Mathematics: A Century Ago—A Century from Now

John Ewing

I am not a historian, but I believe in history. History lends perspective to the way we view ourselves and our future. Oliver Wendell Holmes said:

"When I want to understand what is happening today or try to decide what will happen tomorrow, I look back."

Of course, Holmes also said: "A page of history is worth a volume of logic"…even wise men make mistakes. If we want to understand our profession today, if we want to guess what will happen tomorrow, we ought to study yesterday. It's almost a new century, and so, in a reflective mood, I decided to look back at the turn of the last century for our yesterday. The aim of this article then is to sketch mathematical life in America as it was near the start of the twentieth century and to use that as a background for a peek at what it might be like a century from now. I warn you in advance that I want to draw a moral from this sketch. I am therefore heeding the advice of a real historian, Herodotus (450 B.C.):

"Very few things happen at the right time, and the rest do not happen at all. The conscientious historian will correct these defects."

In preparing this article, I made all the necessary corrections.

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America a Century Ago?

What was happening one hundred years ago? In 1895 America was still in the throes of severe depression, which began two years before. The Supreme Court ruled 5–4 that the income tax was unconstitutional. A new enterprise, Sears-Roebuck, was struggling: A. C. Roebuck sold his interest for $2,000, getting out while the getting was good, or so he thought. The Underwood

Typewriter Company was founded to market a new machine: a typewriter that actually allowed the typist to see what was being typed. A young man named Marconi transmitted a message to his brother on the other side of a hill. People thought it a hoax because there was nothing connecting them. An enterprising young farmer ran boiled wheat through his wife's dough machine and baked the thin film to produce edible flakes. A strange idea, but people bought 10 oz. bags for 15 cents. The farmer was John Harvey Kellogg.

All those things took place in 1895.

The American University

The American university in 1895 was emerging from a quarter century of change that had shaped it into a new form. That new form would continue throughout the twentieth century. It was a frantic period of growth, and universities madly sought both students and money.

During that growth, there was an earnest public debate about the purpose of a university. In 1906 the editor of *Outlook* magazine summarized the popular view: The English university, he said, revolved around culture, the production of gentleman aristocrats. The German university found its life not so much in culture but in scholarship, in erudition, in the production of scholars. But in the American university the emphasis often was neither on culture nor scholarship, but on service, on the preparation of young Americans for active lives of service [7, p. 356].

New university presidents made it their mission to sell their universities to the public, and to a large extent the sales pitch was utility. In 1888 David Starr Jordan, then president of Indiana University and soon to be president at Stanford, gave speeches enticing prospective students. Among the reasons to go to college: the benefit to society, the virtue of hard work, the financial worth of a degree [8, p. 342].

Utility was bundled with another idea, the elective system. A prescribed curriculum, especially a traditional one in the classics and philosophy, did not lend itself to the arguments of utility and service. So during this period the elective system was born—students and faculty were encouraged to choose courses and topics. By 1900 Harvard had virtually no required courses; Cornell had switched to an almost entirely elective system, as did Wisconsin, Michigan, and Yale. The greatest proponent of the elective system, Charles W. Eliot of Harvard, waxed grandly in 1908:

> [Because of the elective system,] teachers and students alike are profoundly moved by the desire to serve the democratic community.… [8, p. 118]

A few held out—Princeton was among them.

The elective system had many consequences. Eliot boasted that it made scholarship possible not only among undergraduates but among graduates and faculty as well. It promoted specialization; teaching special courses required experts in the field, and "fields" might be narrowly defined. The old prescribed course of study was elementary, shallow, superficial. Election permitted faculty to become deep, to indulge their own interests.

This in turn promoted departmentalization [7, p. 400]. In 1893 the biology department at the new University of Chicago split into five: zoology, botany, anatomy, neurology, and physiology. Everywhere, departments of modern languages became departments of German or French or Spanish. Departments of history became departments of European history and American history and political science. Departments of mathematics sometimes became departments of pure mathematics and departments of applied.

In part it was the elective system that led to the notion that a university was a place for research. The idea was reinforced by the German model, but it was characteristically American, based on utility.

Professional requirements changed: by 1893 some amount of graduate work was required to hold a permanent position at nearly every prominent institution. By 1900 the Ph.D. degree became almost universally mandatory. From there, it was a short step to an insistence upon the continued publication of scholarly studies. In 1901 the president of Yale made a statement that ought to sound familiar: Promotion at New Haven, he decreed, depends upon "productive work" which gives the professor "a national reputation". Stanford began publishing an annual list of its faculty's output for each year. Presidents kept index cards for every faculty member with precise lists of publications and talks. It was during this period that the term "publish or perish" was first used. Research had become an essential part of university life [7, p. 404].

There was a continuing debate about the balance of teaching and research. Abraham Flexner, who came down squarely on all sides of the issue, wrote an article in the *Atlantic Monthly* [7, p. 403]. He told the story of asking a college dean who was the best teacher in his institution. The dean named a certain instructor.

"What is his rank?" he asked.

"Assistant professor."

"When will his appointment expire?"

"Shortly."

"Will he be promoted?"

"No."

"Why not?"

"He hasn't done anything!"

Service to the community, utility, electives, specialization, research. It was a logical progression and a major theme in the growth of American universities.

Of course, this is only a partial picture. While universities were marching forward towards research, they were also spending large sums of money on boat houses and gymnasiums. They needed to attract more students. Was there ambivalence about the role of the university? At the turn of the century, President Eliot at Harvard threw away many of Harvard's books rather than spend money on their storage. Columbia University built an ornate new library, mainly designed for holding receptions to honor benefactors. At the singularly research-minded University of Chicago, in 1897 in existence only five years, President Harper complained loudly that his professors were offering entirely too many graduate courses and they had better stop [8, p. 178].

American Mathematics

At the turn of the century, mathematics worldwide was healthy and vigorous and savoring its triumphs—a bit like the present. In the previous quarter century, some major problems had been solved—the transcendence of e and π, the prime number theorem, the finiteness theorems in invariant theory. The German and French mathematicians, and to a lesser extent the British, were in a period of frenetic activity, with Hilbert and Poincaré vying for prominence.

There were many analyses of what everyone viewed as a golden era in mathematics: from 1900 until 1905, the *Bulletin* published a number of articles on progress during the nineteenth century, including a general summary by James Pierpont, who delivered an address to the International Congress of Arts and Sciences in St. Louis. Pierpont's article begins:

> The extraordinary development of mathematics in the last century is quite unparalleled in the long history of this most ancient of sciences…entirely new [fields] have sprung up in almost bewildering profusion, and many of these have promptly assumed proportions of vast extent [*Bulletin*, 1904, pp. 136–159].

Mathematicians were feeling good. And mathematicians in America were taking part in the transformation of universities, accommodating many new roles—utility, service, specialization, research. The elective system had an especially profound effect on mathematics. Before this time, almost all mathematics instruction was broad and shallow, often done by faculty as their obligatory extra course (a bit like coaches in high school who often seem to teach algebra and geometry courses). In 1888 Florian Cajori conducted a survey. Of 118 college mathematics teachers, 73 reported teaching outside mathematics, and 32 of those taught outside the sciences altogether—art, music, languages, and Bible. By the turn of the century, mathematicians taught mathematics.

William Duren wrote a survey of American mathematics for the centennial of the AMS [3, part II, p. 399]. Looking back, he laments the loss of the traditional importance of mathematics, which had been, along with Greek and Latin, part of the required curriculum for many years. The evidence doesn't bear this out: mathematics grew, the profession prospered, and there was greater concern about teaching—all because of the elective system.

The Society

Specialization and research had two consequences: professional societies and professional journals. They arose naturally and spontaneously. There was a new Athletic Research Society, for example, holding annual meetings of coaches and athletic directors to present (and to publish) research papers on athletics [7, p. 403].

In the spring of 1887 an enthusiastic Thomas Fiske was a second-year graduate student at Columbia College. For some reason, his advisor insisted he leave to spend at least six months abroad. He traveled to Cambridge, England, for a semester and immersed himself in the mathematical atmosphere. He later recalled:

> On my return to New York I was filled with the thought that there should be a stronger feeling of comradeship among those interested in mathematics, and I proposed to my classmates and friendly rivals, Jacoby and Stabler, that we should try to organize a local mathematical society [3, part I, p. 13].

They mailed a general letter. Six people responded to form the New York Mathematical Society. By the end of 1890 there were twenty-three members. Things were right. In 1891 they launched the *Bulletin* to record news and mathematical events as well as articles. The number of members had to grow, and Fiske launched a campaign increasing membership from twenty-three to two hundred in a matter of months. In 1893 Felix Klein visited the Mathematics Congress at the Chicago World's Fair and met with the members of the New York Mathematical Society in a special session [6, p. 295ff]. By 1894 the Society changed its name to the American Mathematical Society. By the turn of the century, the Society claimed hundreds of members across the country and continued to grow in spite of its high dues—$5 per annum, nearly ten times those of the equivalent German Society.

Journals

By 1898 members of the Society realized they needed a research journal in addition to the *Bulletin*. The natural course of action (and the easiest) was to assume partial control of the *American Journal of Mathematics*, founded by James Joseph Sylvester and published for a quarter of a century by the Johns Hopkins University. The Council passed a resolution:

Simon Newcomb

> It is recommended that the American Mathematical Society offer to the Johns Hopkins University the following plan of cooperation, for the purpose of enlarging and improving the *American Journal of Mathematics*...[1, p. 56]

The "plan of cooperation" was simple: the AMS would take over editorial control and in return contribute a small sum of money towards yearly expenses. It didn't sound like much of a deal. Because the president of the Society, Simon Newcomb, was on the faculty at Hopkins, people thought it might work.

The negotiations quickly broke down; Newcomb accepted the money but said that editorial control had to remain with Johns Hopkins. Besides, he said, the appearance of the name the *American Mathematical Society* on the title page would be distasteful. This didn't sound like much of a deal for the AMS. Younger members of the Society decided to establish a new journal, but J. E. McClintock—a charter member of the Society, a past president, and an eminent mathematician—objected to any journal that would compete with the *American Journal*. Maxime Bôcher had a stroke of genius: Wasn't the Society supposed to publish its transactions? he asked. Certainly, replied McClintock, transactions were acceptable. And so a second research journal was established. The *Transactions* published its first issue in January 1900.

The *American Mathematical Monthly* had existed for seven years in 1900. It was founded by a man, not an organization, and it was struggling to find an identity. The man who created the *Monthly*, Benjamin Franklin Finkel, was a school teacher with no editorial experience. After moving to a preparatory school in Missouri, he decided to publish a new journal which he called, somewhat grandly, the *American Mathematical Monthly*. Finkel had the ambition to publish a journal devoted solely to mathematics, catering to the needs of teachers of mathematics, largely because of the deplorable condition of high school teaching. The description of the *Monthly* on the first page began: "Most of our existing journals deal almost exclusively with subjects beyond the reach of the average student or teacher of mathematics...The *American Mathematical Monthly* will endeavor to reach the average mathematician...." [*Monthly*, 1894, p. 1] Even then, mathematicians complained that articles were too specialized for general interest.

Finkel's plan was at first a failure. Few high school teachers subscribed. Nonetheless, he had the genius to enlist the help of college and university faculty to provide both funding and written material. The *Monthly* began to thrive, directed towards teachers in colleges and universities rather than high schools.

The *Annals of Mathematics* was founded by Ormond Stone at the University of Virginia in 1884. At the turn of the century, it was transferred from Virginia to Harvard. Its purpose, the *Bulletin* of 1899 reported, was not to compete with the *American Journal* or the new *Transactions*. The *Annals* was meant to appeal to the general mathematical public rather than to specialists; it would publish short expositions and would avoid highly technical articles. The writer complained that there was a scarcity of good exposition. There was great hope that the new *Annals* would entice authors of such expositions.

It seems to me we're still complaining...and hoping.

Libraries

In 1900 mathematics libraries faced a constant struggle. Departments were often brand new and lacked back issues of basic journals. The great depression which began in 1893 had left many institutions short of funds; library budgets were easier to cut than salaries (although those were cut as well sometimes). Mathematics chairs complained of inadequate budgets and used arguments that sound familiar today.

The Indiana University departmental report from 1893 gives the mathematician's laboratory argument: "In our advanced work we greatly feel the need of complete sets of the leading mathematical journals. These are as essential in mathematical work as apparatus and specimens are in scientific work."

The report from 1895 shifts the argument to emphasize students: "There are more students doing advanced work than ever before. This increased demand for advance work makes necessary better library facilities. …Mathematical periodical literature is to the advanced student in mathematics what the modern well-equipped laboratory is to the student in science."

And the report from the next year, 1896, sounds pathetic: "Our work is progressing as satisfactorily as we could wish with our present equipment. We have need of the standard journals and periodicals for our students; we therefore beg of you again a liberal appropriation for the purchase of a few standard journals and current periodicals.…Each year the department offers a course in solid analytic geometry and theory of surfaces. Any instructor is handicapped in his presentation of this subject without the use of models. A complete set of Brill's plaster models would be exceedingly valuable to the department."

By 1899 they returned to the laboratory argument: "Seminar work is a new feature and…emphasizes the necessity of better library equipment. It would seem that this is not unreasonable when it is remembered that our library is also our laboratory.…[The report adds a new plea.] We believe that nothing would do more to encourage and inspire our students than to give them the opportunity of listening to lectures…by well-known mathematicians of other institutions. The expense attached to such a course of lectures would be very small. We therefore ask that $50 be appropriated for this purpose.…"

Money for libraries, equipment, colloquia. This aspect of professional life does not change.

The German Connection

At the turn of the century, American mathematics had sturdy ties to the German mathematical community. Many leading mathematicians had received their Ph.D.s in German universities; those who had not often traveled to Germany for a year or two. Felix Klein, first at Leipzig and then at Göttingen, was a favorite of the young American mathematicians: six of his students became presidents of the AMS; thirteen served as vice-president [6, p. 259]. In 1905 the AMS reported that 10 percent of its members had received Ph.D.s from German universities; 20 percent had done graduate work in Germany.

Why was the German connection so strong? In part, the elective system was the cause. The German *Lernfreiheit* system, in which students could choose lectures and seminars at will, helped to make it comfortable for American students. In part, Klein's magnificent lectures and the carefully crafted seminars, in which students learned not just about mathematics but its presentation as well, attracted students.

But there were more mundane causes [8, p. 129]. Economic conditions in Germany had dramatically decreased the number of German students; foreign students were needed, especially at the graduate level. (Sound familiar?) And Germany was a bargain. In 1890 it was estimated that a year of study in Germany was cheaper by a third than a year at Hopkins, Harvard, or Cornell—and that included travel!

Mainly, however, a German degree was prestige. "One goes to Germany still a doubter as to the possibility of the theoretic life," read an article in *Scribner's* magazine (1891). "One returns an idealist, devoted for the time to pure learning for learning's sake" [8, p. 129].

At the turn of the century, these were the dominant features of American mathematics: a changing vision of universities, the emerging AMS, new journals, a close connection with German mathematics. All these ideas came together in one man, who shaped mathematics in America for much of the next century.


E. H. Moore

E. H. Moore

Eliakim Hastings Moore was born in Marietta, Ohio, in 1862. His father was a Methodist minister, his grandfather, a congressman. Moore went to Yale. He graduated valedictorian in his class in 1883 and two years later received his Ph.D. under Hubert Newton. The title of his thesis was a bit vague: "Extensions of Certain Theorems of Clifford and Cayley in the Geometry of *n* Dimensions."

Moore's advisor, Newton, urged him to spend time in Germany and lent him money to do so. He traveled to Göttingen and then on to Berlin, where everyone, including Moore, worshipped Weierstrass and Kronecker. Moore returned to America in the next year and soon settled into a permanent position at Northwestern University, where he became associate professor. By

1892, seven years after his degree, Moore had five papers and one research announcement—hardly a rocketing career.

Now the same events that drove the expansion of mathematics swept Moore into prominence. In 1892 William Rainey Harper, the president-elect of the new University of Chicago, asked Moore to be the acting head of the mathematics department. Moore accepted and recruited two others, Bolza and Maschke (students of Felix Klein). The next year, in 1893, the Columbian Exposition in Chicago held a Congress of Mathematicians, to which Felix Klein came as the Prussian representative. The new Chicago department and Moore were the natural hosts. Shortly after, Moore organized the Chicago section of the American Mathematical Society, serving as its first chairman, and was then elected vice-president of the Society. When negotiations were held to take over the *American Journal*, Moore was involved. Indeed, he was one of the agitators urging a new journal, and he became the first editor-in-chief of the *Transactions*. By the end of that year, in 1900 at the age of 38, E. H. Moore was elected president of the American Mathematical Society.

Whether Moore was swept along by these events or caused them isn't certain. What is certain is that E. H. Moore dominated mathematics in 1900.

Moore represents one face of mathematics—it's the face we see when we look back at history, because we usually view history with Carlyle, as the biography of great men. But mathematical life in 1900 comprised two cultures, and concentrating on only one distorts our view—and misleads us about our own present.

David Rothrock

David Rothrock

When Klein left Leipzig in 1886, he arranged for his friend Sophus Lie to take his place. Lie had six American students in the next dozen years, including some well-known mathematical figures [6, p. 234ff]. One of the other students of Lie, however, was a young mathematician from Indiana. In a strange way, his academic career explains much about American mathematical life of one hundred years ago.

David Rothrock was born in a small Indiana town in 1864, one state farther west and two years later than E. H. Moore. After completing his public education, he began teaching in a one-room country school house at the age of 17. He was well liked, eager, enthusiastic. In the midst of teaching, he managed to attend Indiana University part-time, majoring in mathematics. "Majoring" was a new idea, recently introduced at Indiana by its president, David Starr Jordan. Rothrock finished his undergraduate work in 1892 and, at the age of 28, immediately became an instructor in the department.

In that year, the department made a report to the board of trustees, pleading for reunification of the departments of pure mathematics and of applied mathematics, and recommending some appointments:

> …the teachers of mathematics in the university have all been graduates of Indiana University and have understood the conditions surrounding the university and the needs of the students.…In order that these conditions may be preserved…we recommend that David A. Rothrock be made instructor in mathematics at a salary of $800. Mr. Rothrock has given excellent satisfaction as a teacher and is one of the best students of mathematics we have had.

Other reports of that year were interesting, by the way. The English department complained in their report about class size: "The correction and personal criticism by the teacher is to the English student what the personal blackboard work is to the student in the Department of Mathematics, and thus there should be a section for every twenty students."

And in that same year, the president filed a report to the board, grumbling that the faculty were leaving due to low salaries, that they needed fewer hours teaching and more time to do scholarly work, that they needed more laboratory equipment, and that the library budget was disgraceful.

Rothrock took leave to attend the new University of Chicago in 1894 and again during the summer of 1896, studying under Moore, Bolza, and Maschke. In 1898 he spent a year at Leipzig, receiving his Ph.D. under Lie. The title of his thesis: "Invariants of the Finite Continuous Groups of the Plane". He returned to Indiana in 1899 and took up a heavy load of teaching, tutorials, and administration.

From the start, David Rothrock was a master teacher. This is from the department's annual report from 1895:

> No better teacher than Assistant Professor Rothrock has ever been in the department. He inspires his students to their best efforts and is greatly beloved by all of them. …He has had charge of a number of advanced classes this year, and in all of them he has done superior work.

Rothrock was one of four spectacular teachers in the department, and together they nurtured students into the undergraduate curriculum and, at least for some, into mathematics. They taught three to five courses each term, three terms per year. They were effective. In 1930 more members of the American Mathematical Society had received their undergraduate degrees at Indiana University than at any other institution except Harvard. During a twenty-year period, from 1888 until 1908, out of more than 2,000 students receiving undergraduate degrees at Indiana, 10 percent had majored in mathematics.

E. H. Moore dominated mathematical life in 1900. In his own way, so did David Rothrock. Rothrock was the other side of mathematics in America. He clearly loved students, but he loved mathematics too. He belonged to the AMS and contributed regularly to a new and rebellious journal, the *American Mathematical Monthly*. He taught advanced seminars on material he brought back from Leipzig and digested exciting new research for the graduate student seminar. But he also taught analytic geometry—every year—and wrote textbooks in algebra and trigonometry. Later, he was an active member in the Mathematical Association of America and served as dean of the college for seventeen years from 1920 until his retirement in 1937.

Academic Standards

What was life like in this other culture? Mathematicians seemed to agree about two things: students were deplorable, and so was teaching.

As the number of universities increased, the competition for students grew keen. Even before 1900 there was a great debate about the nature of a college education and which students should go to college. Articles such as "Boys Who Should Not Go to College" appeared in the *Atlantic Monthly* [8, p. 123]. New institutions opened with an invitation to all comers, no matter how ill prepared; this happened at Cornell, at Stanford, and (to a lesser extent) at Chicago. Colleges needed money: at one college in Ohio, an enthusiastic donor offered a large sum provided the school enroll a certain minimum number of students each year. Not surprisingly, they succeeded [8, p. 357].

Once the students arrived on campus, academic standards varied greatly. In 1903 at Yale a survey showed that the average senior spent less than an hour a day preparing for classes. At Harvard some students received A's in courses they had never attended. An observer of Harvard commented, "Perhaps in no institution is the value of the A.B. degree so indeterminate." [8, p. 359]

Letter grades—or any system of course grades—were not uniform. At Indiana it wasn't until 1908 that the faculty recommended the use of letter grades, six of them: A, B, C, D, conditioned, and failed. A great debate ensued about who should have access to the grades. Surprisingly, the students felt grades should not be shared with anyone, including the students themselves [2, vol. II, p. 47].

Almost immediately faculty worried about grade inflation. After one year at Indiana a comparison was made with Harvard, Missouri, and Wisconsin. Indiana didn't look good, and faculty complained about a few of their colleagues who gave unusually high grades, distorting the statistics for the entire university. The term "snap course" was used for the first time.

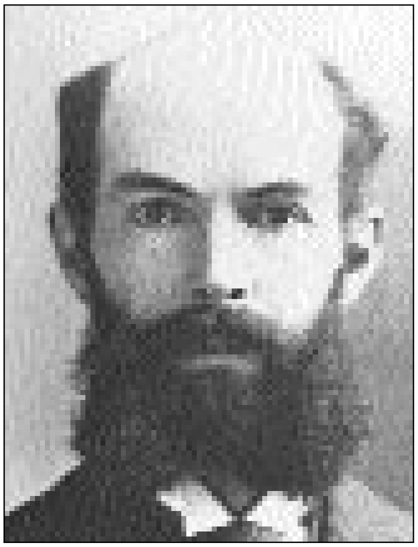
By 1901, amid much impassioned debate, the first College Entrance Board examinations were held, leading eventually to uniform systems of establishing entrance requirements. In 1902 the AMS formed a committee to formulate standard entrance requirements in mathematics; their detailed report was published a year later in the *Bulletin*. Several years after that, the American Association of Universities began accrediting graduate schools.

Reform

One hundred years ago mathematicians complained about the sorry state of teaching. A good sample of those complaints can be found in the report of a panel discussion, held at the summer meeting of the Society in 1900. Here are some excerpts from the *Bulletin* [October 1900, pp. 14–24]:

> I urge…a thorough revision of college courses in differential calculus. The subject is too often taught along traditional lines; the lecturer should ask himself whether certain parts of his subject possess the same importance today that they had in Euler's time. (Harkness)

> The fundamental principles of the calculus must be taught in a manner wholly different from that set forth in the textbooks if they are to be-



William Fogg Osgood

come flesh and blood to the student. (Osgood)

> Our textbooks abound…with proofs that are no proofs and with extremely misleading statements. (Harkness)

> All this time applied mathematics have been out of fashion, but there are many signs that a change is coming, and the intelligent college instructor will do well to anticipate this change.… (Harkness)

The lecture method was hotly debated. Moore, of Chicago, spoke disparagingly about lecturing: he was throughout his life an outspoken proponent of the laboratory method, leading students to discover ideas for themselves. Osgood, of Harvard, was the focus of opposition to such new-fangled ideas. He fulminated:

> As to how early the lecture method may be used with profit I may say that at Harvard the lecture method has been employed with success for over twenty years in practically all of the courses in mathematics.… (Osgood)

H. E. Slaught looked back at the first decade of the twentieth century in the *Monthly*:

> This decade of remarkable activity on the part of teachers of mathematics in this country can be accounted for only on the hypothesis of a widespread and deep-seated feeling of unrest and dissatisfaction with the present methods and results, a conviction that in some way the mathematics teaching in the schools fails to connect vitally with the preparation for living in the twentieth century needed by the vast majority of the pupils in the schools. [*Monthly* 20 (1913), p. 171]

Were there problems? You bet. In the nineteenth century, calculus books were sloppy and fat. There was a five-volume text for calculus. Here's the opening paragraph on series from a well-known and heavily used book on college algebra:

> By the term "series" we mean a sequence of terms connected by + signs. If the number of terms in the sequence considered be finite, the corresponding series is finite; if the number be infinite, the corresponding series is called an infinite series.

A "sequence connected by plus signs?" Here was the progenitor of all modern college algebra texts…the bloodlines are evident.

New teaching journals (including the *Mathematics Teacher*) were formed in the first years of the new century.

Committees toiled tirelessly, producing reports in the *Bulletin* and the *Monthly*. The International Commission on Teaching of Mathematics was formed in 1908. It formed subcommittees to study primary, secondary, and college teaching in various countries. The subcommittees produced great mounds of reports, and those in this country were published by the United States Bureau of Education. They are interesting reading for historical purposes. Six years after its beginning, we are told that the Commission had produced 10,293 pages of reports [5, p. 26]. G. A. Miller, himself a prolific author, reported on these ten thousand pages with a clear sense of admiration.

Research versus Teaching

What was it like to be a mathematician in America at the turn of the century? If you were with E. H. Moore at Chicago in a new department that had quickly come to dominate mathematics in the Midwest, professional life was research with a heady view of a new future. It was exciting. If you were with David Rothrock or at dozens of other universities and colleges across the country, professional life was teaching, but mixed with admiration for the growing mathematical community in America. That was exciting too.

But make no mistake. There were two cultures in the American mathematical community, and the tensions between research and teaching were already felt in 1900. Fiske wrote in a summary of mathematical progress in America, which appeared in the *Bulletin* in 1905:

> Notwithstanding the great progress recently made in America by our science, we are far from being in a position that we can regard as entirely satisfactory.… the most pressing demand seems to be that those engaged in lecturing…at American universities

> should be given greater opportunities for private study and research. At present, the time of almost every university professor is taken up to a very large extent with…the care of comparatively young students. [*Bulletin*, February 1905, p. 245]

Less teaching, more time for research.

The Genius of American Mathematics

One hundred years ago there were two mathematical cultures in America. Call them "research" and "teaching", but the differences are much more complicated. The genius of American mathematics in 1900 was that so many mathematicians succeeded in combining those cultures. In that same *Bulletin* article in which he lamented the burdens of teaching, Fiske went on:

> …we must [also] have improved methods of teaching, better textbooks, and more good treatises on advanced subjects. [*Bulletin*, February 1905, p. 246]

E. H. Moore was the master, and perhaps the leader, in combining mathematical cultures. He was clearly the central figure in research mathematics in America, and yet he spent much effort and time immersed in teaching. From the start, Moore was a special friend to the *Monthly*, and it was largely Moore's influence that shaped the *Monthly* in years to come: not a research journal, but connected to research nonetheless. Moore gave his retiring presidential address at the end of 1902. The topic? Mathematics education.

> The American Mathematical Society has, naturally, interested itself chiefly in promoting the interests of research in mathematics. It has, however, recognized that those interests are closely bound up with the interests of education in mathematics. …Do you not feel with me that the AMS, as the organic representative of the highest interests of mathematics in this country, should be directly related with the movement of reform?…[*Bulletin*, May 1903, p. 412]

And in the same year, as Moore spoke these words, David Rothrock was lecturing to graduate students on new ideas from invariant theory, and Benjamin Finkel was building an expository journal for research mathematicians.

Moore, Fiske, Rothrock, and Finkel found a way to accommodate the two cultures of mathematics, and the mathematical community gained. It was an important lesson. We've forgotten that lesson from time to time during the past one hundred years. Our profession has always suffered from a lapse of memory.

Thomas Fiske

The Future?

What will mathematical life be like one hundred years from now? I began by telling you I wasn't a historian. Well, I'm not a futurist either. Futurists make profound predictions such as this in a recent newspaper: THE FUTURE HOLDS BOTH PROGRESS AND PROBLEMS. I think they are right.

Nonetheless, I will make some predictions. I can't tell you what mathematics will be fashionable in 2095, but I can tell you that our present lists of important mathematics from the twentieth century will look quaint then.

I can't tell you how mathematics will be communicated in the year 2095, but I can tell you that journals as we know them will not exist.

Libraries will not exist either, but the need for storing and disseminating mathematics will. Someone will have to pay for this, and mathematicians will likely be complaining about the lack of money. I hope they will have found a variation on the argument that the library is the mathematician's laboratory.

Will students learn in classrooms, sitting in lectures? Maybe not. But I'll bet that students still learn by interacting with humans—with teachers and with other students. And I'll bet there are complaints in 2095 that students are ill prepared and that teaching is badly in need of reform. Committees will set to work, but being electronic, reports will likely be far longer—a frightening thought.

What about the two cultures? Mathematics is vertical, with the frontiers of research at the top. Most mathematicians are helping students climb from farther down. That will not change, and it's the fundamental cause of the two cultures. The two cultures will still be around in 2095. But I hope we have learned some lessons from the twentieth century. Looking back, we can see the genius of American mathematics in 1900: it embraced both cultures. We ought to recognize the vitality this brings to the profession and the richness it lends to the mathematics itself.

There will be two cultures in 2095, but I predict most mathematicians will find it natural to participate in both. I hope I'm right.

Both E. H. Moore and David Rothrock would approve.

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