

# A Personal Perspective on Mathematics Research in Industry

*Robert Calderbank*

Why should a corporation such as AT&T invest in fundamental research in the information sciences? At a national level, why should the federal government support research in mathematics? We might look at the troubles of the National Endowment for the Arts and conclude that beauty alone may not be enough to guarantee taxpayer support. Indeed the common wisdom is that leadership in mathematics is essential to the nation's competitiveness in science and technology. This chain of reasoning might start from, say, fundamentals of probability and lead first to Shannon's Information Theory, then to the capacity of particular channels, and finally to the modes of transmission that approach these limits.

Within industry, the challenge facing Research Centers in the Information Sciences (which include mathematics, statistics, computer science, and communication theory) is to breathe life into these chains of reasoning. To be successful a center needs to be a community where fundamental research and systems innovation draw strength and inspiration from each other. This is not so very different from the challenge facing university science and engineering faculties: how to fashion a community of scholars that is

more than simply the sum of the constituent departments.

Scientific curiosity that bridges different disciplines is a great spur to innovation in industry, but it also has a broader educational value. Mathematics education is essential to an informed understanding of many aspects of society, including fairness in voting and the means that guarantee individual privacy in the information age. It is important to the long-term future of the profession that college students leave mathematics departments with an appreciation of the different ways that mathematics touches their everyday experience. It is not enough to stop at reforming the teaching of calculus.

Within industry it is essential that an Information Sciences Center have a strategic influence on the development of products and services. A center with about fifty researchers costs \$10-\$15 million per year, and this is difficult to justify on grounds of good public relations. Moreover, it is not enough to wait for someone else to define the application and to provide a technically difficult mathematical analysis in support of that vision. Bricklaying is essential to building, but architects are of greater value. Within academia it is important that research in mathematics have a strategic influence on research in other disciplines. For example, where mathematics and physics connect it is important not only to explore the equations that physicists

---

*Robert Calderbank is a department head responsible for fundamental research in mathematics and communication theory at AT&T Research, Murray Hill, NJ. His e-mail address is rc@research.att.com.*

have written down but to aspire to surprise the physics community.

What constitutes fundamental research in industry? It would be foolish to attempt a list of criteria, and any list is less interesting than a single example. So let us consider the work of Peter Shor at AT&T Research on quantum computers and cryptanalysis. The problem of factoring integers is generally believed to be difficult to solve on classical computers, and this presumed intractability is the basis for the RSA cryp-

tosystem, which is in wide use. Shor has discovered fast algorithms for solving this problem on a hypothetical machine called a quantum computer. This machine, originally proposed by theoretical physicists, is designed to exploit quantum mechanical principles such as superposition of states, and nothing in the laws of physics seems to make quantum computing impossible. Shor's work has provided the first example of an important problem that a quantum computer can solve much faster than a classical computer. This discovery has energized the many research groups around the world that are attempting to discover whether a quantum computer can be built. Mathematicians work on problems like this because of the magic of discovery, and quantum computing is as imaginative and creative as any piece of mathematics.

However, quantum computing also has ramifications that need to be explored for the good of society. If integer factorization can be done fast, then one of the pillars supporting electronic commerce will have been destroyed and new cryptosystems will have to be deployed. Applications notwithstanding, scientific research of this caliber does contribute to a corporate reputation for technical leadership, but in general it is hard to put a dollar value on scientific reputation. What, for example, was the value in the past of the Bell Labs name to AT&T? Nevertheless, public perception of scientific leadership is important not just to marketing and standards activity within North America but to doing business in countries like China and India where scientific partnership is often required.

---

*Within industry  
it falls to  
research to  
imagine the  
future...It is also  
prudent for  
mathematics  
research to  
contribute to  
products and  
services that  
will appear  
before the next  
millenium.*

---

Within industry it falls to research to imagine the future. For example, telephony today involves synchronous transmission of speech and limited volumes of data at kilobit rates. The expectation is that the network that prevails in the next millennium will come to resemble more the Internet with asynchronous transmission of sound and video as well as text files at megabit rates. The impact on society is likely to be enormous; for instance, the retail industry is likely to be transformed as electronic commerce calls the economics of the shopping mall into question. Mathematics research can play a strategic role in the definition of the next network. Algorithms and optimization are essential to network design; even a 5 percent improvement in the layout of a new \$3 billion network pays the rent on an Information Sciences Research Center for more than a decade. Probability, large deviations, and queueing theory are critical to figuring out how to quantify quality of service and how to charge for variable bandwidth—in short, to the design of new services. The mathematics of cryptography is essential to any guarantee of individual privacy and to the feasibility of electronic commerce. And most important of all, in telecommunications the penalty for failing to imagine the future is that you go out of business. Even companies such as Microsoft, which in their early days prospered without investing in research, are now forming research divisions.

Of course it is also prudent for mathematics research to contribute to products and services that will appear before the next millennium. Again we consider a particular example, the V.34 modem standard for point transmission over telephone lines at rates up to 34kb per second. Here data is encoded by a finite-state machine that selects points from a finite-dimensional lattice, and mathematics also enters the algorithms for shaping the signal constellation and for mitigating intersymbol interference. Modem patent revenue may amount to \$10–\$15 million after all the lawsuits have played out, but there can be more important strategic reasons for establishing a standard. Here it created a new commodity market for modem chip sets, and AT&T-ME (Microelectronics) was able to significantly increase its market share.

Fifteen years ago high-speed implementation of complex signal processing algorithms was impractical. Today it is an imperative, and there are great opportunities, for example, in personal communication systems and wireless local access. Here classical analysis has a great deal to contribute, since parts of harmonic analysis are only a small step away from these applications. There are also mass-market opportunities for companies that combine strength in signal processing with strength in integrated circuit design.

One example is magnetic recording, which AT&T-ME has grown into a \$250 million business over the last three years. In this case, the chief technical officer of AT&T-ME, Mark Melliar Smith, wanted to create a strategic partnership involving AT&T-ME, selected hard disk manufacturers, and research in the mathematical sciences. The business advantage derived from sharing innovations in coding and signal processing with these customers is that of a preferred supplier.

It is impossible to imagine a successful Research Center in the Mathematical Sciences that does not include statistics. Important applications include statistical methods for improving the yield of manufacturing processes. Here statistical analysis of patterns of defective chips on silicon wafers or of particle count data in cleanrooms can expose the root cause of manufacturing problems. The fact that companies are incredibly secretive about yield is a good indicator of its importance. Within the telecommunications industry, a new challenge for statistics research is to make sense of massive data sets—for example, the 200 million telephone calls carried every day by the AT&T network. How to visualize this data, how to compute with it in real time—in short to understand how customers are using the network. The strategic purpose here is to contribute to the design of new services. Another purpose is early recognition of telephone fraud, which can save customers significant aggravation and the corporation tens of millions of dollars each year. Statistics research on large data bases is a truly interdisciplinary endeavor that requires leadership in both statistical methodology and computer science. The scarcity of students who can demonstrate leadership in both these fields is something we hope will improve with time.

Industry and government have sound business reasons for investing in research in the mathematical sciences. The value of this investment is strongly coupled to interdisciplinary research. It would benefit the profession as a whole and graduate students in particular if this idea were to take root in more university mathematics departments.

### **Acknowledgment**

I would like to thank Don Lewis and Andrew Odlyzko for their extensive and valuable comments on earlier drafts of this article.