

Carl Herz

1930–1995

The first part of this obituary is written by J. R. Choksi, the second part by W. H. J. Fuchs, and the third part by N. Th. Varopoulos. The first and third parts appeared as an obituary in the December 1995 issue of the Canadian Mathematical Society Notes.

Carl Herz was born on April 10, 1930, the son of Michael Herz and Natalie Hyman in Rockville Center, Long Island. He went to Cornell as an undergraduate in 1946, obtaining his bachelor's degree in 1950. From his junior year he worked as a teaching assistant, actually teaching students who were older than himself. In 1950 he went to Princeton as a graduate student, working with Salomon Bochner. Bochner suggested an initial problem, but the actual thesis, a paper based on which appeared in the *Annals of Mathematics*, was written while Bochner was on leave. This work was most influenced by Alan James.

After completing his Ph.D. Herz returned to Cornell in 1953 as an instructor becoming successively, Assistant Professor in 1955, Associate Professor in 1958 and Professor in 1963. Walter Feit who went to Cornell at the same time recalls how the University President, himself newly appointed, remarked in a speech sometime in 1953–54 that the Math Department had greatly improved since he had become President: "As

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Carl and I were the only new additions to the department in that period, we thought he had great insight; somehow this did not seem to be the consensus of the department." For more on Carl's years at Cornell, see Fuchs' contribution below. Soon after coming to Cornell, with the encouragement of Harry Pollard, Herz started his very fruitful work on spectral synthesis. For more on this see Varopoulos' contribution below.

Carl Herz left Cornell in 1969, spent a year at Brandeis and then moved to McGill in the fall of 1970, where he spent the rest of his career. Of great importance during the sixties and seventies were his visits to Université de Paris Sud (Orsay). He spent the academic year 1962–63 there as a Sloan Fellow and returned regularly afterwards: in fact for many years he spent a month or two of each year in Paris. The results of these visits are described by Varopoulos. He also spent the academic years 1957–58 and 1976–77 at the Institute for Advanced Study, Princeton. In addition he held various summer visiting positions at Warwick (1968), Institute Mittag-Leffler (1970) and Perugia (1975).

Carl Herz joined the Canadian Mathematical Society (then the Canadian Mathematical Congress) when he came to Canada. He played a very prominent part in its affairs over the next 25 years. In 1975 he proposed and helped organize the first Winter Meeting of the CMS. He was subsequently principal organizer of three annual meetings and a seminar in harmonic analysis at McGill in the summer of 1980. He also served on the Board of Directors and the Executive Committees, was Vice-President from

1981–83 and President from 1987–89. He was Jeffery-Williams lecturer in 1985.

Carl Herz was elected a Fellow of the Royal Society of Canada in 1978. He was named Redpath Professor of Pure Mathematics at McGill in 1993 and the same year started a three-year term as Director of the Institut des Sciences Mathématiques in Montreal. Throughout his time at McGill, Herz was deeply concerned about and involved in the renewal of the department. Several of our outstanding young appointments were initiated by him.

Carl Herz was always active in political, social and human rights issues. Chandler Davis recalled that Herz was one of the (not too numerous) people who tried (unsuccessfully) to break the blacklist against Davis by getting him hired in the U.S. Carl married Judith Scherer in 1960; she is now Professor of English at Concordia University. They have two children, a daughter Rachel, born in 1963, and a son Nathaniel, born in 1967. Carl left Cornell in 1969, and became a Canadian citizen in 1981, (soon after the 1980 U.S. presidential election!).

Carl had strong opinions on virtually any subject, and they were clearly and forcefully expressed! Several people have said how much they enjoyed his acid comments on the idiocies of the day. He loved a good argument but he was always open to counter arguments. You could disagree with him, and on occasion make him change his mind. But Carl had a hidden, gentler side which people did not always see. And if he was sometimes hard on people he was always harder on himself.

Carl had a remarkable *joie de vivre* and an exceptionally wide range of interests. Apart from his interest in social and political issues, he had a deep love of literature and music. And in all these Judith shared. A particular common bond was a deep love of the poetry of John Milton. The diversity of Carl's interests was immense. From Shakespeare, to the Ring of the Nibelungs, to Peter Brook's version of the Mahabharata—Carl enjoyed them all and could articulate his feelings about them with zest. He loved to dance with characteristic vigour at department parties. He loved good food of all kinds, all cuisines: it was an experience to go to a restaurant with Carl.

Carl Herz was an exceptionally brave person. All of us marvelled at his attitude—very positive and at the same time quite realistic—when lung cancer struck him in the spring of 1994. He fought the disease, and continued with most of his usual activities for as long as he could, even after a recurrence of cancer in November 1994. He seemed concerned about one thing: he felt he could no longer do mathematical research, and that saddened him. In all other aspects his energies ran unabated. As Director of the Insti-

tut des Sciences Mathématiques, he attacked its problems with his usual vigour, preparing an elaborate and well-thought-out proposal on the ISM to the Québec government in the early months of 1995. Three days before the start of his final illness he was the most vigorous participant in the audience at the Mathematics Colloquium given by Peter Sarnak. This participation was an activity he always thoroughly enjoyed; it was also one which his colleagues found most characteristic! But on the last day of classes (April 10, 1995) he had a seizure and had to be hospitalized; after that he was in and out of hospital. He could not eat and in the last week of April it was found that the cancer had spread.

Carl was not religious. He fought his disease for as long as he could, but he must have known he could not fight any more. He could not control his disease, but in the manner of his passing he could and did control his destiny. Carl died of pneumonia on May 1, 1995.

—J. R. Choksi



Carl Herz

Carl Herz at Cornell

At the invitation of Professor R.P. Agnew my family and I spent the academic year 1948–49 at Cornell. Carl Herz was then a junior majoring in mathematics and the president of the undergraduate math society. The first academic event of the year was his presidential address “A Survey of Integration and Measure Theory”. It was an excellent lecture explaining the McShane approach. But it was also the end of that year's undergraduate math society. Its members were hopelessly befuddled by Carl's rapid and terse exposition.

This little incident shows three traits that were characteristic of Carl: 1) His intellectual brilliance; 2) His unwillingness to make concessions where intellectual integrity was at stake; 3) His refusal to compromise with the demands of the “real world”.

After a brilliant career as a graduate student Carl joined the Cornell faculty in 1953. He had matured mathematically. His research showed the touch of a master: He tackled problems of the highest importance (Spectral synthesis, lattice points in bodies in \mathbb{R}^n, \dots). He made a substantial contribution to everything he touched

and he was a virtuoso in combining a simple fundamental idea with calculations involving a prodigious amount of skill and knowledge.

Carl was not a narrow specialist. His interests were encyclopedic. As an undergraduate he found time to attend courses in the Humanities, among them French History and many English lectures. His best friend at the time was his contemporary Harold Bloom whose brilliance and severe critical acumen matched Carl's set of mind. Through his interest in English he met his wife Judy who was then working in the English Department. They were united in a truly beautiful marriage.

In the 60s and 70s first the universities and later the whole nation stood in the shadow of the Vietnam war and the quest for racial and social justice that the war stirred up. Carl was as absolute and uncompromising in these problems as in all others. He refused to pay income tax while the war was going on. He was one of the leaders of the anti-war protests at Cornell.

But he felt that it was a betrayal of the university's sacred mission to adopt criteria other than intellectual ability for the admission of minority students. His disapproval was so strong that it finally outweighed his great loyalty to Cornell. He resigned in 1969 and went to McGill in 1970.

Much as I missed having Carl as a colleague, I feel that his move to McGill was of great benefit to him. It is very doubtful that he would have matured into the wise, understanding person that he became, had he stayed in his old surroundings.

—*W. H. J. Fuchs*

The Mathematics of Carl Herz

Carl Herz's contribution to mathematics can be separated into several, admittedly intertwined but nevertheless distinct periods. In all these sections of his mathematical life he was, like the rest of us, influenced by what was happening around him; however he was one of those rare mature mathematicians who actually pushed the subject he was working in very much his way and was one of the leading innovators in practically everything that he touched upon.

The first period, 1955–1965, bears the stamp of his Princeton years (A. Beurling, S. Bochner). His work on spectral synthesis (the Cantor set in \mathbf{R} , circle in \mathbf{R}^2 are sets of synthesis [1, 2]) were, and stayed for many years, landmarks in the subject. One should remember that the whole subject of Harmonic Analysis was then dominated by the introduction of Banach algebras, (I. M. Gelfand and the very closely related ideas of A. Beurling), and in particular the Banach algebra proof of N. Wiener's theorem (that the inverse

of an absolutely convergent Fourier series is absolutely convergent). The problem of the ideal structure of $L^1(G)$ and of spectral synthesis (L. Schwartz and P. Malliavin) was then a burning question. Carl's work covered all these problems.

His work on symbolic calculus on positive definite functions [3, 4a, 4b], his work on the Fourier transforms on convex sets [5], his contribution in Potential Theory, etc. became classics; many of us, (I include myself in that group since I was 10 years younger), studied and learned to do mathematics from them. At the end of that mathematical period Carl worked on Bernstein's inequality (on absolutely convergent Fourier series) and on Lipschitz and Lorentz spaces [6, 7]. This is first-rate work: he introduces new ideas and improves and innovates on what was (or even still is!) known in the subject.

It was 1965 when I met Carl Herz at Orsay. I should point out that because of his mathematics Carl Herz had already established very solid ties with French mathematics and mathematicians (Choquet, Deny, Malliavin, Kahane). I was a postdoc at Orsay at the time. He always gave me a lift back to Paris from Orsay on Tuesday afternoon after the seminar in his green DS up the Autoroute du Sud and I think it would be fair to say that I learned more mathematics during that car ride than during the seminar.

I would date the beginning of his second mathematical phase at, more or less, the time that he moved to McGill. He started by a systematic analysis of the $A_p(G)$ spaces (that were introduced by A. Figa-Talamanca in 1965 and as C. Herz showed turned out to be Banach algebras [8, 9]). The work there had a big impact and influence on many people (Eymard, Lohoué and many others). A_p algebras are a natural stepping stone from the commutative to the noncommutative. But I would not say that this was Carl's point of view. His ideas in that area helped both him and others to explore, in a fundamental way, several related areas. His work in the Kunze-Stein phenomenon [10], which is one of the key steps in Cowling's proof of the phenomenon for general semisimple groups, emerged from this. It would on the other hand be fair to say that his interest in the noncommutative started then. This subsequently became his main mathematical interest. In 1972 he recruited S. Drury at McGill and for a while they worked together [11] on a problem on which S. Drury had just done some very important and decisive work, (the union of two Sidon sets is a Sidon set).

It is at that point, say 1973, that Carl moved to his final mathematical period where he contributed to two themes of analysis. One was the H_p theory and BMO, especially the martingale variant [12, 13, 14]. At that time he was in reg-

ular personal contact with R. Gundy and A. Garcia whom he regularly invited to McGill. The original idea of atomic decompositions of H_p spaces is due to Carl [12], and it emerges from his previous work on A_p . As we all know in the years that followed this became a big industry.

The second subject that he contributed to, especially towards the end, was the theory of symmetric spaces, Lie groups and the heat kernel on these; among other things he succeeded in classifying all faithful representations of Lie groups by contact transformations of a compact manifold [15, 16]. It's on these grounds that Carl and I got back together mathematically after a "separation" of several years (that was due to our mathematical "wandering" in various subjects). That reunion started in the summer of 1980 during a very pleasant and successful conference in representation theory and harmonic analysis that Carl organized in Montreal in the month of August. I had visited McGill before but somehow in the past McGill was just another university. This time it was really different. It was a perfect time for a new beginning for both of us and Montreal and McGill was a perfect place for that.

For the next several years I spent a lot of time at McGill where I must say I found that Carl was a perfect host and that the atmosphere in Burnside Hall was very stimulating and congenial. My life there was highlighted by many seminars and other mathematical activities (that involved the McGill analysis group) and by my strong friendship and social life with Carl. In many ways this was the most productive and happiest part of my life. With Carl's death I feel that this part is now closed.

Herz contributed to practically every aspect of harmonic analysis and his contribution in everything he touched was fundamental and of great innovative value. His mathematics like his personality was uncompromising and original. He rejected both in his mathematics and in his life any kind of imitation and, he had a strong distaste for repetition, for reproving theorems either his own or other people's. As I have tried to show in this short aperçu of his mathematics, when Carl had an idea, often a very basic and innovative idea that could generate several papers, he would write one paper and then move to other things. Most of the time these ideas were picked up by other people who beat them to death, forgetting more often than not the real origin of the ideas they were working on. But then Carl, I am sure, did not care and would not have had it any other way.

—N. Th. Varopoulos

Publications of Carl Herz Referred to Above

A complete list of publications appears in the obituary in the December 1995 issue of the *Canadian Math. Soc. Notes*.

- [1] *Spectral synthesis for the Cantor set*, Proc. Nat. Acad. Sci. **42** (1956), 42–43.
- [2] *Spectral synthesis for the circle*, Ann. of Math. **68** (1958), 709–712.
- [3] *Fonctions opérant sur certains semi-groupes*, C. R. Acad. Sci. Paris Sér. I. Math. **255** (1962), 2046–2048.
- [4] *Fonctions opérant sur les fonctions définies-positives*, C. R. Acad. Sci. Paris Sér. I. Math. **255** (1962), 2560–2561.
- [5] *Fonctions opérant sur les fonctions définies-positives*, Ann. Inst. Fourier **13** (1963), 161–180.
- [6] *Fourier transforms related to convex sets*, Ann. of Math. **75** (1962), 81–92.
- [7] *The Ideal Theorem in certain Banach algebras of functions satisfying smoothness conditions*, Functional Analysis (Gelbaum, ed.), Thompson Publ. Co., Washington, DC, 1967, pp. 222–234.
- [8] *Lipschitz spaces and Bernstein's theorem on absolutely convergent Fourier transforms*, J. Math. Mech. **18** (1968), 283–324.
- [9] *Le rapport entre l'algèbre A_p d'un groupe et d'un sous-groupe*, C. R. Acad. Sci. Paris Sér. I. Math. **271A** (1970), 244–246. MR 42:8307a.
- [10] *Synthèse spectrale pour les sous-groupes par rapport aux algèbres A_p* , C. R. Acad. Sci. Paris Sér. I. Math. **271A** (1970), 316–318. MR42:8307b.
- [11] *Harmonic synthesis for subgroups*, Ann. Inst. Fourier **23** (1973), 91–123. MR 50:7956.
- [12] *Sur le phénomène de Kunze-Stein*, C. R. Acad. Sci. Paris Sér. I. Math. **271A** (1970), 491–493. MR 43:6741.
- [13] *Drury's lemma and Helson sets*, Studia Math. **42** (1972), 205–219. MR 46:5939.
- [14] *Bounded mean oscillation and regulated martingales*, Trans. Amer. Math. Soc. **193** (1974), 199–215. MR 50:5930.
- [15] *H_p -spaces of martingales, $0 < p < 1$* , Z. Wahrscheinlichkeitstheorie **28** (1974), 189–204. MR 51:9189.
- [16] *Généralisations de la notion des classes H_p de Hardy*, Théorie du Potentiel et Analyse Harmonique, Strasbourg 1973, Springer Lecture Notes in Math., vol. **404**, pp. 138–148. MR 51:9666.
- [17] *Representations of Lie groups by contact transformations I: Compact groups*, Canadian Math. Bull., **33** (1990), 369–375.
- [18] *Representations of Lie groups by contact transformations II: Non-compact simple groups*, Canadian J. Math. **45** (1993), 778–802.