Views on High School Mathematics Education

This is the first of a series of articles composed from responses of mathematicians, mathematics educators, and teachers of mathematics to a set of questions on secondary education in mathematics. These are the questions:

1. When you consider what high school graduates should understand about mathematics, what do you care about most?
2. What do you think should be essential features of every high school mathematics curriculum?
3. How would we know when high school mathematics education is working well?
4. What do you think is most important in the mathematical background, attitude towards mathematics, and pedagogical approaches of high school mathematics teachers?
5. What first attracted you to mathematics?

The respondents were chosen from those known to us or suggested to us as having dedicated some effort and thought to this subject. We also aimed at obtaining the widest possible set of views and recommendations. All respondents answered all questions, and we have selected those responses for publication which cover the breadth of opinion and ideas presented. Our sense is that rapport among educators of mathematics at all levels is essential to the coherence and success of the enterprise, and fundamental to that rapport is openness of dialogue.

The articles are inspired by the efforts of Al Cuoco and Wayne Harvey of Education Development Center, Inc., to increase the involvement of mathematicians in high school mathematics education. Their work has resulted in a project funded by the National Science Foundation and entitled "Building on Strengths: Stimulating Cooperation among Mathematicians and Mathematics Educators". The highlight of the project is a national meeting, to be held next spring, which will bring together mathematicians, mathematics educators, and K–12 teachers who have strong and differing opinions on basic questions like the ones posed in these articles. After the conference, follow-up activities will be held at regional and national meetings of professional organizations. For further information on the project, visit the Website `http://www.edc.org/LTT/BOS/`.

The next installment will appear in the September 1996 issue of the *Notices*. Apart from these two articles, future articles are planned in which high school teachers will respond to a similar set of questions. The full text of all the responses will be posted on the education home page on e-MATH, at the URL `http://www.ams.org/committees/education/`.

The names of the respondents and their affiliations are: Susan Addington, California State University, San Bernardino; George Andrews, Pennsylvania State University; Richard Askey, University of Wisconsin, Madison; William Barker, Bowdoin College; Hyman Bass, Columbia University, chair, AMS Committee on Education; Curtis Bennett, Bowling Green State University, member, AMS Committee on the Profession; Mark Bridger, Northeastern University; Amy Cohen, Rutgers University; John B. Conway, University of Tennessee; David Cox, Amherst College; Edward Effros, University of California, Los Angeles; Solomon Garfunkel, Consortium for Mathematics and its Applications; Andrew Granville, University of Georgia; Leon Henkin, University of California, Berkeley; John Hollingsworth, University of Georgia; Roger Howe, Yale University; Deborah Hughes Hallett, Harvard University; Fern Hunt, National Institute of Standards and Technology; Raymond Johnson, University of Maryland; Harvey Keynes, University of Minnesota, member, AMS Committee on Education; Kenneth Millett, University of California, Santa Barbara; George Papanicolaou, Stanford University; Stephen Rodi, Austin Community College; Judith Roitman, University of Kansas, member, AMS Committee on Education; Arnold Ross, Ohio State University; Han Sah, State University of New York at Stony Brook; Glenn Stevens, Boston University; Alan Tucker, State University of New York at Stony Brook, member, AMS Committee on Education; H. Wu, University of California, Berkeley.

—*Allyn Jackson and Hugo Rossi*

1. When you consider what high school graduates should understand about mathematics, what do you care about most?

ARNOLD ROSS: We fail to acknowledge fully the destructive power of boredom. Let us note that "boredom" is not the opposite of the common parlance expression of "having fun".

Curiosity is a prevalent (and happy) human trait. It encourages the transition from "look" to "see". In bringing up the very young we must encourage their urge to explore. We must nurture their capacity to observe, to experiment, to project their experience adventurously (conjecture!).

Also we must nurture their capacity to communicate. Here we must recognize that experience comes first and the appropriate language comes second and not vice-versa. Thus we must provide early hands-on experience. The more recent bon mot "mind-on" is more suggestive.

Computers may enter naturally and fruitfully only after youngsters understand and master important algorithms. In the beginning computers should be programmed by the student for each algorithm. Then the computer will be appreciated as a tool of extracting incredibly more information out of each procedural invention.

Learning appropriate intelligent use of computers early is very desirable in our computer oriented environment. In particular it provides a basis for mastering the ever more sophisticated uses of computers in science and technology.

WILLIAM BARKER: I care most about their understanding of mathematics as a process for analyzing and solving problems of consequence in the natural and social sciences. In particular I want students *not* to approach mathematics as a "content-free" exercise in manipulating symbols via memorized rules. I will gladly trade some manipulative skill to have students who approach mathematics as a discipline that requires understanding of the concepts and creative thinking to apply those concepts in the solution of problems that matter. Students should not freeze when confronted by a question that goes beyond the "template problems" of their textbooks. They should view mathematics as a toolbox and be able to select intelligently among their tools. (This is closely related to the "Rule of Three": the analysis of problems graphically, numerically, and symbolically.)

There are, of course, basic manipulative skills that high school graduates should have. Most mathematicians would probably put together pretty much the same list and in the same order of priority; the primary differences would be in the lengths. My list would be among the shortest, but with the requirement that students understand these topics well. When it comes to skills, real ability with a small collection of tools is far better than superficial acquaintance with a large collection.

H. WU: (a) They all must understand that every true statement in mathematics can be logically explained in terms of other true statements and that nothing is ever true just because some higher authority decrees it to be so. This is the basic spirit of openness in any rational discourse, and if math education does not succeed in inculcating this spirit, then school education as a whole has failed miserably.

(b) Concomitant with (a), students must be able to distinguish clearly between what they know and what they don't know to be true. In mathematics and perhaps only in mathematics this sharp distinction can be drawn, and students should profit from it. They should not confuse a heuristic (but incomplete) argument with a conclusive proof. By the same token, once they have learned the proof of something in mathematics, they should be able to savor the satisfaction of unshakable conviction. The possible failure to achieve this goal in school math education is what worries me the most in the present math education reform.

(c) Students should appreciate that mathematics is a language of precision. As a language the mastery of math must include fluency. They must therefore strive for this fluency. Thus certain basic techniques should always be available at their fingertips. In addition, the characteristic feature of precision of this language automatically excludes the kind of vagueness and ambiguity in everyday life. If students really understand this, then they would know that questions such as "What is best?" or "Is this fair?" (which are common in the mathematical problems of the current reform) have no place in any mathematical discussion until these loaded words have been precisely defined.

(d) The purpose of a math education lies not just in teaching students how to solve everyday problems (as some reformers would have us believe) but also in teaching them how to think precisely, logically, and abstractly. The utilitarian aspect of the subject must also be tempered with an appreciation of its cultural aspects: its internal structure and its aesthetic appeal.

GEORGE ANDREWS: Recently Cal Moore, writing on educational reform in California, began as follows:

> While reforms are gradually taking hold, the majority of classrooms still rely on a traditional mathematics curriculum, that, as one cynical observer remarked, is largely composed of eight years of 15th century arithmetic, two years of 17th century algebra, and one year of 3rd century B.C. geometry.

I would suggest that if students really have mastered these subjects, they will have no difficulty with anything thrown at them in college.

Unfortunately, beguiled by ever fancier calculators and computers, teachers appear less and less able to produce students who are masters of these basic topics.

National leadership only muddies the waters. Listen to the NCTM (National Council of Teachers of Mathematics) on technology:

> …the ability of teachers to use the tools of technology to develop, enhance, and expand students' understanding of mathematics is crucial. These tools include computers, appropriate calculators (scientific, graphing, programmable, etc.), videodisks, CD-ROM, telecommunications networks by which to access and share real-time data, and other emerging educational technologies. Exploration of the perspectives these tools provide on a wide variety of topics is required by teachers.

All that's missing are strobe lights and a D.J. The simple, very old-fashioned lesson that "math class is hard" and requires hard work and lots of homework cannot help but get lost in the glitz of gimmicks.

2. What do you think should be essential features of every high school mathematics curriculum?

WILLIAM BARKER: The essential feature of every high school mathematics curriculum should be the existence of a mathematical environment which instills the attitudes cited [in my response to question 1]. Courses should not emphasize the memorization of manipulative rules and their regurgitation on template-problem exams. Students must realize that mathematics is *thinking*, not *mental gymnastics*. Time should be taken for real applications. Better yet, real applications should be used to motivate the need for certain mathematical techniques. For example, trigonometric functions should be introduced, not solely to "manipulate stuff with triangles", but as the primary tool for dealing with repetitive, periodic phenomena. Analytic geometry should be stressed, not simply as a collection of skills to master, but as a profound link between algebraic problems and geometric problems. (Perhaps I missed it, but I don't recall these viewpoints in my high school education. My current students certainly do not come equipped with them.)

I think that high schools need to be very careful about instruction in calculus—I do not think it should always be the capstone goal for their best students. Getting to calculus in high school often means rushing students through the foundational material and developing a "template-problem" mentality to all of mathematics due to shaky understanding of the basics.

Emphasis on varied and effective pedagogical techniques needs to be given as much attention as the curricular content. It is my understanding that great strides have been made in this area in recent years. "Chalk-and-talk" is no longer the only teaching format in the high school arsenal. That's absolutely critical. "Telling is not teaching, hearing is not learning" is a phrase to live by. Collaborative learning situations, both in-class and out-of-class projects emphasizing multi-step problems, discovery learning through calculator or computer labs, written reports—all of these instructional methods should be part of the high school experience. The shift must be towards more opportunities for active learning rather than passive.

Examinations must put more emphasis on conceptual questions and problems requiring thinking, not just template problem memorization. I know this is hard. I have been wrestling with this problem for years at the college level. But the cliché that "exams define the course" is true and cannot be ignored.

H. WU: The answer [I gave] to 1 is a statement of the goals of a school math education. The school math curriculum should therefore focus on the attainment of these goals. One cannot hope to set forth in a few sentences how such a complicated task can be accomplished. The following are at best vague statements. An overall comment is that during the last two or three years of high school, the curriculum for students who expect to be quantitative majors in college should differ from that for students who don't in terms of the technical nature of the instruction. In general:

> (a) Abstractions should be introduced early (in grade school) but in small doses. Students have to learn to handle abstract thinking; and the more they are shielded from abstractions in the early grades, the harder it would be for them to learn it later. For example, if fractions are taught properly (with the requisite amount of abstract reasoning), then fifth graders would already get to see the essence of mathematics at work.
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> (b) Informal reasoning for every mathematical statement should be given from the beginning. By the time of junior high, formal proofs should begin to make their appearance, if only in moderation. In the eleventh and twelfth grades, formal proofs should become routine (at least for students who will be quantitative majors in college).
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> (c) The applications in school mathematics should not overwhelm the curriculum, especially not those arising exclusively from everyday life. The power of mathematics is best demonstrated when it is seen to be essential for the formulation of far-reaching scientific principles rather than just the solution of picayune, mundane problems.
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> (d) Mathematics is precise. For this reason, the possible ambiguity in the solution of real-life problems must be *clearly* traced to the inherent ambiguity in the *interpretation* of the real-life problems and not be blamed on mathematics itself. The sermon that "there is always more than one correct answer to a math problem," so often preached in the math education reform, should be laid to rest for good.

KENNETH MILLETT: A high school mathematics curriculum should present mathematics as an interconnected whole. It should include the study of functions, algebra, geometry, statistics and probability, discrete mathematics, logic, reasoning and communication, applications of mathematics, and numbers (integers, rationals, real and complex numbers, as well as other number systems). Thinking of these as strands of mathematics, a good visual image is of mathematics as a tapestry whose beauty and strength requires the artful interweaving of the strands. Such a high school curriculum is best understood when viewed from a distance so that the unity and proper proportional relationships come into view. The curriculum should be less hierarchical and more multidimensional, with "advanced" issues suggested early on and with "elementary" topics revisited frequently at greater and greater depth. The use of credible mathematical models, problem solving (including the development, use, and analysis of algorithmic approaches), mathematical communication (in all its manifestations and translations between forms), exploration, conjecturing or guessing, and the development of explanations (arguments or "proofs") should be part of the overall experience. A deep conceptual introduction to the mathematics of change is important, but a formal course in calculus is not necessary. Most importantly, thinking about mathematics and reflection on what has been learned should be central to every student's experience in high school. Accuracy and depth of understanding are more important than speed in carrying out narrowly prescribed operations.

ALAN TUCKER: There are core topics in algebra that every student should know. Beyond that, I am at a bit of a loss. The process of rigorous mathematical thinking and exploration can be undertaken in many ways. I tend to believe that the mathematical learning process is more important than specific content learned. I think that the NCTM Standards is a reasonable beginning for one version of a good high school mathematics curriculum, although it needs to go further for more talented students.

3. How would we know when high school mathematics education is working well?

JUDITH ROITMAN: When kids talked mathematics intelligently, as naturally as they talk anything else, whenever mathematics is appropriate. When kids want to talk mathematics.

FERN HUNT: When public debate on issues involving quantitative reasoning are more informed. When more people are comfortable using mathematics in their jobs and in the rest of life.

ROGER HOWE: One way would be for there to be an increased acceptance of mathematics as part of normal dialogue. In the book *Mathematics Tomorrow*, edited by Lynn Steen, there is an article by Neal Koblitz called "Mathematics as Propaganda". It starts with an article about someone from Zero Population Growth talking about environmental impact as being proportional to population. This was on the Johnny Carson show, I think. The guy wrote a linear equation, $D = cP$, D being damage and P being population. Koblitz's point was, writing this equation was an intimidating thing to do. My reaction was to be depressed that something so basic and simple should be perceived as intimidating by the broad public. If we could reach a state where this kind of thing was an accepted and understood part of normal discourse, that would be a positive thing. But I guess that won't happen in the short term. I think the practical short-term answer is to recognize first that "working" will always be a relative term; one can always wish for more, and things could always be worse. Given that, the answer becomes technical: one must have standardized tests which reflect a consensus view of what needs to be known, and [one must] measure progress on these tests. There is another criterion, in terms of manpower: are the math skills offered on the market adequate to the jobs available? But this is a much more contingent criterion, reflecting much more than U.S. education. For example, there is currently a Ph.D. math supply glut. But these supernumerary Ph.D.s are not U.S. citizens.

RICHARD ASKEY: We have many problems in our schools, and mathematics education is only one. However, many of these problems are related, and I do not think we can solve one in isolation. Thus, rather than try to say when I think the whole problem of school mathematics has been fixed and is working well, I think it is sufficient to look at one problem there and see when it has been solved.

The one which is easiest to look at is the problem students taking calculus have with algebra. For far too many, their lack of skills in algebra makes it very hard and, for many, impossible for them to learn calculus. Some have tried to attack this problem by redefining calculus, just as they have tried to attack the problem of poor arithmetical skills causing problems in algebra by changing what algebra is. Neither of these will work, and we need to go back to fundamentals and solve these problems directly. Elementary school is the most important.

In the book *The Shopping Mall High School* (Houghton Mifflin, 1985) the author of Chapter 2, Arthur Powell, describes a treaty between many teachers and many of their students. In short, it is: I will not force you to learn anything in this class if you do not cause any trouble. While this is not true universally, I am afraid it is an accurate description of far too many classes. Many calculus students have told me that they wish they had studied in high school, but no one else did, so they did not either. Some of this talk is for show, but far too much of it is accurate. When students no longer say this, then significant progress will have been made.

HYMAN BASS: We first need to achieve some consensus on the aims of that education. They are multiple—cultural and intellectual (college performance), economic (workplace performance and employability), and social (informed and responsible citizenry). We need some agreement on what balance of priorities these are given and on whether and how the student population should be aggregated in delivering that education. Once we have some semblance of consensus, including a good and collaborative articulation between high school education and college and the workplace, then we can go beyond the current system of national exams—whose value and relevance are conditioned by the extent to which what they measure conforms with the educational aims of the system—and do longitudinal studies of performance of high school graduates in college and the workplace, not only in terms of course grades, but even of their remaining engaged with scientific and technical subjects.

4. What do you think is most important in the mathematical background, attitude towards mathematics, and pedagogical approaches of high school mathematics teachers?

HAN SAH: Most important: know a lot of basic techniques, content, and the proper use of mathematics in the *quantitative* sciences and the *improper* use of mathematics in the more *qualitative* sciences.

Love of teaching and learning as a *calling* and then love of mathematics as a part of this.

Be aware of a variety of pedagogical approaches rather than being dogmatic about a particular approach. Keep in mind that teaching is not for the convenience of the teacher but for the future of the students. Know the background preparation of the students and their tentative goals beyond the immediate present.

JOHN B. CONWAY: Appreciation of the need for theory (instead of fear of it), willingness to get their hands dirty (instead of thinking everything must be so simple), excitement for what mathematics is (instead of boredom), an appreciation of the fact that mathematics is not an assortment of algorithms.

RAYMOND JOHNSON: I believe that a variety of pedagogical approaches can be successful. I suspect that the most important thing is the attitude the teachers bring to mathematics. I know my son suffered from a number of teachers who insisted that mathematics was about right answers and that there was only one way to do a mathematics problem correctly. I think there is a big difference between "only one correct answer" and "only one way to get the answer". My feeling for a strong math background for teachers is based on my belief that the better the math background, the more flexible the teacher is likely to be about mathematical work.

LEON HENKIN: By way of mathematical background, it is most important that high school teachers have a good understanding of the parts of algebra, geometry, and analysis into which high school math will flow when students continue to study math in college. They must also be familiar with a wide spectrum of applications of math that can be made in high school science courses and in nonacademic fields where many high school students may seek jobs after graduation, including the use of computers.

By way of attitude, teachers should enjoy mathematics, let their students see this, and help them to achieve it for themselves. This is much more important than ensuring that the students learn some body of facts in class, for if they truly enjoy working with mathematics, they will go on learning long after leaving the class. Teachers must be sensitive to the fact that students are only learning to use language in the same way as the teachers themselves. Hence, a student may be trying to express a correct idea in words that sounds incorrect to the teacher, and if there is a wrong idea in the student's mind, it may have arisen from a natural misinterpretation of the teacher's words. By way of pedagogical methods, teachers should elevate to first place the use of praise—not only for correct answers, but also for brave guesses even when they turn out wrong.

5. What first attracted you to mathematics?

GEORGE ANDREWS: In high school I was completely unaware of the possibility of being a mathematician. The only really exciting life I could imagine was suggested by the Sherlock Holmes novels. Here was a character, albeit fictional, whose entire career depended on adroit reasoning. Since such a career seemed to be pure fantasy, I decided to become a patent attorney because I was good at science and math and found the law to be something involving some (I hoped) adroit reasoning.

In college at Oregon State my life was transformed by Harry Goheen. He obviously loved mathematics with passionate enthusiasm, and he proselytized vigorously for math majors. He was able to rekindle in me the dream of a career built on the life of the mind. This began in his trigonometry course. By the end of his calculus course, I was a math major.

RICHARD ASKEY: The story my mother tells is of me in a high chair doing a follow-the-dot puzzle and asking for a calendar, since I had finished with the clock. I always liked mathematics, and in high school met a second cousin who had M.A. degrees in mathematics and physics who told me that it was possible to do mathematics. Before then I had wanted to be a physicist.

WILLIAM BARKER: I liked playing with mathematics—*doing* it—and was good enough that I received a lot of encouragement and recognition in high school. Although my high school experience was with a standard curriculum using standard teaching techniques (i.e., lectures), I had teachers who cared about mathematics, were knowledgeable, and showed a real excitement about the subject. Although my instructional preferences might belie it, I am not an applied mathematician. My area is Lie theory, a topic that I love because of its inherent beauty and its ties to so many branches of mathematics.

A criticism often leveled against the instructional reorientation I recommend is that it will not appeal to those students who, "like us", will become interested in mathematics for its own sake and who would languish under a "less rigorous" curriculum. Not so. I wish my own secondary education had been oriented more to conceptual understanding and applications; I think I would have achieved a healthier and more comprehensive foundational view of mathematics and would have ultimately developed as a mathematician much faster. My experience with teaching beginning-level mathematics majors confirms these beliefs. Take the time to set the right foundational attitudes and all our students benefit.

HYMAN BASS: I always enjoyed math at school but had little sense of its scope until my brother, Manuel, in a Navy officers' training program in WWII, came home on leaves and gave me enthusiastic tutorials on the science and engineering courses he was taking. He continued this later as a student at Caltech. When I attended Princeton as an undergraduate, the honors calculus course was run by E. Artin, with Lang and Tate among the instructors. The excitement of that environment won many of us over to mathematics.

JOHN B. CONWAY: Euclidean geometry. Then the power of calculus. Then the elegance of basic analysis.

LEON HENKIN: As a schoolboy I saw that I could be certain, entirely on my own, that I had arrived at a problem solution that was perfectly correct—in mathematics. This set math apart from all my other studies. When teachers and family gave praise for such successes, I was encouraged to go further. But it was only the marvelous depth of mathematics that suddenly opened before me when the deductive method appeared as the core of upper-division college courses that was the decisive component in getting me to consider seriously a career built around mathematics.

ROGER HOWE: I was identified as "good at math" long before I had any idea of a career related to it. A fifth grade teacher told me I would be a mathematician, and my (private) reaction was, "You're crazy, lady." In sixth grade I rejected a proposal to study math beyond grade level. In tenth grade several things happened. I read a popular book about quantum mechanics. It had some mathematical symbols (line integrals) in it, and I had no idea what they meant, so I started to study on my own in order to understand them. For a long time after, through much of college, mathematical physics was a big motivation for learning math. Also in tenth grade I took geometry, which I thought was great. Also, one of my geometry teachers was a real enthusiast and put me on to the books *Elementary Mathematics from an Advanced Standpoint* by Felix Klein. They were *very* hard, and I didn't understand a lot in them, but they showed a completely different world from high school math.

FERN HUNT: An algebra course in the ninth grade and a Saturday course on elementary abstract algebra were the drawing cards for me.

RAYMOND L. JOHNSON: Arithmetic. I started to do well in mathematics when it was arithmetic, and since I continued to do well in high school, I assumed that mathematics was the major for me. If I had known what mathematics was, I might have chosen another major.

KENNETH MILLETT: I was first attracted to mathematics by my experiences in high school mechanical drawing (a vocational course) and Euclidean geometry. The first course involved figuring out the three-dimensional nature of objects from limited visual information and the production of perspective and other drawings. The second course was a traditional "two-column" proof course, during which I experienced the challenges of finding language to "prove the obvious" and the fun of working with my classmates trying to solve problems for which there might not be a solution or which were several levels more challenging than the standard homework. In both courses the teachers provided a rich, challenging curriculum and an environment that encouraged creative thinking, questioning every detail, as well as being respectful to all students and their efforts. Mistakes were seen as opportunities to learn, not a measure of lack of ability. In mathematics I learned the difference between understanding and not understanding. And understanding, once gained, was forever. There was, of course, the sense of adventure. The exploration of unknown intellectual worlds, the exhilaration of discovery, and the fun of sharing these discoveries with others. I felt a greater opportunity to chart new frontiers, to have something of the experience of being the first human to walk on the moon. Mathematics was, and still is, a grand adventure taking me to unexplored places.

JUDITH ROITMAN: The proof that the real numbers are uncountable, encountered in seventh grade. No kidding.

ARNOLD ROSS: I was encouraged to do much reading from a very early age. We made use of a subscription library at the age when the public library was at its infancy. My interests covered all of science, and I did amateur astronomy in my early teens. My early heroes were Faraday and Pierre Curie. I was fortunate to have an early opportunity to study mathematics under mathematicians who were charismatic teachers.

HAN SAH: When I was an undergraduate lab assistant in a physics accelerator lab, I found that I could not understand the purpose of the nuclear physics experiment. I decided to read up on quantum mechanics in the library. To my frustration, I discovered that I could not understand the mathematics used in the texts. Not long after, I asked my best friend in college (a math major) what he was studying in mathematics. I was shocked to find that I could not read past the first few pages of his book with the esoteric title *Theory of Groups.* Since my physics teachers had told me that I had already overdosed on mathematics, I decided to ask my math prof about the propriety of beginning to study some pure math. (I had in mind the vague idea of spending most of my fourth college year to that end.) His answer was, "It is too late to begin studying pure math at the age of 19."

Two weeks later I pigheadedly decided to graduate early and applied to the math department to try studying math full time. I did have a back-up. My chemistry teacher had mentioned during my freshman year, "If you ever get tired of physics, come see us. Our door is always open."

Not long after, as warned by my science teachers, I was firmly seduced by "useless" mathematics. At the same time I retained my interests in science and engineering, and slowly tried to understand the more difficult and fascinating endeavors in the humanities. One does not have to be a genius to become a mathematician. Hard work and an open mind are, however, necessary. What I had found was that my early exposure to science and engineering enabled me to carry out useful collaborations and to discover interesting connections.

ALAN TUCKER: I came from a mathematical family, and when I was three years old, I "knew" I loved mathematics and wanted to get a Ph.D. in mathematics.

H. WU: I had a terrible math education in grade school (in China) and consequently flunked every math course but one up to the seventh grade. Nothing was ever explained to me, and everything was done by fiat. I felt I could never penetrate the secret code used in math. Then in the seventh grade, I had a great teacher. From the first day, he solved every problem in class by reasoning out loud. It then dawned on me that there was no secret code, just the kind of ordinary reasoning that I could do myself. Soon after that, we started on proofs in Euclidean geometry, and this experience consolidated my feeling that math was a learnable subject. I have had little trouble after that.