Mathematics Is an Edifice, Not a Toolbox

In the fall of 1972 President Nixon announced that the rate of increase of inflation was decreasing. This was the first time a sitting president used the third derivative to advance his case for reelection. About five years ago my local newspaper carried an article about research on trends in the mortality rate due to heart disease. The population was divided into four genotypes, and the rates for each genotype over the preceding several decades were plotted on a graph, something like the graph pictured here. The researchers observed that the rate of decrease of these death rates was greater for the upper curves, and postulated that medical research was discriminatory toward the populations they represent. However, by fitting the data to a first-order differential equation, one finds that these graphs are all solutions of a Law-of-Cooling type of differential equation, with the same (more or less) coefficients. The dynamics for each genotype are thus the same, the only difference being in the initial conditions; and further, the steady-state rate for each genotype will be the same. Thus, no indication at all of discrimination or of genetic distinction.

Was President Nixon telling us that the economy was getting better? Did his listeners understand that in fact the inflation rate was still increasing and thus the economy still worsening? How many readers of the above medical research detected in the graph the possibility of consistent rather than differentiating dynamics? What are the implications of such abuse of mathematics and science, and does it lead to the “cultural studies” described in two articles in this issue?

Certainly mathematics provides the tools for creating and dealing with the modern technological environment. It is good that we make sure the public understands this and that we instruct our students in the use of these tools. Mathematics also provides techniques for creating, developing, and testing new ideas and sets a standard for proof and objectivity which is rarely attainable in other sciences and less so in “real life”, but which provides the model to strive for and against which arguments can be measured. But, most important of all, mathematics is a structure providing observers with a framework upon which to base healthy, informed, and intelligent judgment. Data and information are slung about us from all directions, and we are to use them as a basis for informed decisions. Often we are told what those decisions should be; can we reliably evaluate that advice? Can we tell what tools to select if we do not know the relationships among our tools? Do we use a t-test on a genetics problem because we saw it used in genetics elsewhere or because we saw it used on a similarly structured problem elsewhere? The essence of context-free—abstract—mathematics is that it gives us the flexibility to use those tools in a variety of contexts.

Ability to critically analyze an argument purported to be logical, free of the impact of the loaded meanings of the terms involved, is basic to an informed populace. Understanding mathematical ideas and procedures in the absence of context gives the power to remove the context from a “real-life” problem, understand the structure of the problem, and select the right tools to use.

In law schools, students study hundred-year-old cases on contentions that have absolutely no relevance today. This is done so the students can see the structure of the law. Should not mathematics be so presented, at least at critical junctures, so that students can see the structure of science?

Perhaps someday a sitting president and medical researchers will know what room they are in when they enter the edifice of mathematics.

—Hugo Rossi, Editor of the Notices