Letters to the Editor

On Summer Meetings

I am writing in response to Prof. Friedlander's editorial (regarding the summer meeting) in the December 1995 issue of the *Notices*. In particular, I would like to briefly reply to the questions she posed in the last paragraph.

The summer meeting should definitely be continued. It offers faculty (such as me) that are not situated in research institutions the semiannual opportunity to keep abreast of new developments in the discipline. The format should be examined and new ideas tested for their effectiveness. I would like to see several five-day courses/seminars that are designed to introduce mathematicians to fields of interest other than their own specializations. (Texts and manuals should be included for an additional fee.) These courses should resemble some of the seminars at the various summer institutes but be accessible to novices in their presentation. The instructors should be chosen from the best available.

Both the annual and summer meeting should be joint with SIAM. Considering the growing concern expressed in the *Notices* about the paucity of academic positions for new Ph.D.s, I wonder why this wasn't considered earlier.

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(Received December 14, 1995)

Why Do We Appreciate Mathematics?

Though I agreed with almost everything while reading the article "Why Are We Learning This?" by Roman Kossak (*Notices of the AMS*, vol. 42, no. 12), still at the end I felt some dissatisfaction. The feeling was similar to the one I felt when (several times) I taught calculus using one or other of the standard (e.g., American) calculus texts. In most cases, these texts were written very carefully, the authors clearly considered the needs of a large and heterogeneous student population, so the result was usually attractive and convincing. Still I felt that something was missing. Should we put more emphasis on showing students that mathematics is attractive? Certainly we should. In the age of TV, videoclips, videogames, etc., when there are so many tools that can distract our young people from reading, thinking, and studying, we should certainly find new means to revitalize the teaching methods of our several-thousand-year-old profession. Among other things (like calculus reform), it is certainly a good idea Dr. Kossak suggests: to offer special, carefully planned mathematics appreciation courses that emphasize the extremely wide and powerful range of mathematics as it exists in our age, together with its rich history and its role as an indispensable tool in many applications. However, a better use of existing courses (e.g., calculus), taken by hundreds of thousands of students each year, is even more important to promote the same goal.

To find out what improvement can be achieved in existing courses, I think first we should ask ourselves who have chosen mathematics as a profession, Why do we appreciate mathematics? I hope that many of you share my answer: the first and foremost reason is that mathematics has an extremely solid logical structure, unparalleled by any other science or by any other result of human creativity. From ancient times, many generations of mathematicians have worked on extending its range, filling in its gaps, establishing mathematical truth beyond doubt. Though this program clearly can never be completed, still the mathematics of our time has a very complex and solid logical structure that joins essentially the whole into one great entity. This characteristic of mathematics is inseparable from its applications: mathematics is applied because it is able to supply extremely reliable models and methods nothing else can offer.

Here I would like to convince the reader that an introductory calculus course (reformed or not) is especially suitable to reveal the logical structure of mathematics for average students. To clarify my point, I am not advocating endless lecturing on theorems of advanced calculus in an introductory course. I fully realize that this would be illusory in the case of the typical, heterogeneous classes. But there is a beautiful logical "backbone" that leads from the properties of natural numbers to multivariable calculus and which is only partly shown by most standard calculus textbooks and courses. If you show only fragments of a "backbone", your students may miss the point. I believe that the "solution" that many textbooks give in cases of very important but theoretically involved theorems like the max-min theorem or the Riemann integrability of continuous functions, namely, that "the proof can be found in advanced calculus texts", is not helping students to understand and appreciate the logical structure and the reliability of mathematics. So what could be done? As is often the case in real-life problems, there are conflicting facts here, and it is not easy to find a nearly optimal compromise. My proposals are as follows.

(a) A good calculus textbook should carry all of the "backbone" theory (and not only fragments of it) that leads from the properties of natural numbers to the theorems of analysis. The most difficult notions, definitions, theorems and proofs can be placed into optional sections, but the order of presentation should follow a path along which notions and theorems are built on each other. All these should be discussed in "the easiest" possible way. Proofs of repetitive nature can be omitted, difficult proofs could be replaced by the proof of a typical special case, but in my opinion no essential component should be missing. Since the size of a typical calculus textbook is around one thousand pages, my estimate is that all this would result in only a small percent increase in size. It seems good to me if every student may see, and the strongest students may even read, the whole story. The strong ones can grasp its essence, and in this way may serve as sort of "witnesses". Also, such a first encounter with the essence of calculus may encourage our best students to further study of mathematics and may better prepare them for advanced calculus.

(b) A good calculus teacher should emphasize the essential features of mathematics, how theorems are logical, necessary consequences of the axioms. He or she should mention that the specific order in which the material is presented is not arbitrary, that new things are solidly based on old ones all the time. She or he could also reveal the logical structure without giving the difficult details in class, referring the students to the text.

(c) A good calculus exam should ask about the student's understanding of the logical structure of the material too, how concepts and theorems are built on each other, without asking the difficult details.

Certainly there exist textbooks and professors that follow similar methods in case of introductory calculus courses. But there are many that do not, and I believe that the proposed changes could bring at least a small improvement in students' appreciation of mathematics.

Tamas Szabados
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(Received January 24, 1996)

Mathematics Education and Policy

A famous mathematician told me confidentially, "Mathematics education is working towards destroying the underpinnings of mathematics."

California State University, Northridge, is the only four-year college in the San Fernando Valley, which has a population of almost three million people. As a professor in the mathematics department there, I have witnessed the loss of 350 math majors over the last ten years with the concomitant proliferation and utilization of mathematics education ideas. Students are more poorly prepared than ever before. It is a mistake to believe this will change by further watering down the content of courses such as calculus and linear algebra. I don't accept the notion that we have no choice but to let education experts dominate what we, as mathematicians, will be teaching in the future.

Professor Dubinsky maintains in the February 1996 *Notices* that fields such as statistics, computer science, applied mathematics, and even functional analysis experienced resistance from the mainstream mathematics community at their inceptions. Considering how valuable these fields are now, Professor Dubinsky suggests that the field of mathematics education should be given the same chance to contribute to mathematics as these other fields have.

Mathematics education does not belong in the same company as statistics, computer science, applied mathematics, and functional analysis. If these disciplines had failed to produce useful and important results (which of course didn't happen), there would have been no widespread negative effects on the education of students. By contrast, educational theories can and do turn into educational policy, with negative results.

Here are some examples. At the instigation of "education experts", it was decided in California about nine years ago to alter the way basic language skills are taught to elementary school children. A teaching method called "Whole Language Learning" required students to learn to read and write by exposing them to stories and literature. The method deemphasized spelling and phonics. Whole Language Learning was adopted by many elementary schools in the state of California. As reported in the lead article of the *L.A. Times*, September 13, 1995, the results have been disastrous. In a follow-up article in the January 22, 1996, edition, the *Times* reported, "60 percent of California fourth-graders are not able to read at basic levels." In response to this debacle, California has mandated that elementary schools teach spelling and basic skills. In the January 22 article, it was reported that schools were not complying. More recently still, the *Times* ran an editorial ("Reading Lessons That Won't Wait") insisting that the schools immediately return to teaching phonics and basic skills. It will not be easy to undo this mess.

The *L.A. Times* has been tracking problems in math education as well. In its December 19, 1995, issue an article appeared entitled "Parents Skilled at Math Protest Curriculum". The article reported that groups of parents in areas such as Palo Alto (including Stanford faculty members), La Jolla, and Torrance, where there are large numbers of scientifically trained people, are protesting the "trend of getting away from algebra and solving equations and moving to a more touchy-feely, guess and check method."

In 1982 the math education people in my department (at that time) decided to drop our advanced calculus course from the required list of classes for students in our secondary education mathematics degree program. This course did little beyond justifying the algorithms in first-semester freshman calculus. Surely no luxury for future high school calculus teachers! It was removed as a requirement to attract more students into this option, and the experts believed advanced calculus was too difficult for these majors. Ironically the number of majors in this area has remained more or less constant and quite low.

My department is currently debating whether to adopt the *Harvard Calculus* text for our main calculus sequence—the sequence for math, science, and engineering majors. Encouraged by the momentum of calculus "reform", many of my colleagues are satisfied that future teachers graduating with bachelor's degrees in mathematics from our department will have never even seen the definition of a limit. Future public school children will bear the burden of these mathematics education ideas.

Dubinsky wants the field of mathematics education to be judged by what it has accomplished. I agree. The "Harvard Calculus Movement" has taken out what little theory existed in elementary calculus. Linear algebra reformers have reassigned the definition of a linear transformation as an optional topic in beginning linear algebra courses, emphasizing row operations on matrices. Education experts seem to ignore the fact that most math majors develop their skills gradually and that they need at least some exposure to abstraction early in their training. If students don't get any rigor in calculus or beginning linear algebra, it will come as a real shock in later courses.

In addition to its negative contributions, there are general principles which lead me to be suspicious of the field of mathematics education. By its very existence, it tends to overemphasize pedagogical issues in the context of the poor performance of American students. Social, political, and environmental factors (e.g., underpaid teachers, large classes, low levels of lead poisoning in poor neighborhoods) are consequently deemphasized as contributors to our education problems. With an active corps of professionals in mathematics education, politicians are likely to throw money in their direction instead of addressing the larger problems affecting our students. Moreover, as academicians, mathematics educators, like all academicians, are under pressure to come up with new ideas, e.g., to carry out research (over 25 percent of the total number of presentations—which exceeded 1,200—at the last annual joint meeting of the AMS and MAA were devoted to areas of mathematics education). An individual's stature in this field is surely increased if his/her new education ideas are actually implemented in teaching institutions. Whether good or bad for the students, there are necessarily careerist pressures to change the status quo. Considering how rare good ideas are, this is a dangerous situation.

Before further damage is done, I urge the mainstream mathematical community to get involved in these matters. Proposed changes in educational policy, both national and local, need to be scrutinized by practicing mathematicians.

Jerry Rosen
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Increasing the Need for Exposition

It is particularly important now, in these days of budget cuts and disfavor of basic research, to work to increase the understanding and appreciation of mathematics. Perhaps this is why the mood seemed quite tense at the recent JPBM panel discussion on mathematics coverage in the media (Joint Meeting, Orlando, 1996). Questions at this session all contained the implicit statement that mathematics coverage was disproportionately low and put the panelists on the defensive. The responses from the journalists on the panel were extremely good, brutally honest, and can hopefully help us solve what so many see as a problem.

For example, in her explanation of why mathematics does not receive much coverage in the *New York Times* science section, Gina Kolata said that one problem was the fact that mathematics research is rarely discussed in weekly science journals such as *Science*, *Nature*, the *New England Journal of Medicine*, and the *Journal of the American Medical Association*. Since I am a member of the American Association for the Advancement of Science, which publishes *Science* and which has a mathematics section, I took particular notice when she mentioned low mathematics coverage in this journal as one of many factors affecting coverage in the broader media. In fact, *Science* is read not only by science journalists looking for stories but also by officials in government agencies responsible for funding science and by scientists in other fields. If the coverage of mathematics there is truly disproportionately low, then the effects could be more serious than lack of media coverage. In this letter, I will argue that mathematicians can and should attempt to alter the coverage of mathematics in *Science*.

In the years that I have been reading *Science*, there have been a few published research articles which could be considered examples of mathematics research. However, these articles were not written by mathematicians (they were written by chemists, computer scientists, and engineers). Furthermore, the subject of each of these articles is optimization of algorithmic processes. There is certainly nothing wrong with these articles, and I don't mean to be criticizing their authors; I am merely concerned that readers of the journal will not be aware that there is also mathematical research of interest being conducted by mathematicians and that the "Traveling Salesman Problem" is not the only question being investigated. The coverage of mathematics in the "news" section of *Science* tends to focus on mathematics as a school subject. When mathematical research is discussed, the "Traveling

Salesman Problem" and other easily explained puzzles are once again the center of attention. It would not be fair of me to create the impression that other areas of mathematics are never discussed in the news section; I only wish there were more. (*Science* also contains a book review section which provides relatively frequent and interesting reviews of mathematics books.)

Certainly, most mathematics research of even the highest quality is not appropriate for publication in *Science*. However, journals such as the *Journal of Mathematical Biology*, *Communications in Mathematical Physics*, and the many SIAM journals contain articles by mathematicians which I could imagine being published there.

As for the news section, until and unless some formal system is established, we must take it upon ourselves to contact the *Science* news staff when a development has occurred in our field of research. I know, for instance, that recent developments in mathematical physics can be found on the Internet at the many e-print locations or in John Baez's "This Week in Mathematical Physics". These developments must be relayed to the news staff and stated in such a way that they will understand the importance and interest of the result. This is particularly important, since, quoting Kolata, mathematics articles have to "compete with more easily understood articles in other fields." Unlike the research articles, the news articles probably don't need any immediate scientific application to be publishable. Take a look at the news articles published in other fields for an idea of what is involved: they don't have to be tremendous results worthy of a Nobel prize, they can be in obscure fields of research unfamiliar to most *Science* readers, they only need to be interesting and timely. (See Ian Stewart's book *The problems of mathematics* for many examples of conveying the interesting aspects of mathematical results in nontechnical terms.)

In summary, it could benefit the entire mathematical community to increase and improve the coverage of mathematics in widely read journals such as *Science*. I believe this could be achieved if mathematical research articles were submitted to *Science* for publication, if mathematicians kept the news editors informed about interesting developments in their fields, and if the members of the mathematics section of the AAAS worked to ensure that the magazine presented a more diverse, less "watered-down" picture of mathematical research.

Alex Kasman
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On the Review of *A Mathematician Reads the Newspaper*

Reading Gina Kolata's review of my book *A Mathematician Reads the Newspaper* in the March issue of the *Notices* reminded me of an incident related to me by a friend. He went into a large bookstore looking for a copy of Wittgenstein's *Tractatus* and, not finding it in the philosophy section, asked a clerk where it might be. He was nonplussed to discover that the clerk had placed it in the section on cars and trucks! Like the store clerk, Ms. Kolata seems to take titles a little too literally. *A Mathematician Reads the Newspaper* is not a handbook for journalists or mathematicians but an admittedly quirky, somewhat oblique approach to the myriad ways in which mathematical and quasimathematical notions underlie topical issues ranging from crime and cults to sex and the Senate. As a self-described "inveterate reader of book reviews", Ms. Kolata must realize that her effort is an outlier among reviews of my book. Newspapers as distinguished as the *Washington Post*, *LA Times*, and *NY Times* and journals as disparate as *New Scientist*, *Chance*, and the *Journal of the American Medical Association* have reviewed it quite favorably. Hundreds of letters and very robust sales are further testament to its appeal. I request anyone with a possible interest in the book to get a second opinion on it, preferably their own.

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Editor's Note

The April issue of the *Notices* carried a Letter to the Editor from William J. Davis which stated that e-MATH, the AMS World Wide Web home page, had no link to the listserv for discussion of calculus reform issues. In fact, the calculus reform listserv has been available on e-MATH for some time now; from the top-level of e-MATH, follow the "Mathematics on the Web" link and then go to "List Servers". However, at the time Davis wrote, the calculus reform listserv was not linked to the Committee on Education home page. That link has now been added.

About the Cover cont'd.

The colors at a point are indexed by the number of iterative applications of F, starting at the point, needed to escape from a specified region that contains the attractor. This image also represents the attractor of an iterated function system. For a more detailed discussion see *Fractals Everywhere, Second Edition*, Academic Press, 1993. The image was created by Michael Barnsley and John Herndon. Copyright Michael Barnsley 1988, 1993.