New Directions at the IAS

The Institute for Advanced Study (IAS), with its serene, sloping lawns and dignified buildings, is a shrine to intellectual achievement. Some universities—like the one across town in Princeton—might rival the Institute in the distinction of their faculties, but for sheer focus on higher intellectual pursuits the IAS wins out. The Institute has no degree programs, no courses, and very little bureaucracy to distract its thinkers from their thoughts. Long known as the ultimate ivory tower, the Institute, under the directorship of Phillip Griffiths, is applying its high standards to a broader range of goals. Nowhere is this more evident than in the programs of the IAS School of Mathematics and in the IAS/Park City Mathematics Institute, for which the IAS served as the host site this summer.

Applications a New Focus

For many years the the School of Mathematics at the Institute was known primarily as a center for research in pure mathematics, especially areas such as representation theory and number theory. More recently the school has broadened its agenda to include what Griffiths calls "theoretical applied mathematics". The shift began about ten years ago, when the Institute hired Luis Caffarelli and Thomas Spencer. Caffarelli is an expert in partial differential equations and is well known for his work on free boundary problems and fluids (he recently moved to Courant Institute). Spencer, a mathematical physicst specializing in statistical mechanics and random media, has recently become interested in the statistical description of solutions to partial differential equations. For the past few years the Alfred P. Sloan Foundation has provided crucial finanacial support to support the Institute's initiatives in several aspects of applied mathematics.

This past year, the research program focused on wave (or weak) turbulence, such as one might find on the surface of a calm ocean. The problem "sounds simple, but it's not," Spencer notes. While there are relations to the problem of full turbulence and the usual descriptions given by the Navier-Stokes equations, the mathematics is very different. In particular, the conventional methods of equilibrium statistical mechanics do not work, and one needs to develop new approaches.

The previous year the Institute hosted a research program that examined properties of semiconductors and lubricants, substances that have some fluidity but are not well described by fluid mechanics. The main tools are kinetic equations, the best known among them being the Boltzmann equations. "Kinetic theory is an area where American mathematicians are not so active, but there is a lot of activity in Europe," Spencer notes, "so we had a lot of participants from Europe." Although the researchers worked on some problems coming from industrial applications, the thrust was nevertheless theoretical. The work contrasts with that of, say, the Institute for Mathematics and its Applications at the University of Minnesota, where there are direct contacts with people from industry and the work is much closer to the specifics of given applications.

Another major push in applied directions, headed by IAS faculty member Enrico Bombieri, has centered on combinatorics and theoretical computer science. Many of the activities in this area have been organized in conjunction with DIMACS, the NSF-sponsored center on discrete mathematics and computer science which is a consortium of Rutgers and Princeton Universities and AT&T Research, Bellcore, and Lucent Technologies. This coming year there will be a program on those areas of discrete mathematics that are close to statistical mechanics, and efforts will be made to connect people who work in combinatorics with those working in mathematical physics.

Robert MacPherson, who joined the IAS mathematics faculty two years ago, has become interested in some problems in theoretical computer science and combinatorics since his contacts with the researchers in these areas. In fact, last year he solved a combinatorics conjecture posed by Gil Kalai, who was visiting the IAS from the Hebrew University of Jerusalem, and they ran a seminar on the solution. "Some would worry if this would fit into the Institute because we're very theoretical," says MacPherson. "But these people have talked to others in almost every area of mathematics about the problems that are growing out of computer science."

This year the IAS is launching an ambitious, three-year program in mathematical physics, headed by IAS faculty members Pierre Deligne, from the School of Mathematics, and Edward Witten, from the School of Natural Sciences. The goal of the program is to familiarize mathematicians with the latest ideas in theoretical physics, such as string theory and mirror symmetry. Over the years Witten's intuition about physics has allowed him to come up with bold conjectures that mathematicians seized upon to prove some spectacular theorems. The most recent example came in late 1994, when Witten and his colleague Nathan Seiberg of Rutgers University proposed that certain equations contained much of the information of Donaldson theory. Now known as the Seiberg-Witten equations, they have sparked a revolution in low-dimensional topology.

The IAS program will not try to mine the Seiberg-Witten equations for more mathematical theorems. Rather, the program's focus will be on developing mathematicians' knowledge of physics so that they too can tap into physical intuition and eventually gain insight into mathematical problems. "Mathematicians will learn about path integrals, renormalization, gauge theory—some physics, basically," Spencer explains. "What they can get out of it is a new way of thinking about mathematics—and it won't be a rigorous way of thinking." For this reason, the participants will be people "who will not insist on proving theorems at every step," says Spencer. MacPherson calls it "unlike anything anyone has ever tried before."

This less rigorous approach has been the cause of some controversy in the mathematical community. In the article "Theoretical Mathematics", which appeared in the *Bulletin of the AMS* in 1993, Arthur Jaffe and Frank Quinn laid out the concerns that many have to taking such a nonrigorous approach in mathematics research. In setting up the new program, the Institute seems to be taking to heart the views of Sir Michael Atiyah, who, in a rebuttal to the Jaffe-Quinn article, advocated a "more buccaneering style" in mathematical research.

Linking Research and Education

The late Hassler Whitney, who was a professor at the IAS from 1952 until his retirement in 1977, worked hard on problems of mathematics education for the last twenty years of his life. Following in Whitney's tradition, the IAS this summer played host to thirty-seven high school teachers as part of the IAS/Park City Mathematics Institute (PCMI). Whitney occasionally brought teachers to the IAS, but, according to MacPherson, the PCMI constitutes the largest organized contingent of high school teachers ever brought to the Institute.

The PCMI grew out of the Regional Geometry Institute that started six years ago in Park City, with funding from the NSF. This is the third year that the IAS has served as institutional sponsor, and the first year that the PCMI has met in Princeton rather than in Park City. The PCMI brings together high school teachers, undergraduates, graduate students, and researchers, with parallel sessions for each group and informal discussions and social events for everyone. The unifying theme of the program is the spirit of research and discovery. For example, Gregory Lawler of Duke University and Emily Puckette of Occidental College, who served as instructors in the PCMI program for undergraduates, say they covered material that is off the beaten track of usual undergraduate courses in probability and that is closer to the frontier of research. The courses combined lectures and informal group discussions with the students. In computer laboratory sessions, students got to explore some open problems in random walks.

The idea of integrating research and education has become a hot topic, particularly with the NSF. In fact NSF Director Neal Lane has made it a theme of his tenure. Lane visited the Institute during the PCMI and was the keynote speaker at a special event there entitled "A Celebration of Mathematics: Teachers and Researchers Working Toward Excellence in Mathematics Education". In an interview, Lane said that the idea of integrating research and education is based on the assumption students benefit by learning in an environment in which research is going on. "In the classroom is a person who in other hours of the day is probing fundamental, difficult, challenging questions" in science or mathematics, he said. "This creates a certain attitude toward inquiry and discovery, a frame of mind, a different take on the world. It's a combination of enthusiasm, openness, and creative thought processes." Sometimes professors can bring into the classroom things they have been doing in their research. Said Lane, "There's an element of romance about it."

In higher education, such integration can happen fairly easily. The PCMI tries to siphon some of the excitement off to the high schools by linking teachers and researchers. But do the various groups at the PCMI really mix? According to John Polking of Rice University, who serves as convener of the PCMI Steering Committee, the mixing of the groups is "the overriding purpose" of the program. "Everyone in mathematics has their own job, things they are good at, things they have to do," he explained, so it is not possible that all of the groups will be interested in the same things. However, "they gain mutual respect because they are all doing what they do as well as possible." A random sampling of PCMI participants report that the amount of mixing is variable. There was plenty of interaction between the graduate students and researchers and some between the graduate students and the undergraduates, but less between the high school teachers and the other groups.

One reason the teachers are more isolated is that their program is more structured than the programs for the other groups, so they become a more cohesive clan. Groups of teachers are drawn from specific geographic sites and attend the program for two consecutive summers. Each site has a director, usually a mathematician in a local university or college. During the academic year, the teachers meet as a group and with site directors. By contrast, the topic for the PCMI research program, and hence the participants, changes each year; this year the topic was probability and next year it will be symplectic geometry. The graduate and undergraduate programs follow suit and therefore attract a different group of students each year.

In addition, the teachers' activities in the PCMI provide a sharper contrast with their day-to-day work than those of the other groups. For many of the teachers, the PCMI is an oasis from the pressures of teaching high school. Jo Vaccaro, a teacher at Stafford High School in Houston and a participant in the Rice University PCMI Site, said that at least twice during the school year ambulances visited her school to rescue students who had overdosed on drugs or attempted suicide, and several other students had suicidal tendencies or severe behavioral problems. "Many of the students have terrible home lives," she noted. "Many have little motivation to put a lot of effort into their schoolwork." For the teachers, "It is sometimes difficult to keep ourselves motivated." Programs like the PCMI "help me recharge my enthusiasm and refocus my love for teaching," she said.

What the PCMI teachers' program tries to do is get the teachers thinking deeply about and exploring mathematics. Polking taught a course on geometry which included such things as Girard's theorem about the sum of the angles of a spherical triangle. After the teachers had experimented with drawing triangles on inflatable beach balls, they took a crack at discovering the formula in Girard's theorem, which says that the sum of the angles is $\pi +$ (area of triangle) $/r^2$. As Polking was planning to discuss the generalization to the Gauss-Bonnet theorem (in which the triangle is a geodesic triangle on a more general surface, and the term (area of triangle) $/r^2$ is replaced by the integral of the curvature over the triangle), he was clearly enthusiastic: "This is such a winner. How could anybody not like this?" In the class, the teachers enjoyed the discussion even when a few giggles indicated that the presentation was sailing over some heads. Questions about the mathematics mingled easily with suggestions about how to use models to demonstrate notions of curvature in the classroom.[1]

The program also includes sessions that focus more exclusively on classroom practice; these are led by Naomi Fisher of the University of Illinois at Chicago and Cindy Hays of McCallum High School in Austin, Texas. As Fisher points out, these sessions are very different from the usual "in-service" program for training teachers in the use of a new kind of teaching module or technique. Indeed, much of what they do in the PCMI program is not immediately transferrable to the high school classroom. "What we are aiming for is to encourage teachers to take a fresh look at familiar mathematics and standard high school topics in order to consider ways to rearrange their own curricula, and introduce new ways to present standard topics to make their teaching more coherent and absorbing for their students," Fisher explained. In particular, they try to get the teachers to make connections between separate parts of the mathematics curriculum, especially between geometry and other areas. She observes that some of the teachers make slow, evolutionary changes shifts, while others make bold, radical changes. Said Fisher, "What is exciting to me is to see teachers gain the confidence to make these decisions for themselves and carry them out successfully."

Clashing Views

One thing that comes out in some of the PCMI activities is that often the researchers and the teachers have very different views on high school mathematics education. In a lunchtime conversation with Elton Pei Hsu of Northwestern University, who led a graduate course at the PCMI, Michael Cranston of the University of Rochester, who was in the research program, and undergraduate instructors Gregory Lawler and Emily Puckette, there was general agreement that it is essential that high school students have a solid grasp of basic techniques such as factoring polynomials. There was also considerable wariness of student dependence on calculators and sympathy for the notion that calculators should not be allowed on exams. (And it's easy to see why: Lawler said that he had seen on an e-mail discussion group the comment that now that one has calculators there is no longer a need for such formulas as $\sin 2x = 2 \sin x \cos x$.)

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[1]*Notes from this class may be found at the web site* http://math.rice.edu/~pcmi/sphere.

Such views make many teachers shake their heads and cluck their tongues. Having tasted a richer and more exciting menu of mathematics than the one they were accustomed to, they are skeptical of views that smack of "back to basics". To those wary of the trend toward nontraditional approaches to teaching mathematics, Shirley Hill has this to say: "We certainly didn't do it awfully well the other way." Hill is a retired mathematics professor from the University of Missouri at Kansas City who serves on the PCMI Oversight Board. If the goal is to produce a few top-notch mathematicians, perhaps no changes are needed in the teaching of mathematics, she said. "But I think you've got to look at a broader population that's got to understand a lot more mathematics. And now you have to ask, Would the past programs produce that? I don't think so. What's happening now may not be fully evolved, but we don't have a great record of success to fall back on."

Tim Giesbrecht is a teacher in Franklin Junior High School in Pocatello who is a member of the Idaho State University PCMI site. He said that in order to capture the attention of his students he has to present mathematics in such a way that it relates to their lives in some fashion. Visualization helps a great deal, he said. For example, he worked with his students on the mathematics behind a simple harmonic oscillator. Being able to model the motion on a graphing calculator helped to spark the students' interest. Giesbrecht compared students' interest in mathematics to their interest in cars. "It's difficult to get students interested in the details of learning how to fix cars," he said. "What they want to do is drive cars. In the same way, we want to show them how to 'drive' mathematics, not 'fix' it."

For teachers to be able to take that kind of exploratory approach, they need to be confident about their mathematical knowledge—and to be comfortable knowing that they don't have all the answers. Cindy Hays from McCallum High School in Austin, Texas, was on a panel, held in conjunction with "A Celebration of Mathematics". which focused on connecting mathematics researchers and teachers. One of the best things about the PCMI, Hays noted, was that it "got me in touch with how fun it is not to know the answers." In talking to the research mathematicians attending the PCMI, she found that they don't know all the answers either. "I have much more freedom to talk about different topics with the students," she declared. "I won't necessarily understand all of hyperbolic geometry or spherical geometry, but I can give my kids a glimpse of the magic and the power of mathematics without knowing all the answers."

—*Allyn Jackson*