistence that you consider our work on its own terms keep you from judging us and doing so according to the highest quality standards of scholarly work.

  Although we feel that our field has made considerable progress in its development, we welcome your criticisms because they can only help us improve. One of the unique and admirable properties of mathematics is its ability to maintain high standards by means of criteria that are independent of irrelevant factors. Anyone who proves a good theorem gets appropriate recognition—no matter who he or she is. Help us to emulate this beautiful feature of mathematical culture.

- Look a little more closely at our work and our motivations for doing it, and try to see if we are not worthy of your respect.

For the work, I can offer nothing other than our publications and our course developments. I think we have done some worthwhile things. Work in this field has led to a greater understanding of how students learn (or don't learn) various concepts in mathematics; of how students develop (or don't develop) the ability to make proofs and find counterexamples; and of the value (or lack of value) of various pedagogical strategies such as lecturing, the use of technology, and cooperative learning. Details about some of these can be found in various reports, such as those by A. and J. Selden [5], W. G. Martin and G. Harel [4], P. Dunham [9], and Hagelgans et al. [10]. In the case of pedagogical strategies such as cooperative learning, research has also led to practical advice for the working faculty member [10].

Regarding our motivations, I would like to point out that for some years there were rules in the NSF against *any* support for postsecondary mathematics education.

Many of us began working in this area in spite of the lack of any *possibility* of support. When this changed and NSF support for calculus reform began and research in teaching and learning was opened up to mathematical topics beyond high school, it was not a horde of newcomers that received the first grants, but mainly people who had been working for several years with no support at all.

- Work actively to influence good students *for whom it is appropriate* to think about the field of mathematics education.

  I am not suggesting that anyone be discouraged from doing mathematical research. I am thinking of the large number of graduate students who learn a great deal of mathematics and develop high standards and good taste about what is important, but for one reason or another decide not to do research in mathematics. I hope that you will think of mathematics education as one alternative for such individuals.

- Think carefully about the pure vs. applied dichotomy.

  We don't want to open any can of worms or revisit controversies some consider to be settled, but there are parallels, and history can inform us. Too often, a mathematician will argue forcefully against excessive concern with goal-oriented research in mathematics but will insist, just as strongly, that research in education should concentrate, more or less exclusively, on what will help the mathematics instructor in her or his classroom—next week. All of the arguments, some of which are marshalled by Krantz in his editorial, for the importance and ultimate effectiveness of undirected research in mathematics can and should be applied just as strongly to mathematics education.

I do not wish to suggest here that support from the mathematical community for the field of collegiate mathematics education is totally lacking. On the contrary, in recent years an impressive record has been built. The AMS instigated, and MAA and SIAM joined in, the establishment of *UME Trends*, a newsletter reporting on the field. The AMS and MAA have established a Joint Committee on Research in Undergraduate Mathematics Education. The CBMS publishes RCME, an annual volume of research papers in collegiate mathematics education. Perhaps most importantly, the winter and summer math meetings now feature, on a regular and growing basis, research papers, hour talks, panels, and other activities in postsecondary mathematics education. And as a final item, one which I find personally very gratifying, there occurred in November 1995 the first of what I hope will be many Oberwolfach conferences in collegiate mathematics education.

As a practitioner of both pure and applied collegiate mathematics education, I look to the mathematical community for help in establishing what I believe to be an important new field that has made, and will increasingly make, significant contributions. As one who still considers himself a mathematician, I am proud of the contributions that have been made by our community to the emergence and development of several new fields—including postsecondary mathematics education. I call for, and look forward to, a continuation and growth of the application to collegiate mathematics education of the honorable custom of the mathematics community being an environment in which important new fields are born and nurtured. It adds to the very important social contribution that we make just by doing mathematics.

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