Reporter's Notebook: A Seattle Sampler

The Seattle Mathfest, held August 10–12, 1996, on the campus of the University of Washington, provided an occasion to sample some fine mathematics while taking in spectacular views of Mount Rainier. And if three days of talks and panel discussions were not enough, there was a symposium immediately after the Mathfest, entitled "In Celebration of the Centenary of the Prime Number Theorem: A Symposium on the Riemann Hypothesis". The symposium, sponsored by the American Institute of Mathematics (AIM), featured a rare public lecture by Fields Medalist Atle Selberg, one of the major figures in the field. (An article about AIM and the symposium is planned for an upcoming issue of the *Notices*.)

The AMS has decided to get out of the business of this kind of summer meeting, at least till the year 2000; starting in 1997, the Mathematical Association of America will be the sole sponsor of the Summer Mathfest. The Society will continue to offer specialized summer seminars and institutes, but has ended its participation in this kind of general summer mathematics meeting. The fact that the Seattle Mathfest was quite successful, attracting about 1,200 attendees, must have left a few AMS hearts wistful. What follows is a small sampler of the offerings in Seattle.

A Bit of Bubbly

The Seattle meeting featured a number of sessions that broke with traditional formats. A talk on the "double-bubble conjecture" by Joel Hass of the University of California, Davis, included a panel that kibitzed on the lecture as it proceeded. Frank Morgan of Williams College moderated the panel consisting of Jenny Kelley and Jean Taylor of Rutgers University and Helen Moore of Bowdoin College. The program said that the panel would add their "two cents' worth" after Hass spoke, but in fact the panel spoke before the lecture, during a break midway through, and also at the end. Such a panel could be disruptive, but as it turned out the different viewpoints and the questions of the panelists helped to illuminate the topic.

The topic of Hass's lecture was a variant on the isoperimetric problem, which goes back to the time of the ancient Greeks. As Virgil recounts in the *Aeneid*, Queen Dido cut a deal with chieftains in North Africa in which she would be allowed to keep as much land as she could enclose inside a fence of a given length. That her choice of a circular fence was the one that gave her the most land was not established mathematically until the nineteenth century, when it was proven by Weierstrass. The 3-dimensional isoperimetric problem poses an analogous question: Among all shapes of a given surface area, which encloses the maximum volume? Or, said differently, given a certain volume, what is the smallest-area surface enclosing the volume? In 1882 Schwarz proved it is a sphere.

The question Hass explored is, What is the smallest-area surface enclosing two given volumes? Intuitively, it seems it would be a double-bubble: two identical spheres that meet at an angle of 120 degrees, with a flat disk separating their interiors. That this is the case was proven last year by Hass and Roger Schlafly of Real Software of Santa Cruz. Their proof relied on work of Michael Hutchings, a graduate student at Harvard who participated in the Research Experiences for Undergraduates program at Williams. Hutchings showed that each of the two pieces of the bubble have to be connected. Another important piece of information came from joint work of Frank Morgan of Williams and Brian White of Stanford, who proved that whatever the most efficient surface was, it had to be a surface of revolution. The field of solution candidates was then reduced to two: the double-bubble or one of a family of "torus bubbles"—a torus bubble is a sphere with a torus around its middle.

The Hass-Schlafly proof was unusual in that a computer carried out some of the critical steps. The torus bubbles can be classified according to two parameters, an angle and a mean curvature. Using geometric and other arguments, they were able to reduce the possible values of each parameter to a closed interval, so that the the parameter space became a rectangle. All that is left to do is to check the areas and volumes of all of the candidates to see if any of them is more efficient than the double-bubble. The problem is, of course, that there are infinitely many candidates. How can the computer handle such a calculation?

Hass explained that their program used what is known as "interval arithmetic", in which the computer performs calculations not on floating-point numbers but on intervals with endpoints that are floating-point numbers. For each 2-dimensional interval in the parameter space, the program produces an interval of values of the volumes of the torus bubbles represented by that interval. If the range of the volumes of each component are unequal, the program tosses out that interval, since only equal volume solutions are being sought. Other tests are used if volume comparison does not apply. The remaining intervals are then subdivided again, and the same elimination is carried out. This process produces strict upper and lower bounds on the volumes of each component of the torus bubble. From these and similar bounds, the proof follows. (For an expository presentation of this result, see "Bubbles and Double Bubbles", by Joel Hass and Roger Schlafly, *American Scientist*, September-October 1996, pages 462–467. This and related papers may be found on the World Wide Web at `http://www.math.ucdavis.edu/hass/bubbles.html`.)

The panel asked Hass if this method would work for the analogous "triple-bubble" problem, which is still open. Could it be three spheres stuck together, or maybe a double-bubble with a torus around its "waist"? It is not even known whether these are the only possible candidates. Hass said he did not know whether his and Schlafly's proof was adaptable to this problem. But he did mention that this result should apply to spaces with metrics that are "close to" the Euclidean metric. Hass, Hutchings, and Schlafly are working to generalize their result to bubbles enclosing unequal volumes and bubbles in manifolds.

The Ghost of Rochester

Although the crisis at the University of Rochester has been resolved, its ghost still haunts the mathematical community. Last fall the Rochester administration announced it would cut its mathematics department faculty by half and eliminate the mathematics graduate program. The move produced a strong outpouring of protest, a good deal of it generated by the AMS Task Force on Rochester, and by March the administration and the mathematics department reached an agreement in which the graduate program would be reinstated and the cuts in faculty made less severe. Despite the relatively happy ending, the fact that a highly regarded research university was seriously questioning the value of having a mathematics graduate program left a deep impression on the mathematical community.

"How Can You Defend Your Graduate Program in Mathematics?" was the title of a panel discussion sponsored by the AMS Committee on Education and organized by committee member Harvey Keynes of the University of Minnesota. The three panelists took rather different tacks on the question. John B. Conway, head of the mathematics department at the University of Tennessee at Knoxville, said he is not convinced that graduate programs are under attack. "If Rochester is $n = 1$, then what is $n = 2$?" he asked. He pointed out that it is very difficult to ferret out why the Rochester crisis occurred; one would have to examine the situation two, three, or even ten years earlier to understand how it came to pass. Nevertheless, he had a number of suggestions to offer to protect one's graduate program: teach calculus better, examine precalculus, improve the undergraduate major program, and, finally, connect to engineering and science departments. "If the crunch comes and darkness is at your door," he said, "these are the only allies you can have."

Bus Jaco of Oklahoma State University saw a number of threats to mathematics graduate programs. One of them is the mathematical community itself: overproduction of mathematics Ph.D.s has fueled comments that the lesser lights among graduate programs should close up shop. For Oklahoma State, which has been improving in recent years but finds its backwater image hard to shed, such comments hit close to home. Jaco suggested that some sort of national accreditation might be in order, though he opposes “Ph.D. birth control” on a national scale. He also suggested that, rather than large changes in the graduate program, what may be needed is a change in faculty attitude, with more attention paid to such things as exploring career options for new Ph.D.s, discussing professional and teaching issues with students, and insuring that students get early experiences in mathematics research.

Echoing this idea was William Rundell, chair of the mathematics department at Texas A&M University. Rundell’s vision is for mathematics departments to “own the boundary” of the discipline so that, for example, mathematics majors are prepared to enter Ph.D. programs in a variety of areas. Unfortunately, many departments pay scant attention to undergraduates and simply complain about their academic weaknesses. Rundell pointed out that, judging by SAT or GRE scores, other departments see mathematics majors as “Rolls Royces”. “We complain about our students as being miserable,” he said. “But if we do that, we’ll look foolish.” One result is that mathematics loses many students to management and engineering.

One of the most pernicious threats to graduate programs in mathematics is simple economics. During his presentation Rundell quoted figures about the cost-per-student-credit-hour of certain courses offered at Texas A&M. He implored the audience not to write them down, lest the figures end up in the hands of his state legislature. Suffice it to say that the difference between the cost-per-student-credit-hour of certain business courses and that for graduate mathematics courses is two orders of magnitude. With the popularity of the business major and the employment troubles of mathematics Ph.D.s, the mathematics graduate program can seem the right place to cut. “Despite the prevalence of overall departmental cost/student data, these particular figures are rarely asked for,” says Rundell. “They could be dangerous in the hands of those looking for simplistic solutions.”

During the question period, one member of the audience said that after hearing the panelists he wasn’t sure he was at the right session; he thought he had come to hear about how to defend one’s graduate program against complaints of the *students*. Many of his students have to moonlight during graduate school, he said, and once they finish they cannot find jobs. Some he hires on as instructors, and they take the jobs because they pay marginally better than the community college across town.

Rundell had one question to ask: How much can these students program in Java? Currently, he notes, there is a real demand in major computer software industries for a combination of traditional mathematical sciences training and specific skills such as Java programming—and the salaries far exceed that of an assistant professor. Being willing to take advantage of such opportunities could give students a very different perspective on graduate school in mathematics. In his response Conway took a more traditional tack. A while back he realized that most of the Ph.D.s from his department were getting jobs at four-year institutions. “Our graduate programs have ignored this reality,” he noted, as they tend to focus on preparing students for positions in research institutions. To address this problem, Conway is trying to set up a program whereby his students can teach at local community colleges to get some experience that might help them in landing jobs later on. For the mathematical community more generally, the difficulty is that no one wants to give up any part of the graduate program to do anything different. Said Conway, “We are sometimes our own worst enemies.”

Whimsical Mathematics

Mathematical objects usually have names like x, $\mathbb{Q}$, or maybe $\aleph$, if you want to get fancy. For Colin Adams and Edward Burger, these plain-Jane names are not good enough. At their talk in Seattle, these two Williams College mathematicians favored names like Bubba, Bosco, Olive, and Carlo. One object was likened to a tumor, while a deformed torus was called a “quasimodonut”. Just what were they doing with all this whimsy?

Believe it or not, they were proving theorems. For their AMS-MAA Joint Invited Address, Adams and Burger wrote, produced, and starred in a play called “Casting About: About Casting”. The characters, Sam and Buddy, were workers at the Acme Casting Factory, a metal casting plant in Allentown, Pennsylvania. The show opened with slides of Adams and Burger getting ready for work, complete with hardhats and plaid shirts, while the song “Allentown” by Billy Joel played in the background. After this introduction, the two appeared onstage in their workers’ getup. As they chatted during their lunch break, the mathematics crept in slowly: amid talk about work at the plant Sam mentions a new treat at the local donut shop called a “glazed handlebody”.

The jokes flowed fast and corny, inspiring roars of laughter among the folks packing the 800-seat auditorium. The humor was tailor-made for this audience: where else would talk of downsizing at the “Rochester casting plant” have even caused a titter?

BUDDY: (Stunned) What?! (Pause) Rochester downsized?

SAM: Yeah. The company almost cut the entire casting division, but with some pressure

Williams College professors Edward Burger (left) and Colin Adams (right) in character and costume as Buddy and Sam from their math play "Casting About: About Casting".

from the MAA and the AMS, they decided to back off.

BUDDY: The MAA?

SAM: Yeah, the Metalworkers Association of America.

BUDDY: Oh, yeah, and the American Molders Society. Boy, those are powerful organizations, I'm telling you.

Well, you had to be there.

Before this sort of thing could wear too thin, Sam and Buddy start discussing what kind of objects one can create from a two-piece casting mold (with each mold deformable into a ball). Buddy bets Sam $5 that the only possibilities are a ball, a donut, a "glazed handlebody", or "an apple with wormholes". The two stay in character, with Sam naming the object to be cast after himself and the two parts of the mold "Bubba" and "Bosco". He asks skeptical questions as Buddy goes through the proof, and the result is an intuitive explanation that is perhaps more understandable than a more straightforward lecture.

After a 5-minute intermission, Sam bets Buddy $5 that he can prove that one can cast *anything* using a 3-piece mold (provided that the object has only one boundary component). This time Sam goes through the proof of this rather surprising result, with Buddy asking the questions. The two theorems proved in the play are new results by Adams and Burger. At one point one of them says somewhat disappointedly that the results are all "theoretical". "Imagine if that's what you did all day, just sitting around chewing the cud on theoretical nonsense?" laughs the other. "Yeah, and what if they actually paid you to do it? Ah, we're being silly!"

Buddy and Sam also discuss a third result by Adams about tiling of 3-space. First they discuss tiling space by tetrahedra ("Tetrawhatdra?" one of them asks) and reminisce about when Acme Casting made bronze tetrahedral mementoes for a mathematics meeting. Adams's surprising result, which follows from the theorem proved after the intermission, says that 3-space can be tiled with knotted tori. Saying that he read about the result in *Better Homes and Gardens*, Buddy claims that this method of tiling space is all the rage for decorating bathrooms. Indeed, why tile just the bathroom floor when you can tile the whole space?

BUDDY: So now the entire bathroom is filled with these knotted doughnut shaped tiles.

SAM: I love it. Seems totally pointless, but I love it. We should make and sell tiles that look like that.

BUDDY: Hey, you know what we should do? The next time there's a big math conference, we could go there and sell these knotted tiles! Heck, they went for those silly bronze tetrahedra; they'd gobble these things up.

SAM: Hey, yeah, but I got a better one. [Laughing] Maybe at their next conference, we could go and give a presentation.

BUDDY: Yeah, right [laughing]; you and me talking to an auditorium filled with mathematicians about casting and tiling. That'd be a good one. Come on, lunch is over; we'd better get back to work.

—Allyn Jackson