How We Got Where We Are:

An International Overview of Mathematics in National Contexts (1875-1900)

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In the last thirty years or so the mathematical community internationally has observed a remarkable number of centennials. To name just a few, the Moscow Mathematical Society entered a new century in 1964, with the London Mathematical Society (LMS) following one year later; the *Mathematische Annalen* turned one hundred in 1968 four years before the Société Mathématique de France (SMF); the *American Journal of Mathematics* and the Circolo Matematico di Palermo (CMP) reached their century marks in 1978 and 1984, respectively; and the American Mathematical Society (AMS) celebrated its centenary in 1988 just two years before the Deutsche Mathematiker-Vereinigung (DMV).[1]

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[1]*For publications honoring the centennials of the LMS, AMS, and DMV, see [6], [8], and [11], respectively. Hélène Gispert detailed the history of the SMF from 1872 to 1914 in her book [12], while Aldo Brigaglia and Guido Masotto chronicled the early history of the CMP in [5].*

At the very least, these milestones suggest that the mathematical endeavor developed in important ways in diverse national settings during the closing quarter of the nineteenth century, but … how? How did we get where we are? How did we become an international mathematical community, working together across political boundaries on common problems using shared techniques?

Prior to the nineteenth century, scientists (as opposed to the modern notion of "specialists"), formed communities centered around institutions like the Royal Society of London or the Académie des Sciences in Paris. These institutions, together with the courts of various monarchs and the salons of wealthy patrons, encouraged research, sponsored general journals like the *Philosophical Transactions*, and supported scientific communications. The universities, on the other hand, taught an essentially medieval "liberal arts" curriculum within the context of faculties of philosophy, law, and medicine. Euclid's mathematics formed a key part of this curriculum, with the calculus added on at some universities in the eighteenth century, but the emphasis was on imparting a body of knowledge, not on imparting a body of knowledge in order to further knowledge, not on imparting the "latest" knowledge in the field, and not on actively training others to make original research contributions. These latter ideals, so characteristic of the university today, only became part of its educational mission during the nineteenth century, first in Germany[2] and, by the closing quarter of the century, in many other countries as well. Educational reform—whether spurred partially by political unification in the case of Italy, by the loss of the Franco-Prussian War in the case of France, by the philanthropy of wealthy industrialists in the case of the United States—is a kind of international common denominator relative to the formation of mathematical communities in individual national contexts between 1875 and 1900. What other attributes transcended national boundaries? An answer to this at least hints at "how we got where we are".

Von Humboldt's vision of higher education… stressed the importance of pure research over utilitarian concerns…

The Mathematical Scene in Nineteenth-Century Germany

The opening decade of the nineteenth century was one of great political reorganization in the German states owing to the effects of the Napoleonic Wars. In Prussia, for example, the years from 1806 and the Prussian defeat at the Battle of Jena to 1810 and the founding of the University of Berlin witnessed a series of fundamental political, socioeconomic, and educational reforms. The latter, spearheaded by Wilhelm von Humboldt, the elder brother of the celebrated naturalist and traveler Alexander von Humboldt, came quickly to dominate not just the educational system in Prussia but those in the other predominantly Protestant German states as well.[3] Von Humboldt's vision of higher education, which reflected the influences of both idealist philosophy and neohumanism, stressed the importance of pure research over the utilitarian concerns perceived as dominant within the post-Revolutionary educational system in France. This emphasis on research accompanied and complemented a strong insistence on academic freedom which developed into the ideals of *Lehr- und Lernfreiheit*, that is, the freedom to teach and to learn without political or religious interference. Such educational reforms aimed not only to support the faculty's search for new knowledge but also to train independent-minded, creative, and original thinkers within an atmosphere of disinterested, scholarly pursuit.

Although the full philosophical range of von Humboldt's educational vision was never realized, the latter, more immediate aims became characteristic of the Prussian system. Beginning with philosophy and soon extending to the natural sciences, mathematics, and other developing disciplines, teaching *and* research defined the university professor's mission. As Gert Schubring has argued, "[t]he transition from laying the main emphasis on teaching, which compelled the teachers to seek additional part-time teaching posts, to a dual activity in which teaching comprised only the lesser part of the remunerated activity marks the decisive step towards professionalization" [23, p. 123]. In the case of mathematics, moreover, this research ethic ultimately brought with it increasing specialization in the field, as mathematicians and mathematicians-to-be tended to focus their studies in an effort to make their own personal contributions. At the same time, the emphasis on disinterested—as opposed to more applications-oriented—research resulted in the evolution of a fundamentally purist approach to the discipline.[4] Perhaps nowhere were these interrelated aspects of the development of mathematics in Germany more in evidence early on than at the University of Berlin.

Under the influence of Dirichlet beginning in the 1830s, the University rather swiftly established itself as the dominant force in mathematics in the German-speaking world.[5] This domi-

[2] *As Gert Schubring has pointed out [24], there was, in fact, no one German model. Observers from other countries actually focused on the Prussian model, as defined not only by the policies of the Prussian Ministry of Education but also by the specific programs in place at universities like Berlin and, after 1866 when it became part of Prussia, Göttingen. After the German unification, this model increasingly influenced the very different educational environment which had developed in the predominantly Catholic southern German states.*

[3] *This panoply of issues has also been thoroughly examined. See, for example, the references provided in [18, pp. 24–26] and [23]. Wilhelm von Humboldt was both an influential statesman within the Prussian government and a philologist, aestheticist, and philosopher.*

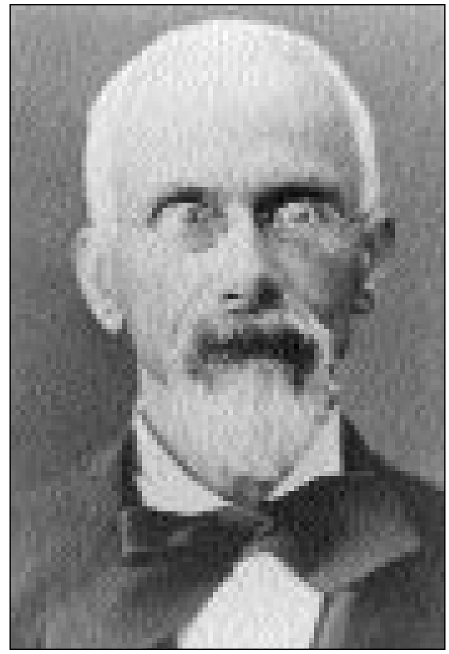
[4] *The processes underlying these developments as well as the interrelations between pure and applied mathematics are, however, much more complicated than these statements might suggest. See, for example, [23] and [24].*

[5] *Just as American would-be mathematicians traveled abroad in the near absence of high-level training at the end of the nineteenth century, Dirichlet had journeyed to Paris in the 1820s for his mathematical education. From 1800 to the 1830s at least, France was Europe's acknowledged leader in mathematical research. For more on the unparalleled strength of the French mathematical community in the opening decades of the nineteenth century, see [14]. France, however, lost its place to Germany after midcentury. See below.*

nance only intensified during the last half of the nineteenth century owing to the triumvirate of Kummer, Weierstrass, and Kronecker. Their Berlin advocated a highly formal and rigorous approach to mathematics which produced stunning results, particularly in analysis, algebra, and number theory [3]. By the century's closing quarter, however, the University of Berlin's position at the head of the mathematical hierarchy in Germany came under increasing challenge from the formerly Hanoverian university in Göttingen.

Opened in 1737, Göttingen University, too, had quickly embraced the ideals of *Lehr- und Lernfreiheit* and had developed strong research-oriented programs in, first, philology and somewhat later the natural sciences. In mathematics, the near-legendary Gauss is often perceived by the twentieth-century mathematician as representing the beginning of a long, distinguished, and continuous research tradition at Göttingen which included the names of Dirichlet, Riemann, Clebsch, Klein, Hilbert, and Weyl, among others. Yet the establishment and maintenance of a true research tradition hinges not only on gifted, research-minded professors ready to impart their knowledge at a research level but also on a corps of talented students willing and able to put those lessons to use. A man of eighteenth-century sensibilities, the austere Gauss had no real desire to train future researchers, and, consequently, few students benefitted directly from his guidance. Dirichlet, Riemann, and Clebsch, although committed to the new nineteenth-century ideals of teaching and research, each had careers at Göttingen cut short and so were able to influence few students. It was Felix Klein, who, after assuming the Göttingen chair in mathematics in 1886, established that university as a rival standard-bearer of mathematics in Germany [21; and 18, pp. 151–154].

As a professor, Klein brilliantly employed the seminar, an institution which had evolved in the German context as the principal vehicle for the active training of young researchers, to animate a thriving mathematical community in Göttingen [18, pp. 189–234, 239–254].[6] Moreover, as a mathematical activist, he lobbied energetically and successfully for mathematics with the officials of the Prussian Ministry of Education, edited the *Mathematische Annalen*, supported and participated in the activities of the Deutsche Mathematiker-Vereinigung [22], and generally served as an advocate for the field in his efforts to stimulate further the German mathematical community as a whole. These institutions—the graduate seminar, the specialized journal, the specialized society—together with the twin values of teaching and research largely defined the profession and, in subtler ways, the discipline of mathematics as it had developed in Germany by the end of the nineteenth century. These same institutions and values informed the emergence of mathematical research communities in a number of other countries as well and thereby served to build a common foundation for the subsequent internationalization of the field.



Photograph courtesy of the London Mathematical Society

Enrico Betti

The Reverberations in Mathematics of Educational Reform

It is not accidental that this brief sketch of the context of German mathematical developments in the nineteenth century opened with a mention of educational reform. Changes in higher education and in its overall objectives naturally spurred changes at the level of the individual disciplines. Educational reform also tended to affect mathematics all the more directly, since one of the key features distinguishing the mathematical endeavor of the nineteenth century from that of the preceding hundred-year period was its venue, namely, the university as opposed to an academy of sciences, a royal court, or elsewhere [23, p. 111]. Its effects were not always positive relative to the development of *research-level* mathematics in a given national setting though, as a comparison of the situations in Italy and France with those in Spain and England underscores.

Prior to the diplomatic recognition in 1861 of a unified Italy, several of the Italian states had supported schools with mathematicians whose research attracted the attention and earned the respect of those whom they viewed as standard-bearers in the field in France, Germany, and Great Britain. In the 1850s, for example, Enrico Betti pursued researches in Pisa on questions in both Galois theory and the theory of substitutions which favorably impressed Charles Hermite, while Francesco Brioschi did work in Pavia on the theory of determinants and, more generally, on the theory of forms which met with approval from James Joseph Sylvester. These two

[6] *Obviously, Berlin and Göttingen did not support the only active graduate programs in mathematics in late nineteenth-century Germany. Paul Gordan and Max Noether at Erlangen, Sophus Lie at Leipzig, and others throughout Germany contributed to the overall profile of higher education in mathematics. For more on the Leipzig seminar in particular, consult [2].*

Photograph courtesy of the London Mathematical Society and the Istituto Nazionale de Alta Mathematica

Francesco Brioschi

cases suggest, and a look at the publications in Barnaba Tortolini's *Annali* confirms, that the 1850s had already witnessed a certain concentration in Italy on algebraic research at a high level [4, p. 272].[7] A sense of the importance and desirability of research was only strengthened further in the mathematical circles surrounding Betti, Brioschi, and Felice Casorati after these three mathematicians returned from their 1858 pilgrimage to the mathematical centers of Göttingen, Berlin, and Paris.

Following Italy's political unification, a number of mathematicians, but most notably Brioschi, held posts within the new government which allowed them to influence developments in politics generally and in education specifically. As the general secretary of the Ministry of Public Instruction from 1861 to 1862 and as a member of that Ministry's Executive Council for some thirty years, Brioschi exercised his influence in the decision-making process regarding educational reform at all levels. In particular, beginning in 1863, he guided Milan's newly founded Istituto Tecnico Superiore in its educational mission of training engineers and brought to its faculty not only Casorati and himself but also his former student Luigi Cremona from 1867 to 1873. Brioschi fostered an atmosphere conducive to research within this academic setting—as evidenced by Casorati's work in Riemannian function theory as well as by Cremona's continuing work in algebraic geometry—at the same time that the school itself provided new job opportunities for those who could meet the challenges of both teaching and research [4, pp. 275–276].

Similarly, at Pisa's Scuola Normale Superiore, an institution founded in 1808 by Napoleon on the model of Paris's École Normale Supérieure, Betti animated an active circle of researchers during his almost thirty-year-long tenure as the school's director through his promotion especially of Riemann's ideas in complex function theory and in the geometry of *n*-dimensional space. The group of mathematicians influenced by Betti at Pisa included, among others, his student and later colleague Ulisse Dini as well as Vito Volterra, Salvatore Pincherle, and Federigo Enriques [4, pp. 279–280]. Although changes such as these were taking place at certain schools under the sway of specific individuals, it was not until 1875 that the minister of education, Ruggiero Bonghi, officially introduced reforms aimed explicitly at bringing Italy's institutions of higher education closer to the German model with its emphasis on teaching, research, and graduate training. His successor, Michele Coppino, rolled back these reforms to a large extent, but the ideals they reflected were—thanks to the efforts of Brioschi, Betti, and others—already fairly firmly entrenched at the faculty level throughout much of Italy by the closing quarter of the nineteenth century.

These developments in Italy did not go unnoticed elsewhere in Europe. In a France awakened from complacency by its loss of the Franco-Prussian War in 1870–1871, Gaston Darboux had already had cause to remark that "we need to mend our [system of] higher education. The Germans get the better of us there as elsewhere. I think that if that continues, the Italians will surpass us before too long" [12, p. 19].[8] In fact, spurred largely by the military defeat and its implication that the so-called *grandes écoles* were perhaps not grand enough to prepare the French adequately for times of crisis, leaders of the newly formed Third Republic sought to strengthen their political position, at least in part, by fostering an intellectual élite associated not with the *grandes écoles* as had been the case with previous regimes but with alternate institutions of higher education. In order to realize this objective, these other institutions, principally the *facultés* in each of France's administrative regions, needed a new focus, which the French politicians and educational reformers found in the German model [25, pp. 302–303].

In a series of major reforms which took place between 1876 and 1900, the French, first, instituted a scholarship program in order to encourage a greater number of strong students to enroll in the *facultés*; second, established new chairs in the various scientific disciplines, including mathematics, as well as the salaried position of *maître de conférence* for both the support of more junior scholars and the enhancement of the overall graduate training program; and third, weakened the traditional ties between the *facultés* and the secondary system of education, thereby allowing for and fostering a commitment to teaching *and* research. The reformers believed that by consciously adapting various aspects of the German model to the French setting, "the French science faculties could out-perform the German institutions" [25, p. 303]. Indeed, the creation of new jobs (principally in the provinces), the loosening of the old

[7] *Tortolini founded the* Annali di Scienze Matematiche e Fisiche *in 1850. In 1858 the journal changed its name to the* Annali di Matematica Pura ed Applicata *in emulation of Crelle's German journal and Liouville's French publication of the same name.*

[8] *All translations of quotations originally in languages other than English presented here are my own.*

disciplinary boundaries through the creation of chairs in various subdisciplines (as exemplified by Camille Jordan's chair, not in mathematics but in higher algebra *per se*), and the adoption of research productivity as a criterion for determining salary, all contributed to the rise of a more specialized, research-oriented mathematical profession in France on a par by 1900 with that in place in Germany [12, pp. 59–63]. As Hélène Gispert has characterized it in her study of the Société Mathématique de France, France, which had entered the Dark Ages relative to mathematics during the decades from 1830 to 1870, experienced an emergent period between 1870 and 1900 that resulted in a new golden age after the turn of the twentieth century [12, p. 113].[9] That new golden age owed, in large measure, to the stimulus provided by educational reform.

Still, such reform did not necessarily have a positive impact relative to the development of mathematics at the research level. In 1857 Spain adopted a centralized educational system modeled on the one put in place in France under Napoleon around the turn of the nineteenth century. Under Madrid's firm control, further change came only slowly. In mathematics, that control translated into the dominance in the advanced curriculum of the projective geometry which Karl von Staudt had developed around midcentury and which Madrid's Eduardo Torroja y Caballé embraced beginning in the 1870s. Although Torroja did advocate *doing* mathematics in his courses at Madrid, he clung doggedly to an area which, over the closing decades of the nineteenth century, grew increasingly distant from, for instance, the more purist Riemannian frontiers of geometrical research [13, p. 1508]. In so doing, Torroja and his adherents in Madrid obstructed the efforts of others in Spain, like García de Galdeano [15, pp. 112–114], to encourage the sort of mathematics being done elsewhere in Europe and particularly in Germany, France, and Italy [1, pp. 162–163].

In England, on the other hand, educational reforms in 1858, 1871, and 1877 brought an increased number of professorships, the advent of fellowships based purely on merit, and the end of the religious tests for faculty members and students. They also resulted in the strengthening of the university structure from the resources of the associated colleges, with an eye specifically toward improving training in the sciences.

In Milan, Brioschi fostered an atmosphere conducive to research within the academic setting

Despite such changes, the fundamental goal of the English system remained the liberal education of gentleman- (and, beginning in the 1870s, lady-) scholars, and the pedagogical emphasis still lay largely on the passing of set examinations. Throughout the nineteenth century, the English educational system continued to focus primarily on the diffusion and not the advancement of knowledge, and, in mathematics as in the other sciences, this tended to militate against the definition of a mathematical profession in terms of teaching and research.

As the examples of Italy, France, Spain, and England illustrate, widespread educational reforms in the last quarter of the nineteenth century affected the development of mathematics in countries throughout Europe (and the United States could be cited here as an example as well). The creation of new academic chairs and institutions, the addition of new grades of instructors, the direct emulation of the German ideals linking teaching, research, and the production of future researchers—these aspects of reform complemented one another in those countries where mathematics at the research level came to define the professional standard. The absence of one or more of them, however, tended to thwart that sort of development.

The Production of Future Researchers

In turn-of-the-century France, Émile Picard summarized well the key role educational reforms had played in the professionalization of high-level mathematics. "Beyond their mission of making the sciences known and understood," he wrote, "the institutions of higher education … have another [mission], just as noble as all the others, that of advancing science and of continually initiating new generations of researchers to the methods of invention and of discovery" [12, p. 60]. As he clearly stressed, a sense of the importance of the training of future researchers represented one crucial byproduct of these Ger-

[9] *Although the circumstances surrounding their "emergent periods" were certainly quite different, France and the United States were influenced by many of the same external factors in this same thirty-year period. Compare [12] and [18].*

man-inspired reforms. Thus, educators and mathematicians in other countries who looked toward Germany and France for their inspiration and guidance in the final quarter of the nineteenth century increasingly conceived of this "noble mission" as an integral part of their endeavor. The United States and Russia provide just two of a number of examples of this sort of influence.

E. H. Moore, with Bolza and Maschke, implemented a training program in mathematics at Chicago which rivaled that of many of their German competitors.

The years between 1875 and 1900 represented a period of growth and financial prosperity in the United States which had important repercussions in higher education. As great fortunes were made on the railroads, the telegraphs, and industrial expansion in general, individuals like Johns Hopkins and John D. Rockefeller endowed universities through their private philanthropy. The presidents of these new schools, well aware of the educational scenes abroad and especially in Germany, France, and Great Britain, crafted their new institutional philosophies informed by the examples of those foreign systems. In particular, many of them adopted the production of research and of future researchers as explicit missions for their faculties and schools [18, pp. 261–294].

At the University of Chicago, for example, a university financed by Rockefeller and opened in 1892, a strong emphasis was placed from the very beginning on securing a talented research faculty. In the words of the university's first president, William Rainey Harper, this faculty would seek "not to stock the student's mind with knowledge of what has already been accomplished in a given field, but rather so to train him that he himself may be able to push out along new lines of investigation" [18, p. 278]. In mathematics, Harper succeeded in bringing together three men who embodied these same ideals.

The American Eliakim Hastings Moore and the two Germans, Oskar Bolza and Heinrich Maschke, implemented a training program in mathematics at Chicago which rivaled that of many of their German competitors [18, p. 367]. This comes as no surprise in light of the facts that Bolza and Maschke had learned their trade from Felix Klein and that Moore had spent a year abroad studying mathematics in Göttingen and Berlin. In addition to the regular lecture courses they offered in the established areas of late nineteenth-century mathematics—invariant theory, the theory of substitutions, elliptic function theory, among others—the Chicago mathematicians also incorporated the seminar into their overall pedagogical approach. As especially Bolza and Maschke knew from firsthand experience, the seminar served as a fertile seedbed for the germination of new mathematical ideas along more specialized lines. The Chicagoans further augmented this learning device with what they called the "Mathematical Club", a series of biweekly meetings throughout the academic year in which speakers, both faculty and students, presented expositions of the recently published results of other mathematicians or of their own evolving ideas. The atmosphere fostered by this faculty and through these means produced in short order a number of first-rate mathematicians, notably Leonard E. Dickson, Oswald Veblen, Robert L. Moore, and George D. Birkhoff (see [18, pp. 372–393] for a brief overview of some of the research these mathematicians did within this mathematical environment).

Although the broader cultural and political circumstances in Moscow could perhaps not have been more different than those in the Chicago of the late nineteenth century, Moscow University, like the University of Chicago, supported a mathematics program under an activist influenced by contemporaneous mathematical developments in both Germany and France. Nikolai Vasil'evich Bugaev journeyed to Berlin in 1863, where he studied under Kummer and Weierstrass, and then continued on to Paris to hear the lectures of Liouville, Chasles, Serret, and others. After some two and a half years abroad, Bugaev returned to Moscow, where he took a professorship in 1867, the year after earning his doctorate there for work on numerical identities involving the exponential function e^x. From 1867 until his death in 1903, he continued his work in number theory, striving in particular to develop general methods in a subject given to clever solutions of often limited applicability [16, pp. 198–200].

Bugaev, however, did more at Moscow than his own mathematical research. He influenced a corps of colleagues and students through his broader conception of mathematics. For Bugaev, mathematics involved communication, which he fostered through his vigorous support of the Moscow Mathematical Society and, after 1866, of its journal, *Matematicheskii Sbornik*. It also hinged on its university setting, which he worked to strengthen and enhance at Moscow through his efforts first as secretary and then as dean of its faculty of physics and mathematics. Most importantly, it depended on training students capable of contributing to its further development. To the latter end, Bugaev taught a wide range of courses—in, for example, number theory, the theory of elliptic functions, the calculus of variations, and the theory of analytic functions—which aimed to introduce his students to these subjects at the research level. He also fostered and contributed to a philosophical atmosphere in which mathematics was interpreted essentially as a theory of functions and where the theory of discontinuous functions played a key role. This conception not only proved conducive to the acceptance of Georg Cantor's novel set-theoretic ideas but also served as the foundation of the Moscow school of function theory, spearheaded in the early decades of the twentieth century by Bugaev's student, D. F. Egorov [7], and perpetuated by Egorov's disciple, N. N. Luzin. This school, which also included such influential twentieth-century mathematicians as P. S. Aleksandrov, A. Ya. Khinchin, and D. E. Menshov, made seminal contributions to the advancement of measure theory and the general theory of functions of a real variable [19].

The cases of both Moscow University and the University of Chicago drive home the obvious point that the success of the mathematical endeavor in a given national context depends crucially on the process of training talented students in areas rich in interesting open questions. At its core, mathematics undeniably involves proving theorems, and these students not only learned how to carry out that creative process successfully but also embraced the belief that they should pass on their insights to a subsequent generation. As they had been trained, so should they train—this philosophy came to characterize the mathematical mission internationally in the latter quarter of the nineteenth century. Moreover, in concert with the other factors examined here, it encouraged the formation of self-sustaining mathematical communities, that is, interacting groups of people linked by common interests.

The Establishment of Lines of Communication

The formation of a community, however, also turns upon the ability of its members to communicate effectively. The time period 1875–1900—one in which telegraphy, railroad systems, steamships, and the printed word linked nations internally and with each other—witnessed the widespread creation of at least two sorts of communications vehicles dependent on this new level of mobility: the mathematical society and the specialized mathematical journal.

Although the Moscow Mathematical Society predated it, the London Mathematical Society, which first met under that name in January of 1865, served as a model for mathematical organizers throughout Europe and in the United States. Not only did it bring together mathematicians in and around London and eventually throughout England for the presentation and discussion of mathematical results, but it also published from the outset the *Proceedings of the LMS* for the further dissemination of original research [20; and 6, pp. 577–581].

Photograph courtesy of the Department of Mathematics, University of Chicago

Eliakim Hastings Moore

Photograph courtesy of the London Mathematical Society

Michel Chasles

Looking across the Channel, the Society's first foreign member, Michel Chasles, called for his own countrymen to follow the British example. In his 1870 report on the progress of mathematics in France, Chasles strongly advocated the formation of a society specifically for mathematicians, in contradistinction to one in which mathematicians represented only one of the scientific constituencies. Such an organization would serve to focus mathematicians on their discipline *per se*, its technical development as well as its broader structural needs. He also underscored the importance of liberal membership policies which would permit all mathematicians to join and participate. In his view, the élitist and exclusionary membership policies of institutions like the Académie des Sciences did little to promote the overall French mathematical endeavor. Moreover, the extant journals—the *Comptes Rendus* of the Académie, the *Annales Scientifiques* of the École Normale Supérieure, and even Liouville's *Journal des Mathématiques Pures et Appliquées*—all limited access to publication for reasons independent of mathematical quality. A French mathematical society could provide, in a sense, a free and independent outlet for the publication of its members' work [12, pp. 14–17]. The institutional void which Chasles sensed in French mathematics was ultimately filled in 1872 following the formation of the Société Mathématique de France and its *Bulletin*.

Similarly, the example of the English mathematicians, at least to some extent, informed initiatives taken in Palermo and in New York City for the promotion of research-level mathematics. Giovan Battista Guccia, a student of both Brioschi and Cremona, established the Circolo Matematico di Palermo in 1884, inspired by the examples of both the French Association for the Advancement of Science and the LMS. His objectives for the new society, however, were in some sense more outward-looking than those of either of the societies which served as his model. Guccia sought to create an organization which united mathematicians internationally at the same time that it spurred advanced work in Italy by drawing it into the wider international arena [5, pp. 51–75]. He worked to achieve these goals by actively soliciting foreign members as well as by publishing, beginning in 1885, the Circolo's *Rendiconti*. Guccia recognized the distinct advantages of an international as opposed to a strictly national posture for the overall vitality of mathematics at the research level.

The time period 1875–1900… witnessed the…creation of two sorts of communication: the mathematical society and the specialized mathematical journal.

Although the vision of his contemporaries in New York City was initially not as far-reaching as Guccia's, the founders of the New York Mathematical Society also had the general furtherance of mathematics as their primary goal when they met for the first time in 1888. In six years, this group, which had started out as little more than a mathematics club for Columbia College with a total membership of only six, had begun publishing its *Bulletin*, had assumed a nationwide purpose in changing its name to the American Mathematical Society, and had grown to well over two hundred members [18, pp. 267–268]. As with the SMF in France, the AMS came to define the primary locus of professional mathematical activity at the research level in the United States by 1900.

As this brief discussion of the establishment of national mathematical societies highlights, mathematicians during the closing quarter of the nineteenth century recognized the importance of communication both in person and in print for the advancement of their discipline. Their organization of societies further reflected their growing sense of mathematics as a profession, while their creation of new publication outlets reinforced the standards adopted for that profession.[10] By the late nineteenth century, to be a mathematician meant the same thing internationally: namely, to produce and to share the results of original research with like-minded members of an extended community of mathematical scholars both at home and abroad.[11]

An International Overview: Some Common Denominators

This brief comparative inquiry now suggests a number of the parameters crucial to the development of the mathematical endeavor during the last quarter of the nineteenth century. First and foremost, the establishment internationally of a mathematical *profession* during this time period largely—although not exclusively—hinged on changes in higher education in the various national settings. Although these changes came about often through very different sequences of events—political, financial, philosophical, pedagogical—and so for very different reasons in different national contexts, they nevertheless tended to provide increasingly conducive settings within higher education for the study and pursuit of research-level mathematics. In particular, the adoption of nationally tailored versions of the twin German principles of *Lehr- und Lernfreiheit* in various countries brought with it a redefinition of the role of the professor of mathematics, which increasingly encompassed the dual activites of teaching *and* the production of original research. Education at a graduate level thus developed in order to train students capable of realizing these two goals, and this training took place in the lecture hall as well as within the context of another keystone borrowed from the German educational system, the seminar.

As the discovery of new mathematical results came to set the standard of entry into the evolving profession, the discipline defined itself in in-

[10] *Here, we could clearly also cite many examples of journals founded during this time period which were independent of the mathematical societies formed: the* Mathematische Annalen *founded by Alfred Clebsch in Germany in 1868, Gaston Darboux's* Bulletin des Sciences Mathématiques *started in 1870, and the* American Journal of Mathematics *begun by James Joseph Sylvester in the United States in 1878, to name just three of the earlier periodicals.*

[11] *For a quantitative sense of the depth of the American mathematical research community, see, for example, [9] and [10]. [12] provides an analysis of the broader French mathematical constituency, and [17] gives some indication of the situation in Spain.*

creasingly specialized terms. Universities split their chairs of mathematics *and* physics or of mathematics *and* astronomy and even created chairs in specific mathematical areas such as geometry and higher algebra. This specialization resulted both in the sharpening definition of subdisciplines within mathematics and in an increase in the number of positions available in the field. This latter aspect of the evolution of a profession was also influenced by the establishment of new grades of instructors under the professor (*Dozenten*, *maîtres de conférences*, assistant and associate professors, etc.). As individuals sought out this graduate training, as they assumed these new positions, as they adopted these values of teaching and research, they banded together in national or broadly based mathematical societies and shared their new research through specialized journals targeted at an appreciative and understanding audience. The individual nationalization of mathematics was thus well under way by the end of the nineteenth century; and since the model emulated was largely the same in the various national contexts, this implies that the internationalization of the field was likewise in process. Perhaps no one piece of evidence supports this latter conclusion better than the fact that Zürich hosted the first International Congress of Mathematicians in 1897.

An international perspective on the development of mathematics over the period from roughly 1875 to 1900 thus uncovers a number of factors common to particular national settings which strictly nationally oriented studies tend perhaps to obscure. It also provides at least a sense of the complexity of the process of the internationalization of mathematics. Mathematicians today tend to take the international nature of the field for granted, but it is really an aspect of the discipline that has come about primarily in the twentieth century as a result of dynamic changes, especially during the closing quarter of the nineteenth century.

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