Models That Work: Case Studies in Effective Undergraduate Mathematics Programs

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Success in undergraduate mathematics depends on more than the curriculum.[1] With NSF support, the Mathematical Association of America (MAA) created a committee to study a small group of successful undergraduate mathematics programs and to identify features and practices of those programs that might serve as the starting point for program improvements elsewhere. This report complements recent undergraduate curricular recommendations from the MAA with a discussion of other aspects of the undergraduate enterprise.

The heart of this study is a set of site visits to ten mathematics departments. The institutions visited spanned the spectrum from two-year colleges to research universities and were chosen because they were seen as having undergraduate mathematics programs that are particularly successful in several of the following areas: (i) attracting and preparing large numbers of mathematics majors, (ii) preparing students to pursue advanced study in mathematics, (iii) preparing future school mathematics teachers, and/or (iv) attracting and preparing members of underrepresented groups in mathematics.

The report describes general attitudes and strategies as well as particular activities that are effective. Based on the site visits, the report provides suggestions for other institutions to consider as they try to create and sustain an environment that will foster such attitudes and activities.

1. *General Attributes.* The following features, found at most of the programs visited (and at many institutions), seem to underlie these programs' success. Note: This is not to imply that such features are either necessary or sufficient for success.

- No matter how successful their current programs are, faculty members in the visited departments are not yet satisfied with the programs. Experimentation is continuous.
- There is a great diversity of instructional and curricular approaches, varying from one visited department to another, and even varying within a single department.
- Faculty members believe in the value of their work as collegiate educators, enjoy teaching, and care about their students.
- Faculty members communicate explicitly and implicitly that the material studied by their students is important and that they expect their students to be successful in mathematical studies. Courses are designed to meet the needs of the program's students, not the program's faculty.
- Extensive student-faculty interaction characterizes both the teaching and learning of mathematics, both inside and outside of the classroom.

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[1] *"In this report, the terms 'mathematics' and 'mathematical sciences' are used synonymously."*

2. *Attracting Students to Study Mathematics: In-Class Experiences*. Most faculty in the programs visited approach all courses with a primary focus on the general mathematical experience rather than the particulars of the individual subject. In every class they try to motivate their students to learn and to be interested in mathematics. The particular course syllabus is a context for achieving these broad goals. All programs visited gave considerable attention to the teaching of first-year calculus. Many faculty considered it the most important teaching assignment they had. They believed the best inducement for beginning students to take another mathematics course is to have an excellent teacher in their current course. As a consequence, the departments were more selective about who was allowed to teach in beginning calculus than in higher-level courses.

Despite teaching classes with a variety of student abilities and a mixture of those eager to learn more mathematics and those required to be there, faculty in programs visited seemed to be effective reaching virtually all students in their classes. Faculty were seen by all students to be setting high but achievable standards and then helping them meet these standards. Almost all the programs visited put a large amount of effort into developing and validating good placement tests, whose results students tended to follow closely. This heightened the faculty's confidence that in introductory courses all students could master the material. It also contributed to lower failure rates in these courses.

While most programs visited had an honors calculus course for more mathematically talented students, faculty generally viewed the regular calculus course rather than the honors calculus as the prime source for recruiting mathematics majors. Graphing calculators were widely used in introductory mathematics courses at the programs visited, but most programs visited made little use of computers.

Many of the programs visited were experimenting with new approaches to precalculus, often using a combined two-semester precalculus/calculus course. Several students interviewed said that their precalculus course convinced them to become mathematics majors. Two programs visited had innovative alternatives to freshman calculus that led into post-calculus courses at the sophomore and higher levels. An innovative sophomore "laboratory" course consisting of a set of mathematical explorations which students wrote up in 12–page papers seemed worthy of consideration by other institutions.

Few faculty at the programs visited consciously try to recruit mathematics majors in their courses, but their good instruction and their personal interest in their students' mathematical and personal growth prove to be very effective indirect recruiting strategies.

3. *Attracting Students to Study Mathematics: Out-of-Class Experiences*. The faculty in most departments visited were generally available to students anytime they were in their offices. Office hours were a two-way street, where faculty got valuable feedback to help them be more effective in class at the same time that faculty helped students with their problems. At two- and four-year colleges visited, all mathematics faculty participate in advising mathematics majors. At universities visited, the advisors are a subset of the faculty chosen for their record of good rapport with students. Upper-division mathematics students at most programs visited played a major role in formal recruitment activities, whether it was organizing informational sessions and social events or helping to prepare written materials about the major.

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4. *Attracting High School Students to Study Mathematics.* Most programs visited have an above average number of entering students expressing an interest in mathematics. This interest is a result of praise for the program that current students have communicated to their high school teachers and of the goodwill of graduates from the programs who are now high school teachers. All the programs visited had activities for teachers or students: continuing education programs, student competitions, and special programs for disadvantaged students or talented students. Mathematics majors often played a significant role in assisting in or organizing these efforts.

5. *Organization of the Mathematics Major and Supporting Activities*. Most of the effective mathematics major programs visited have an inclusive view of trying to serve a broad range of student objectives with general training in the modes of reasoning and techniques of mathematical sciences. Some effective programs have a very focused goal, such as the preparation for doctoral study. The prime objective of the mathematics courses at programs studied was to train their majors to think mathematically. Help-

ing students master a particular list of concepts and techniques was less important.

A very successful style of mathematics major used at one institution is a contract major in which a student and faculty advisor negotiate the courses in the mathematics major. As well as giving students a sense of ownership of their major, this format forces faculty to justify the value of their vision for a mathematics major. A highly successful approach used at a few universities (not visited) has been to design a professional (terminal) B.S. program with an applied flavor and little theory, geared to be a double major with engineering or business.

At two- and four-year colleges visited, many or most faculty have played a significant role in the development and maintenance of the successful mathematics program. The mathematics department chair at these institutions typically focuses on maintaining a friendly, supportive atmosphere around the department for faculty and students. At universities, a few faculty, frequently including the chair, are responsible for the success of the undergraduate program in mathematics.

Most of the programs visited had active student organizations. Students ran these activities with little faculty help. A monthly student-oriented colloquium was common, including talks by alumni about how they used their mathematical training in their careers. Most of the programs visited had a significant number of students participating in special individual or small-group learning experiences, such as independent study during summers or the academic year, an internship, senior thesis, or a (small-group) directed reading course.

While most of the programs visited have received external funding for instructional innovation, the most important support typically came from local campus administrators in the form of additional faculty positions and favorable treatment in other budgetary and administrative decisions.

6. *Effective Programs for Preparing Students for Advanced Study*. Two very different strategies were observed for increasing the number of students continuing on to advanced study in mathematics: (i) develop a major with an inclusive goal of preparing students for a wide variety of careers to attract a large number of students, and then motivate some of this large cohort to pursue graduate study, and (ii) have a major focused primarily on preparation for graduate study; this approach requires a selective student body and excellent instruction to avoid scaring most potential mathematics majors away.

An environment similar to that found in graduate school was a common theme at programs effective in preparing students for graduate study. Such an environment typically included frequent mathematical discussions outside of class among students and faculty (often in a departmental commons room), independent research opportunities, and seminar-type courses.

7. *Effective Programs for Preservice Preparation of School Teachers*. The syllabi and viewpoint of postcalculus courses enrolling substantial numbers of preservice secondary school mathematics teachers give consideration to the special needs of these students. Instructors of these courses should have a basic familiarity with the NCTM Standards. The instructional style in the mathematics program serves as a model for good teaching practices. The mathematics methods courses develop more than the mathematical foundations of school mathematics. They take a broad view of contemporary mathematics education issues. There are out-of-class activities and interactions with mathematical education faculty that enhance the in-class education and foster an intellectually active atmosphere of inquiry about new educational trends.

8. *Effective Programs for Underrepresented Groups in Mathematics*. Effective instruction and extensive student-faculty interactions in the mathematics programs visited appears to "disproportionately benefit" women, but not minorities. Special additional efforts seem to be required to obtain fuller participation of underrepresented minorities in mathematics. The mathematics programs at historically Black colleges and universities (HBCUs) visited had especially nurturing atmospheres for students. Mathematics majors at HBCUs showed a heightened degree of interest in professional career opportunities. Minority students with mathematics aptitude seem to be more numerous at non-HBCU institutions with engineering schools.

The institutions visited in this project were: Lebanon Valley College, Miami University of Ohio, Mount Holyoke College, Saint Olaf College, Seattle Central Community College, Southern University, Spelman College, University of Chicago, University of Michigan, and University of New Hampshire. For further details about the case study findings, readers are referred to the project report, "Models That Work: Case Studies in Effective Undergraduate Mathematics Programs", available from the Mathematical Association of America.