Hans-Joachim Bremermann, 1926–1996

R. Michael Range and Russell W. Anderson

The Work of H. J. Bremermann in Complex Analysis

Hans-Joachim Bremermann studied mathematics in Münster, Germany, during the late 1940s–early 1950s. This was an extraordinary period for the famous Münster school in complex analysis, centered around Heinrich Behnke since the 1920s. H. Cartan visited Münster in 1949 for the first time after the war, sharing a wealth of new ideas from the French school. F. Hirzebruch studied there before moving to Zürich in 1949 to work with H. Hopf. In 1951 Karl Stein created what became known as Stein manifolds, and together with Behnke he introduced the first "complex spaces". H. Grauert and R. Remmert began their studies in Münster during that time, laying the foundations for their pioneering work. Naturally Bremermann too learned complex analysis and went on to make significant contributions to the field, most notably his 1953 solution of the Levi problem and his wide-ranging and profound work on plurisubharmonic functions.

The Levi problem had been a long-outstanding central problem. A region G in n-dimensional complex Euclidean space is called a domain of holomorphy (or regularity region in the older terminology) if it is the precise domain of existence of some holomorphic (complex analytic) function, that is, if there exists a holomorphic function on G which has no holomorphic continuation across any of the boundary points of G. In dimension one, every open set is a domain of holomorphy, but already in 1906 F. Hartogs had discovered a simple 2-dimensional domain with the property that every holomorphic function on it has a holomorphic extension to a strictly larger region. Shortly thereafter, E. E. Levi recognized that every domain of holomorphy satisfies a local boundary condition (now called "pseudoconvexity"), and he asked whether, conversely, pseudoconvexity of G implies that G is a domain of holomorphy. Levi's problem defied solution until 1942, when K. Oka, culminating a series of brilliant ideas, proved that in dimension two the answer is affirmative [55]. The restriction to dimension two was critical, and the higher-dimensional case remained open. But after the difficult immediate postwar years, time was ripe for the solution of the Levi problem in arbitrary dimension. In his 1951 dissertation [1, 2], Bremermann first solved the problem within the class of so-called Hartogs domains. Shortly thereafter he handled the case of semi-tubes [3], thereby extending earlier work of S. Bochner, and finally he produced the solution for the general case [4]. Bremermann's proof followed in broad outline Oka's 1942 approach, improved by new methods in order to overcome the limitations to two variables. The solution of Levi's problem was an international event: it was also solved around the same time in Paris by F. Norguet and in Japan by Oka.

Plurisubharmonic functions were a key tool in all solutions to the Levi problem. Now Bre-

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mermann turned to investigate them in their own right. These functions were introduced by P. Lelong and Oka in the early 1940s as the appropriate several complex variables generalization of classical subharmonic functions in the plane, which had first appeared in Hartogs's work on the region of convergence of power series in two variables. Plurisubharmonic functions are those upper semicontinuous functions whose restrictions to all complex lines are subharmonic (where defined). In dimension $n > 1$, this condition is considerably more restrictive than $2n$-dimensional (real) subharmonicity. Among the simplest examples of plurisubharmonic functions are the logarithms of moduli of holomorphic functions. The class of plurisubharmonic functions is a positive cone closed under the natural analytic operations of taking (bounded) suprema and decreasing limits of sequences. Bochner and Martin had conjectured in 1948 that every plurisubharmonic function on a region G can be generated in this way, starting from logarithms of moduli of holomorphic functions on G. In 1956 Bremermann [5] proved this conjecture for domains of holomorphy and showed that it fails for general regions. This work continued to stimulate new investigations (see, for example, [51] and [47]). Shortly thereafter, Bremermann published another major paper on plurisubharmonic functions [6]. Oka and Lelong had shown that pseudoconvexity can be characterized in a simple and elegant way as follows. An open set G is pseudoconvex if and only if $-\log \operatorname{dist}(z, bG)$ is plurisubharmonic on G, where the distance from z to the boundary bG is taken with respect to the Euclidean metric. Bremermann gave a new proof of this result in which in particular he replaced the Euclidean metric by an arbitrary norm. This improvement has been useful in a number of applications, and it further demonstrated the fundamental nature of pseudoconvexity. Bremermann's proof was adopted by L. Hörmander in his 1966 classic [52], and it thereby became widely known to several generations of complex analysts. Another major contribution in this paper was the thorough investigation of the remarkable formal analogies between pseudoconvexity and plurisubharmonic functions on the one hand and (Euclidean) linear convexity and convex functions on the other hand, a phenomenon which continues to intrigue and present deep problems to complex analysts and which even today is not yet fully understood. At the root lies the simple observation that subharmonic functions are the formal generalization of convex (or "sublinear") functions of one real variable. (Note the obvious fact that harmonic functions, i.e., solutions of the Laplace equation, in one real variable are precisely the linear functions!) Also, convex functions in several variables are precisely those functions whose restrictions to all real lines are convex (where defined)—by analogy, we might call these functions "pluri-convex". Bremermann exhibited the formal analogy in numerous ways, many of them quite deep, surprising, and nontrivial. Suffice it to mention the following result, apparently not previously known, that is proved in [6]: An open set G in R^n is convex if and only if the function $-\log \operatorname{dist}(x, bG)$ is convex on G.

While working on the paper [6], Bremermann already had an eye towards holomorphic functions on infinite-dimensional Banach spaces. Building upon the methods he had introduced there, he developed these ideas in three papers [7, 8, 11]. Bremermann based his work on the notion of Gateaux-holomorphic functions, i.e., those functions on an open subset of a complex B-space whose restrictions to all finite-dimensional complex affine subspaces are holomorphic (where defined). In particular, he extended several classical results to this setting, including the fact that domains of holomorphy are pseudoconvex and that pseudoconvexity is again characterized by the plurisubharmonicity of $-\log \operatorname{dist}(z, bG)$. Infinite-dimensional holomorphy eventually grew into a major separate research area, which in turn has played an important role in finite-dimensional complex analysis, for example, in proofs of existence of complex structures on abstract "moduli" spaces, where finite dimensionality is not known a priori.

In 1959 Bremermann broke new ground with a generalization of the Dirichlet problem [10]. Given a continuous function $u(z)$ on the boundary of a domain G in C^n, he asked for conditions which would imply the existence of a plurisubharmonic function $p(z)$ on G, which takes on the prescribed boundary values in an appropriate sense. He completely solved the problem for two basic classes of domains, the strictly pseudoconvex domains and the analytic polyhedra, proving that the boundary values could be prescribed on the full topological boundary of G in the former case and on the so-called distinguished boundary in the latter case. More generally, he showed that on a large class of pseudoconvex domains the problem was generally solvable, provided the boundary values were prescribed only on the Shilov boundary of the Banach algebra of holomorphic functions with continuous boundary values. In case the boundary is smooth, he gave a complete description of the Shilov boundary in terms of the Levi form, a well-known fundamental differential invariant. Based on his work in [8], he also introduced methods to construct and estimate the plurisubharmonic extension. Ideas introduced in this fundamental paper have inspired later investigations, for example, in 1987 N. Sibony [56] introduced the class of B(remermann)–regular pseudoconvex domains and showed their equivalence to a class of domains introduced by D. Catlin, which in turn is intimately connected with fundamental questions in the theory of the $\bar{\partial}$-Neuman problem.

In the meantime, Bremermann's interests in applications outside of mathematics had started to take form. Already in 1957, together with two physicists, he had applied extension properties of holomorphic functions to investigate analytic properties of two-particle scattering amplitudes in quantized field theories [9]. In 1961 he began to investigate a number of questions relating complex analysis to Schwartz distributions [12, 16]. This work culminated in the monograph [17], which provides an excellent introduction to distributions and a detailed exposition of Bremermann's contributions to the subject. In particular, the idea of representing a distribution as the difference of the boundary values of two holomorphic functions is thoroughly explored in different settings. By the end of the 1960s Bremermann's interest had shifted from complex analysis to mathematical questions in biology.

H. Bremermann's last contribution to complex analysis is a beautiful expository article on "Several Complex Variables" (in *Studies in Real and Complex Analysis*, Math. Assoc. of Amer. and Prentice Hall, Englewood Cliffs, NJ, 1965). It provides an excellent overview of some of the basic fundamental results in the field, suitable for a wide audience. It is most highly recommended reading for anyone who wants to learn more about the subject. It should help to remind us of Bremermann's important work.

— *R. Michael Range*

Bremermann's Contributions to Mathematical Biology

Already an accomplished mathematician, in the late 1950s Bremermann's interests shifted to the nascent fields of artificial intelligence and mathematical biology. Bremermann pioneered several new areas of research, including complexity theory, genetic algorithms, neural networks, fuzzy logic, optimization theory, pattern recognition, and evolutionary biology [46, 49]. He continued to develop mathematical modeling as a tool to understanding complex (especially biological) systems for the rest of his life. His intellectual journey was marked by brilliant insight and foresight.

His interest in computation and practical algorithms developed early from seminars on Turing machines in Münster and his own frustrating experiences programming von Neumann's computer, MANIAC [46]. Bremermann developed a keen sense of the physical and practical limitations of brute computation. This work led to the development of the "Bremermann limit", an estimate of the computational capacity of matter in the universe and a seminal contribution to the emerging field of complexity theory [25, 28, 33, 48]. Around the same time, he laid out an agenda for artificial intelligence in a monograph, *The Evolution of Intelligence* (1958). Sponsored by the Office of Naval Research, these early publications were rapidly translated into Russian, and Bremermann's name became well known across Eastern Europe.

He was among the first to develop and analyze evolutionary/genetic search procedures employing all of the elements of modern genetic algorithms—binary and continuous genetic coding, random mutation, selection, and sexual recombination [13, 14, 15, 18]. He was the first to suggest applying such algorithms to training multilayer perceptrons [19], which he finally implemented in 1989 [42]. Genetic algorithms have since become a popular method of training neural networks [50]. In 1995 he was awarded a lifetime achievement award by the Evolutionary Programming Society, and the proceedings of this year's annual meeting are dedicated to his memory.

Early on Bremermann recognized the limits of rule-based artificial intelligence approaches to pattern recognition [22]. With Lotfi Zadeh he supervised several theses on fuzzy logic [23]. With his student, Richard Hodges, he attacked the problem of pattern recognition with deformable prototypes [27]. He also developed a global optimization algorithm [21], inspired by his analysis of bacteria chemotaxis [26]. The "Bremermann optimizer" has been applied to determining nucleic acid sequences [24], analysis of spectra [20], parameter estimation for the Calvin photosynthesis cycle [29], and optimal neuromuscular control [53].

Interestingly, his experience with genetic algorithms led him to consider the evolutionary significance of sexual recombination [32, 36]: he proposed a resolution to the long-standing evolutionary question: Why would two well-adapted individuals endanger their genetic heritage with the radical genetic gamble of sexual recombination and crossover [46]? Rejecting group-selectionist arguments of the era, Bremermann considered the individual benefits of sexual recombination at the molecular level [30, 31, 32, 36, 39]. Bremermann's analyses showed the error-correcting benefit of genetic mixing to be too small to be primary [30, 32]. He saw the answer in the co-evolutionary interactions between complex and long-lived hosts and rapidly mutating parasites [32, 39]. These ideas are now gaining empirical support in controlled experiments with clonal and sexual fish populations [54]. Incidentally, the dispute over the utility of the recombination "operator" continues in evolutionary computation, bitterly dividing the field [50].

Bremermann was then drawn into mathematical studies of parasitism and disease, to which he devoted the majority of his energies for the past 15 years. He analyzed pathogenic strategies [40], host/parasite interactions [35, 38], virally induced cancers [30, 34], catastrophes in cultivated crops [37], and the epidemiology [41] and pathogenesis of AIDS [43].

Bremermann continued his researches into his retirement and illness. In July 1995 he gave an invited lecture at the Dalai Lama's 60th birthday celebration in India. He recently published a series of papers on HIV pathogenesis. Using mathematical and immunological arguments, he helped expose fundamental paradoxes in the conventional, direct viral-killing model and advocated the view that AIDS is a disease of HIV-induced immune activation [44, 45].

Hans Bremermann is not only remembered for his genius but also for his warmth, generosity, courage, integrity, humility, and love. He is survived by his loving wife of forty-two years, Maribel, a native of Spain and professor emeritus of romance literature at San Francisco State University.

— Russell W. Anderson

Biographical Sketch

Hans-Joachim Bremermann, professor emeritus of mathematics and biophysics, University of California at Berkeley, died of cancer on February 21, 1996. Born in Bremen, Germany, on September 14, 1926, he earned his Ph.D. in mathematics from the University of Münster. After teaching mathematics for a year at Münster, he held teaching and research positions at Stanford University, Harvard University, again at Münster, and the University of Washington. In between he was twice invited to conduct research at the Institute for Advanced Study in Princeton, first as a mathematician (in 1955) and again as a physicist (in 1958). In 1959 he moved to the University of California, Berkeley, where he remained until his retirement in 1991. At Berkeley he held professorships in mathematics and in biophysics. He also was a member of the graduate group in bioengineering at the University of California, San Francisco. He was a founding editor of the *Journal of Mathematical Biology*. A Fellow of the American Association for the Advancement of Science, Bremermann was a member of the AMS as well as a number of other scientific societies in mathematics, artificial intelligence, and biophysics.

— Allyn Jackson

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Selected bibliography of H.-J. Bremermann are referenced [1] through [45].

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