Mathematics: the Science of Patterns

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Mathematics: the Science of Patterns
Keith Devlin
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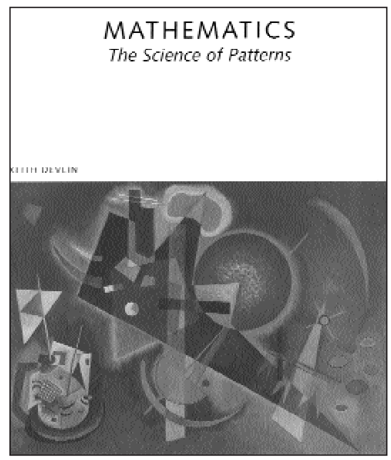
Among the variety of activities which constitute the career of the professional mathematician, communicating the nature of mathematics to those not in the profession is surely one of the most frustrating; so frustrating, in fact, that many of us are unwilling even to try it. Where do the difficulties come from? First of all, for many people, their relationship to mathematics has been defined by their educational experience. They take mathematics only as long as it is required, and neither the experience itself nor its recollection tends to be happy. Try to tell them about mathematics? They will simply shut you out. Admittedly, the same distaste is often characteristic of science as well. Science, however, is more in the habit of making its case for research in terms of usefulness than is mathematics. The applicability of mathematics is very real but often hard to see in specific areas; the topics on which we actually work may not easily lend themselves to communication. So a second major difficulty in explaining what we do is to find a way to begin, to make a connection with some previous experience or human need. Our standards for communication are yet another problem: It is our professional habit to be as precise as we can, and if we gloss over some fine points in the interest of clarity, we will criticize ourselves severely if our colleagues have not done it for us already. All in all, a classic no-win situation.



Nevertheless, the mathematical community's concern with popular communication is on the rise. The first international conference on popularization was held in 1989 under the auspices of the International Commission on Mathematical Instruction, and the subject continues to be represented in national and international meetings. Television, museum exhibits, and to a growing extent, software, can be used to reach a mass audience, but the more traditional media of books and periodicals continue to dominate communication with the amateur mathematician and the educated lay public, two favorite targets of opportunity.

There have, over the years, been many "popular" books for these audiences, and the nature of such books is interesting. As a typical example, look at one of the most successful, *The Enjoyment of Mathematics*, by Hans Rademacher and Otto Toeplitz. Its subtitle is *Selections from Mathematics for the Amateur*. It was originally published in 1933 (in German), and the English version dates from 1957.

It contains a series of developments, averaging about eight pages in length, of separate topics in mainly geometry, combinatorics, number

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theory, and topology. They presuppose a first-rate high school education, and each example is self-contained, reaches closure, and is elegant, a term whose meaning we all recognize but do not find easy to explain.

The problems were not chosen to be current, or applicable, or especially important: they are beautiful problems and embody beautiful mathematical thought.

Keith Devlin's *Mathematics: The New Golden Age*, published in 1988, serves as a second example. It is intended for the interested layperson. Each development is perhaps three times as long as in Rademacher and Toeplitz, and there is much less actual proof and much more informal description, exposition, and history. The intent is to make it possible to appreciate eleven separate newsworthy developments in mathematics during the twenty or so years preceding the book's publication.

These two books are excellent examples of their genres and are typical of popularization for the amateur mathematician and the educated layperson respectively. What is missing in both books, and in the many other popular books I have seen, is the feeling for the connectedness and the continuity of mathematics, the way in which mathematics develops and progresses. This is not a complaint about those two truly excellent books; that is not what they were trying to do. But if you really want to get across where mathematics has been and where it is now, this continuity of thought is fundamental. The first book I know which truly attempts to convey the logic behind the development of mathematics is Keith Devlin's *Mathematics: The Science of Patterns.*

The six chapters of the book are entitled "Counting", "Reasoning and Communicating", "Motion and Change", "Shape", "Symmetry and Regularity", and "Position". Each chapter takes the reader from the beginnings of the subject, be they ancient times or the eighteenth century, up to the present time. The continuity in each chapter is truly remarkable and exemplifies not only a command of the subject matter but a style of writing of which a mystery writer could be proud. Take the chapter on position, for example. The lead-in to the notion of topology is the familiar map of the London underground. The history of the subject begins with Euler and the Bridges of Königsberg and is followed by a transition to networks and the Euler formula in the plane and on the sphere. Möbius's definition of a topological transformation leads to surfaces and naturally to their orientability and nonorientability. The Euler characteristic is identified as an invariant which together with a number of crosscaps is used to classify surfaces. Then surfaces are generalized to manifolds, and homotopy is defined and illustrated. Do homotopy groups distinguish manifolds that are not topologically equivalent? What follows is a history of the Poincaré conjecture up to the present time.

The continuity in the chapter so far is evident, but the style is also marvelous. Here is the paragraph which ends the discussion of the Poincaré conjecture:

> It should be said that the expectation that the one remaining case of the Poincaré conjecture will turn out to be true is certainly not based on the fact that all the other cases have been proved. If ever topologists had thought that all dimensions behaved in more or less the same way, they were forced to change their views radically by an unexpected, and dramatic, discovery made in 1983, by a young English mathematician named Simon Donaldson.

Devlin then reminds us of differential calculus and of the definition of a manifold and goes on to ask about differentiation structures on Euclidean n-space. This is then followed by the story of knots and their invariants, the significance of the Alexander and Jones polynomials, and the connection with the work of Atiyah and Witten.

At this point Devlin is ready to return to Fermat's Last Theorem, which had naturally come up in Chapter 1, "Counting". There is discussion of the associated curves and surfaces, of Mordell's Conjecture, and the outline story of the sequence of discoveries which led to the proof by Wiles (which was not complete at the time the book was finished). This ends the chapter on topology and the book as a whole. It may seem surprising that the discussion of Fermat's Last Theorem comes in a chapter on topology, but both the chapter and the theorem reflect the book's overall theme of the unity of mathematics.

The continuous flow of exposition which is so characteristic of this book is achieved by rigorous control of not only what is put in but also what is left out. Enormous self-control is evident in its resistance to mathematical and pedagogic temptations. For example, in the presentation of the method of generating primitive Pythagorean triples, the conditions are $s > t$, s and t have no common factor, and one of s, t is even, the other odd. The temptation to ask what happens if both s and t are odd would be overwhelming to me: It would be so nice for the readers to see that in that case the solution would not be primitive—and why not. But that would interrupt the flow, and it is not done. Devlin does not even suggest this as an exercise for the reader—that is not the point of the book.

As I said, there is bound to be some sniping from mathematical colleagues, and I will try to fulfill this obligation. The constant in the formula for the volume of a truncated pyramid on page 14 is $1/3$, not $1/2$. There is the usual demonstration that the harmonic series diverges, but then the completeness axiom on page 98 reads as if the partial sums of that series had to have a real number as a limit. The discussion of the differential and integral calculus in Chapter 3 comes close at several points to leading the reader to infer that a function is a formula. In Chapter 2, it is not clear (p. 39) whether the predicate includes the verb itself. We are told that S denotes Socrates and P denotes the predicate "is a man", but in the next paragraph the proposition is restated as "S is P", which strictly speaking would have "is" in it twice. Might a second distributive law be mentioned for a Boolean algebra (p. 43)? The discussion of the meaning of i^i on page 101 omits the possibility that it could have an infinity of values.

Not having Devlin's self-control, I have allowed these last comments to interrupt the flow of this review. *Mathematics: the Science of Patterns* is a wonderful book for the layperson about the development and structure of mathematics. Devlin succeeds in keeping the reader enthralled with this story by taking the patterns of mathematics as they are, neither neglecting the applications of the subject nor relying on them to hold his audience. Physics and cryptography and codes for communication and wallpaper design have their place but are not the glue that keeps the story going. It is a great story.

Mathematics education is not the subject of this book, but one cannot help bringing it to mind as one reads and marvels at the story of mathematics as the science of patterns. The traditional memorization of formulas and tricks does not come very close to communicating the spirit of mathematics. Has anyone tried to develop the big ideas, as Devlin has done so successfully? One example comes to mind, namely, *On the Shoulders of Giants* from the Mathematical Sciences Education Board and edited by Lynn Steen, who has done much to get people to think of mathematics as the science of patterns. Several of the main themes in the two books coincide. Devlin's book is hugely successful in introducing the lay reader to the real spirit of mathematics and in bringing that reader to some appreciation of the research frontier. Let us hope that in the future mathematics education will lay a good foundation for this appreciation.