

Review of Roland Omnès, *The Interpretation of Quantum Mechanics*

William Faris

The Quantum World

The first chapter of the *Feynman Lectures on Physics* contains the following proud claim on behalf of the scientific enterprise:

If, in some cataclysm, all of scientific knowledge were to be destroyed, and only one sentence passed on to the next generations of creatures, what statement would contain the most information in the fewest words? I believe it is the atomic hypothesis (or the atomic fact, or whatever you wish to call it) that all things are made of atoms—little particles that move around in perpetual motion, attracting each other when they are a little distance apart, but repelling upon being squeezed into one another. In that one sentence, you will see, there is an enormous amount of information about the world, if just a little imagination and thinking are applied.

Contrast Feynman's confidence with the following phrase from Wittgenstein's *Philosophical*

William Faris is professor of mathematics at the University of Arizona and was, during the writing of this article, Fulbright Lecturer, Moscow Independent University. His e-mail address is faris@math.arizona.edu.

Roland Omnès, *The Interpretation of Quantum Mechanics*, Princeton University Press, Princeton, NJ, 1994.

Remarks: "The table I see is not made of electrons."

Those familiar with the successes of modern physics might scoff at such a statement. However the author of the book under review, a physicist who is evidently as familiar with Feynman as with Wittgenstein, quotes it approvingly. In his words: "This agrees well with the impossibility of identifying in quantum logic a classically meaningful property with a more complete set of elementary properties referring to each constituent particle in the object."

In the view of quantum mechanics that he presents, the particles are there, they have properties, but the properties are typically neither true nor false. In his terminology, facts are true properties, but they are restricted to the macroscopic world. As for other properties: "One must distinguish between the facts, the microscopic properties that may be said to be true, and also the vast number of microscopic properties that cannot even be said to be true or false." This raises the issue of just how well we understand the world on the atomic scale.

According to the current picture, the world consists of particles; these include electrons,

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protons, and neutrons. Protons and neutrons are themselves made of constituent particles, quarks. Particles move under the influence of forces. There are short-range forces (strong and weak interactions), and there are also long-range forces (electric and gravitational interactions). The electric, weak, and strong forces dominate on the atomic and subatomic scales; there is considerable progress toward a unified theory of these forces. The description of all these particles and forces is via quantum mechanics.

Quantum mechanics is not just another physical theory; it is supposed to be the framework for all physical theories. Its structure is independent of the details of what kind of particles exist and what kind of forces make them interact. In the words of the author of the book, this theory “has known a progress with no analogue in the history of science, finally reaching a status of universal applicability.”

Quantum mechanical calculations pervade the description of the world. The numerical factor that enters into such calculations is Planck’s constant \hbar . Consider, for instance, the size of an atom. The Bohr formula for the radius of an atom is

$$a_0 = \frac{\hbar^2}{me^2}.$$

This involves not only the mass m and charge e of the electron, but also Planck’s constant. When the numerical values are inserted, this gives the familiar fact that atoms have a diameter of the order of 10^{-8} cm.

The mystery of quantum mechanics begins when one looks more closely at its foundations. It seems reasonable that properties of the atomic world may not be the ones that we are used to. For example, an atom is much smaller than the wavelength of visible light, and so it is meaningless to speak of the color of an atom. However, one might expect that properties of particles such as being located in a certain region or having a certain energy would be meaningful. It turns out that such properties have a peculiar structure.

For each property the state of the system determines the probability that the property is true when an appropriate measurement is made. However, quantum mechanics does not fit comfortably into the ordinary framework of probability theory. The reason is that in general there is no natural way to combine properties. One can ask for the probability that the electron is in a certain region in space. One can also ask for the probability that its momentum is in a certain range. But one cannot ask for the probability that the electron is in the region and the momentum is in the range. It is sometimes said that a measurement that will decide about one property in-

terferes with or precludes a measurement that will decide about the other property. However, the usual analysis says that the combined property is simply meaningless. Furthermore, the truth or falsity of the individual properties are not defined until the point at which the measurement that will establish one of the properties is performed. At that point the selected property becomes true or false.

Is this a satisfactory picture of the world, or is it merely a description of the results of experiments? This sort of question is a source of unease among physicists. One of the earliest and most influential answers was the Copenhagen interpretation, a circle of ideas associated with Bohr and other pioneers of the quantum theory. According to this view, the interpretation of quantum mechanics must refer to our experience in the macroscopic world of everyday experience that is described by classical mechanics. The classical world is thus in opposition to or at least complementary to the quantum world. Such a view is congenial to a physicist who takes the view that the job of physics is only to predict experimental results.

Heisenberg, von Neumann, and other scientists contributed their own perspectives to the interpretation of quantum mechanics. Textbook accounts often give an uneasy review of the opinions of these masters and then move on to the mathematics, where there is no doubt about what to do. The more careful authors attempt to give a quantum mechanical account of the measurement process, usually following von Neumann. The typical conclusion is that the result of an experiment on a system must be described in terms external to the system. However, this leads to puzzling questions. Can the universe be described by quantum mechanics? There are many atoms in the universe; why should they not form an aggregate that obeys the laws of physics? Yet here there is no external system.

This and other questions have led some physicists to a reevaluation of the interpretation of quantum mechanics. Roughly speaking, there are two camps. One camp would like a conceptual revolution that sweeps away the orthodox way of thinking of quantum mechanics. The other camp would like to keep quantum mechanics much as it is but seeks an interpretation that does not depend on having an external system. This camp is willing to question the dogmas of Bohr and the other masters. However, its goal is conservative: to find a reformulation that maintains the spirit of the orthodox framework. The book *The Interpretation of Quantum Mechanics* by Roland Omnès represents the conservative camp. It builds on various newer ideas, including decoherence and the notion of consistent histories. The author refers in particular

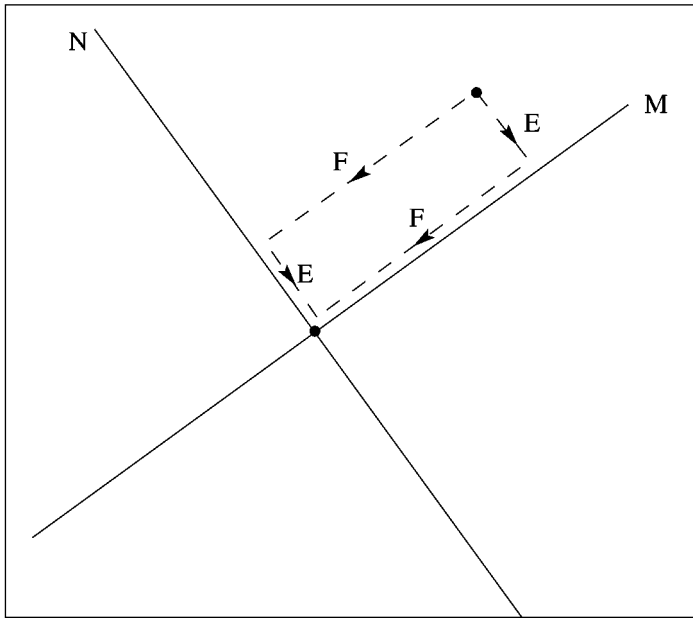


Figure 1. Compatible properties. Projections commute.

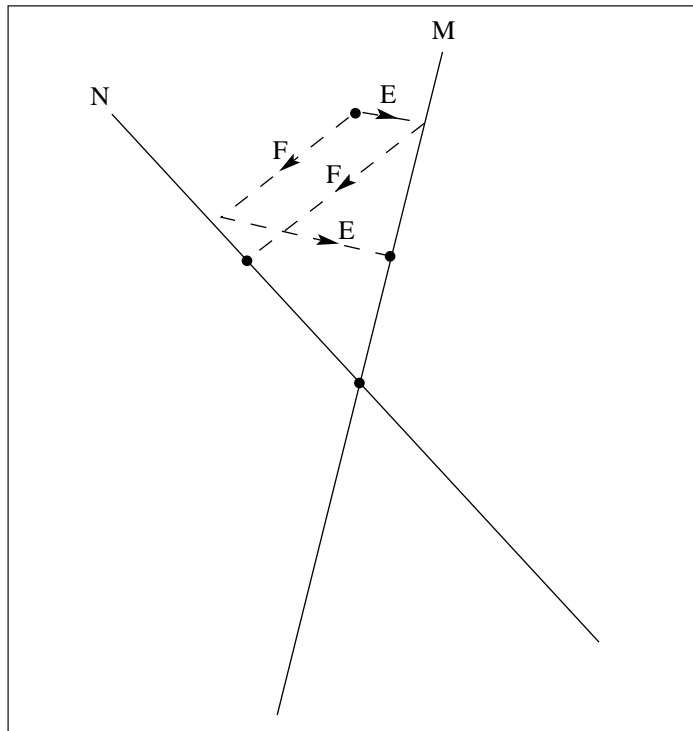


Figure 2. Incompatible properties. Projections do not commute.

to related work of Griffiths (on consistent histories) and by Zurek (on decoherence) and by Gell-Mann and Hartle (another synthesis). The importance of the book is that it represents a self-contained statement of one variant of what may become the new orthodox position. Does this position carry conviction? The following more detailed description may suggest an answer.

Properties and States

The mathematical framework is standard quantum mechanics. There is a Hilbert space \mathcal{H} . This is a complex vector space with an inner product; with its norm it is a complete metric space. Every closed subspace is itself a Hilbert space; we shall refer to it simply as a *subspace*. Each subspace M of \mathcal{H} specifies a *property* of the quantum system. The entire space is the *sure property*, and the zero subspace is the *impossible property*. If M is a subspace specifying a property, then the orthogonal complement M^\perp specifies the *negation* of the property. If M and N are two subspaces with orthogonal complements M^\perp and N^\perp , then the four intersections $M \cap N$, $M \cap N^\perp$, $M^\perp \cap N$, and $M^\perp \cap N^\perp$ are each subspaces. If the direct sum of these four subspaces is the entire Hilbert space, then M and N are *compatible*. If M and N are compatible, then the *conjunction* of each pair of properties is defined to be the corresponding intersection. Otherwise the conjunction is not defined. When M and N are compatible, the *disjunction* of M and N is also defined and is the direct sum of the first three of the four subspaces. When M and N are compatible and their conjunction is impossible, then the properties are *mutually exclusive*. In this case the disjunction of M and N is just the direct sum of orthogonal subspaces M and N . The logic of quantum mechanics is thus much like ordinary logic, except that conjunction and disjunction are defined only for compatible properties.

This logical structure is often formulated in another language. An *operator* is a linear transformation of \mathcal{H} to itself. There is a one-to-one correspondence between subspaces of \mathcal{H} and orthogonal projection operators; the subspace is the range of the operator. Call such an operator a *projection*; it is a self-adjoint operator E with $E^2 = E$. In this language the projections I and 0 correspond to the sure property and the impossible property, and the projection $E' = I - E$ corresponds to the negation of the property specified by E . The properties associated with E and F are compatible if E and F commute (Figure 1). In general the projections do not commute, and the properties are not compatible (Figure 2). In the case when the projections E and F commute, the projection $EF = FE$ represents the conjunction, and the projection $EF + EF' + E'F = E + F - EF$ represents the disjunction. When $EF = 0$, the properties are compatible—in fact, they are mutually exclusive—and the disjunction is represented by the sum $E + F$. From now on we identify properties with projections.

A state provides a specification of a probability $p(E)$ for each property E . This specification must be additive. This refers to the situa-

tion where there are projections E_1, \dots, E_n onto orthogonal subspaces with $E_1 + \dots + E_n = I$. The additivity requirement is that

$$p(E_1) + \dots + p(E_n) = 1.$$

This says that the probabilities of a set of mutually exclusive properties whose disjunction is sure add up to one. It is not necessarily assumed that every property can be measured or that all these probabilities are empirically meaningful. The state is a mathematical specification of probabilities for all properties, whether they have physical meaning or not.

In quantum mechanics states are determined by vectors in the Hilbert space in the following way. A *pure state* is determined by a unit vector ψ in the Hilbert space. The probability of a property E when the system is in a pure state is given by the inner product

$$p(E) = \langle \psi, E\psi \rangle.$$

Since $\langle \psi, E\psi \rangle = \|E\psi\|^2$, this number is between 0 and 1 (Figure 3). In general a *state* is defined as a pure state or as a randomized family of pure states.

Two unit vectors determine the same pure state if and only if they belong to the same one-dimensional subspace. Therefore, the space of pure states is really the complex projective space consisting of all these one-dimensional subspaces. This suggests a representation of pure states as projections on one-dimensional subspaces. In this review I follow the physicist's convention that inner products are conjugate linear on the left and linear on the right. Thus the projection on the one-dimensional subspace spanned by ψ is

$$P = \psi \langle \psi, \cdot \rangle.$$

This leads to a particularly elegant expression for the probability as a trace of a product of a state projection P with a property projection E :

$$p(E) = \text{tr}(PE).$$

This expression follows immediately from $\text{tr}(PE) = \text{tr}(PEP)$ and $PEP = \langle \psi, E\psi \rangle P$. This trace can be thought of geometrically as the square of the cosine of the angle between the two subspaces.

The physical interpretation of properties is also reflected in the geometry of the corresponding projections. Let E and F be projections representing properties. The geometrical relation between the two corresponding subspaces [1] is expressed by certain angles θ that lie in the range from 0 to $\pi/2$. They are defined so that the spectrum of the self-adjoint operator EFE acting in the range of E consists of the numbers

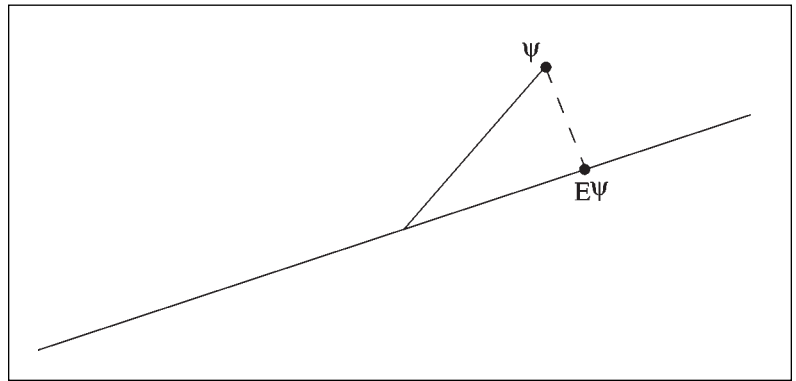


Figure 3. Probability of a property E : $p(E) = \|E\psi\|^2$.

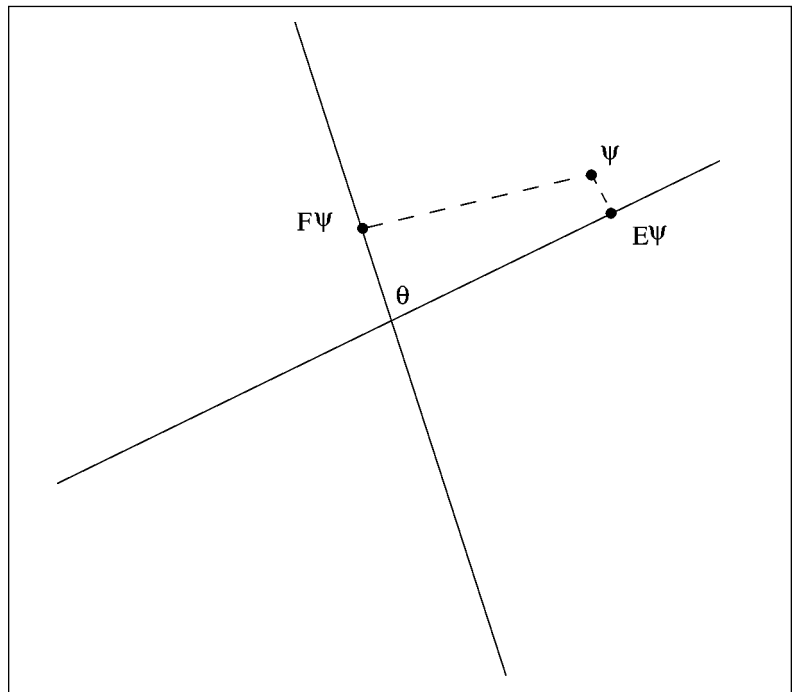


Figure 4. An uncertainty principle: $\|E\psi\|^2 + \|F\psi\|^2 \leq 1 + \cos \theta$.

$\cos^2(\theta)$. The spectrum of the self-adjoint operator $E + F$ consists of the numbers $1 \pm \cos(\theta)$, with the same angles θ . Let $\hat{\theta}$ be the infimum of the angles. The norm of $E + F$ is $1 + \cos(\hat{\theta})$. In the quantum mechanical interpretation this says that, for every state p , the sum of the probabilities of E and F satisfies the inequality (Figure 4)

$$p(E) + p(F) \leq 1 + \cos(\hat{\theta}).$$

This is a special case of a characterization [2] of all points with coordinates $p(E)$ and $p(F)$. It is convenient to estimate the right-hand side in terms of a trace, since it is easier to compute. The trace of EFE is the sum of all the $\cos^2(\theta)$, so

$$\cos(\hat{\theta}) \leq \sqrt{\text{tr}(EFE)}.$$

Suppose that E corresponds to the position of a particle being in a particular interval of length

Δq and that F corresponds to the momentum of the particle being in another interval of length Δp . Then the trace is given explicitly by

$$\text{tr}(EFE) = \frac{\Delta q \Delta p}{2\pi\hbar}.$$

If the product $\Delta q \Delta p$ is much smaller than \hbar , then the trace is close to zero, the minimum angle $\hat{\theta}$ is close to $\pi/2$, and consequently the probabilities $p(E)$ and $p(F)$ cannot both be close to one. This is a form of the *uncertainty principle*: it is impossible to simultaneously specify position and momentum to a greater precision than given by Planck's constant \hbar .

There are two ways of combining pure states to give new states, superposition, and mixture. A *superposition* is obtained by taking linear combinations of the vectors. Thus if ψ_a is an orthonormal family of unit vectors and c_a are complex coefficients with $\sum |c_a|^2 = 1$, then the superposition is the pure state given by the vector $\sum_a c_a \psi_a$. The corresponding one-dimensional state projection P is

$$P = \sum_b \sum_a \bar{c}_a c_b \psi_b \langle \psi_a, \cdot \rangle,$$

and the probability of a property E is

$$p(E) = \sum_b \sum_a \bar{c}_a c_b \langle \psi_a, E \psi_b \rangle.$$

The terms $\bar{c}_a c_b$ with $a \neq b$ are called *interference terms*. If one thinks of the pure states as points in a complex projective space, then the superposition of states is given by a geometrical construction in this space. For example, the possible superpositions of two states lie on the projective line (Riemann sphere) containing the two states (see [3] for an elementary discussion).

The other way of combining pure states is as a *mixture* obtained by taking linear combinations of the corresponding one-dimensional projections. The coefficients are probabilistic weights $w_a \geq 0$ with $\sum_a w_a = 1$. The mixture is the state obtained by randomization with these weights, so the probability of a property E is

$$p(E) = \sum_a w_a \langle \psi_a, E \psi_a \rangle.$$

There are no interference terms. Let ρ be the *density operator* defined by combining the projections with these weights, so

$$\rho = \sum_a w_a \psi_a \langle \psi_a, \cdot \rangle.$$

This is a positive self-adjoint operator with trace equal to one. The probability of the property in

the mixed state is the trace of the product of the state operator ρ with the property projection E :

$$p(E) = \text{tr}(\rho E).$$

Superposition has characteristically quantum mechanical features (interference terms), while mixture is just an ordinary probabilistic process.

Decoherence

The most famous puzzles of quantum mechanics have to do with the notion of superposition. These arise when the classical world of macroscopic properties is coupled to the quantum world of atomic properties. The conventional account of this refers to an experimenter who couples an atomic system to the fate of a cat. However, it is convenient conceptually to dispense with the experimenter, and present practice is to substitute a less attractive life form, perhaps from another phylum [4]. For instance, the decay of a single radioactive atom might trigger a volcanic eruption that kills a passing cockroach. The collective variables describing the fate of the cockroach are coupled to the environment that consists of the world on the atomic scale. The state determining the health of the cockroach might well turn out to be a superposition of two states, one where the cockroach is alive and the other where it is dead. This involves more than just uncertainty about the fate of the cockroach; the two possible fates are entangled in a more profound sense.

Such a situation has its formulation in the framework of quantum mechanics. Consider two quantum mechanical systems described by Hilbert spaces \mathcal{H}_C and \mathcal{H}_E . We can think of the first system as a collective system (describing a cockroach or perhaps a counter in some physics laboratory). The second system is the environment. These two systems determine a combined system determined by the tensor product Hilbert space $\mathcal{H} = \mathcal{H}_C \otimes \mathcal{H}_E$. For each pair of vectors ϕ in \mathcal{H}_C and χ in \mathcal{H}_E there is a tensor product vector $\phi \otimes \chi$ in \mathcal{H} . Such a tensor product vector represents a pure state in which the two systems are independent. Let ϕ_i and ψ_j be orthonormal bases for \mathcal{H}_C and \mathcal{H}_E . The tensor product vectors $\phi_i \otimes \psi_j$ form an orthonormal basis for \mathcal{H} . The general vector in \mathcal{H} is a doubly indexed linear combination of tensor product basis vectors

$$\Psi = \sum_i \sum_j d_{ij} \phi_i \otimes \psi_j.$$

This may be partially factored as

$$\Psi = \sum_i \phi_i \otimes \left(\sum_j d_{ij} \psi_j \right) = \sum_i c_i \phi_i \otimes \chi_i,$$

where the $\chi_i = (1/c_i) \sum_j d_{ij} \psi_j$ are unit vectors. The general vector in \mathcal{H} is a singly indexed linear combination of tensor product vectors; it is not possible to factor it any further. The corresponding pure state is a superposition that represents a complicated quantum mechanical dependence of the two systems. In some of the states ϕ_i the cockroach might be alive, in others it might be dead. The superposition represents an intricate combination of these possibilities.

The *decoherence* effect is a tendency for the state to evolve in time to a state where the unit vectors χ_i are orthogonal. This is a dynamical effect that tends to occur when the environmental system is very large. The importance of decoherence is that it makes the collective system obey classical probability. More precisely, a measurement on the collective system is unable to distinguish a superposition from a mixture.

This can be seen by looking at projection operators associated with the collective system. For each such projection F there is a corresponding projection \hat{F} for the combined system determined by the formula $\hat{F}(\phi \otimes \chi) = F\phi \otimes \chi$. With the pure state given by Ψ we have

$$\begin{aligned} p(\hat{F}) &= \langle \Psi, \hat{F}\Psi \rangle \\ &= \sum_a \sum_b \bar{c}_a c_b \langle \phi_a \otimes \chi_a, F\phi_b \otimes \chi_b \rangle \\ &= \sum_a \sum_b \bar{c}_a c_b \langle \phi_a, F\phi_b \rangle \langle \chi_a, \chi_b \rangle. \end{aligned}$$

In general this probability involves the quantum properties of the combined system; there are interference terms arising from the superposition. However, if decoherence makes the environmental vectors χ_a orthogonal, then the terms are zero for $a \neq b$, and the probability of interest is

$$p(\hat{F}) = \sum_a |c_a|^2 \langle \phi_a, F\phi_a \rangle.$$

This is just the result that one would obtain by a probabilistic mixture of the pure states given by the ϕ_i . The cockroach is alive or dead with certain probabilities, but other than that there is no particular quantum mystery. Of course there are still remaining questions: when does decoherence take place, and, even more fundamentally, what is the actual fate of the cockroach?

Consistent Histories

The change of a quantum system over a time interval is given in conventional quantum dynamics by a unitary operator. An operator U from \mathcal{H} onto itself is *unitary* if it preserves the inner product. In the *Schrödinger picture* this is

regarded as changing state vectors ψ to state vectors $U\psi$. In the *Heisenberg picture* this is regarded as changing properties F to corresponding properties F^U defined by $F^U = U^{-1}FU$. Since

$$\langle U\psi, FU\psi \rangle = \langle \psi, U^{-1}FU\psi \rangle,$$

these two pictures give the same predictions for the probabilities. Theoretical physicists spend much effort computing the unitary dynamics for a given situation.

In some variants of the Copenhagen interpretation there is another kind of change, a so-called *reduction*, or *collapse*, of the state vector. In the simplest circumstance it takes the following form. With the state p given by ψ the probability of property E is $p(E) = \langle \psi, E\psi \rangle = \|E\psi\|^2$. If, after a measurement, E turns out to be true, then the state vector changes to $E\psi/\|E\psi\|$. With this new state \bar{p} the probability of another property F would be $\bar{p}(F) = \langle E\psi, FE\psi \rangle / \|E\psi\|^2 = \|FE\psi\|^2 / \|E\psi\|^2$. Since reduction provides a new kind of dynamics, it seems that one should either explain its relation to the unitary dynamics or eliminate it from the theory.

The book under review has a proposal in this direction. The idea is that it is useful to assign probabilities to ordered conjunctions of properties. These properties need not be compatible. Let E and F be projections representing properties. Consider the operator EFE , representing an ordered conjunction of E and F . Regard the state p as a linear function defined on operators, and define the probability of the ordered conjunction of E and F to be $p(EFE)$. For a pure state this probability is $\langle \psi, EFE\psi \rangle = \|FE\psi\|^2$ (Figure 5). Similarly, the probability of $E' = I - E$ and F is $p(E'FE')$. These should add up to the probability of F . The additivity condition

$$p(EFE) + p(E'FE') = p(F)$$

is equivalent to

$$p(EFE') + p(E'FE) = 0.$$

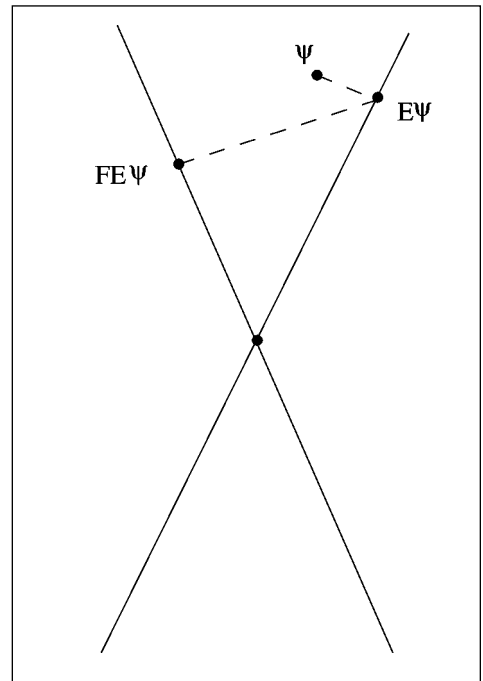


Figure 5. Probability of E and then F :
 $p(EFE) = \|FE\psi\|^2$.

When this is satisfied, the properties are said to be *consistent* with respect to the state p . More generally, let E_1, \dots, E_m be a family of exclusive properties such that $E_1 + \dots + E_m = I$. Let F_1, \dots, F_n be another such family. Define the probability of the ordered conjunction of E_i and F_j as $p(E_i F_j E_i)$. The consistency condition for additivity is now

$$p(E_i F_j E_k) + p(E_k F_j E_i) = 0$$

for $i \neq k$.

If properties are compatible, then they are consistent with respect to every state. However, for special states there may be properties that are consistent but not compatible. A simple example of consistency is when the state is a probabilistic mixture of pure states given by orthogonal unit vectors ϕ_a with weights w_a . If the projections E_i project onto the vectors ϕ_i and the projections F_j are arbitrary, then the properties are consistent, and the probabilities of the conjunctions taken in order are

$$p(E_i F_j E_i) = w_i \langle \phi_i, F_j \phi_i \rangle.$$

This example provides a possible framework for discussion of the decoherence effect.

Omnès, following Griffiths [5], is motivated by examples where E_i and F_j are properties associated with two different times $s < t$. When the properties are consistent, this is an example of what is called *consistent histories*. This notion may be generalized to any finite number of reference times. Consistent families of histories are rare objects, but they are central to the proposed theory.

One appeal of consistent histories is that they might replace reduction of the state vector. In the state p given by ψ the probability of E_i is $p(E_i) = \langle \psi, E_i \psi \rangle = \|E_i \psi\|^2$, and the probability of E_i and then F_j is $p(E_i F_j E_i) = \langle \psi, E_i F_j E_i \psi \rangle = \|F_j E_i \psi\|^2$. The conditional probability of F_j given E_i is the quotient $p(E_i F_j E_i)/p(E_i) = \|F_j E_i \psi\|^2 / \|E_i \psi\|^2$. This is the probability of F_j that would have been given by reduction if the previous measurement had made E_i true.

A consistent family of histories has no more quantum mystery; it defines an ordinary stochastic process. A stochastic process is a probability measure P on a space of functions of time. If there are only two instants of time $s < t$, then an element of this space is a function ω defined on $\{s, t\}$ such that $1 \leq \omega(s) \leq m$ and $1 \leq \omega(t) \leq n$. Such a function is an *outcome* of the probability experiment. An *event* is a set of outcomes; it is usually defined by a condition involving an arbitrary outcome ω . The probability measure P assigns a probability to each event. It is characterized by

$$P[\omega(s) = i, \omega(t) = j] = p(E_i F_j E_i).$$

For each i there is a new probability measure \bar{P}_i defined by

$$\begin{aligned} \bar{P}_i[\omega(s) = i, \omega(t) = j] &= \frac{P[\omega(s) = i, \omega(t) = j]}{P[\omega(s) = i]} \\ &= \frac{p(E_i F_j E_i)}{p(E_i)}. \end{aligned}$$

This is the conditional probability given the event $\omega(s) = i$. The calculation above shows that it coincides with the probability measure given by reduction of the quantum state.

We are accustomed to the fact that probabilities do not determine the actual outcome of an experiment. No amount of mathematics can foretell the conclusion of an evening at the roulette tables. On the other hand, partway through the evening we know how we are doing. Our stochastic process is like any other probability model; the probabilities given by P do not determine the outcome ω . This is only known when the experiment has been completely performed. If the experiment has been conducted only up to time s , then it may be known only that $\omega(s) = i$, for some particular i . This partial knowledge of the outcome of the experiment is equivalent to a knowledge of the probability measure \bar{P}_i , since \bar{P}_i determines i . In fact, \bar{P}_i assigns probability one to the event that $\omega(s) = i$. If one keeps a fixed quantum state, the probability measure is fixed, and the partial outcome is new information. On the other hand, if a reduction of the quantum state is specified, then this determines the new measure and hence the partial outcome. Thus there are two different mechanisms that can describe evolving reality for a consistent family of histories: a direct description of the evolving outcome or an evolving reduction of the state vector. The next issue is whether one or the other of these mechanisms is appropriate to the interpretation of quantum mechanics.

Actualization

The bedrock of the Omnès theory is Rule 1: *The theory of an individual isolated physical system is entirely formulated in terms of a specific Hilbert space and a specific algebra of operators, together with the mathematical notions associated with them.* Omnès emphasizes: "The word 'entirely' that occurs in it will be taken in its strongest sense, to mean that not only dynamics, but also the logical structure of the theory and the language one uses when applying it to observations and experiments will be cast into the mold of Hilbert space." Again: "What is important about the first rule is that it assumes that

everything that might be said about the physical system should take place in its mathematical framework. This includes in particular the understanding of empirical properties and the whole of interpretation." He shows no enthusiasm for the notion that "other significant data could exist, completing or replacing the wave function."

Rule 2 requires unitary dynamics, and Rule 3 deals with the description of composite systems by tensor product Hilbert spaces. Everything must fit into this structure, including the world of our familiar experience. One must make the most of the Hilbert space structure, and this is where consistent histories are to be put to use.

Omnès elevates their role to a "universal role of interpretation," given as Rule 4: *Every description of a physical system should be expressed in terms of properties belonging to a common consistent logic. A valid reasoning relating these properties should consist of implications holding in that logic.* Such a logic is defined by the probabilities associated with the histories; a property implies another property if the conditional probability of the second property given the first property is equal to one.

The first task of consistent histories in the Omnès theory is to explain classical properties. These are properties defined by requiring that collective variables belong to cells in classical phase space of a size that is large with respect to Planck's constant \hbar . For systems in which there is no significant interaction of the collective variables with variables on the atomic scale, and for initial states that specify a single cell, these classical properties are at least approximately consistent and obey deterministic dynamics. So the classical world has its familiar properties while being part of the quantum world. In particular we can talk about the usual trajectories given by classical mechanics. In more general situations there is the possibility of interaction between the collective system and the atomic-level environment. Then decoherence eliminates interference effects and allows the collective system to be described by ordinary probability.

Omnès refers to classical properties of macroscopic objects that arise from this theory as *phenomena*. Phenomena are described by probabilistic laws. Classical properties are called *facts* when they actually occur in reality. The passage from phenomena to facts does not emerge from the internal structure of quantum mechanics; Omnès postulates it as an additional rule, analogous to the reduction rule of the Copenhagen interpretation. The postulated passage from phenomena to facts is called *actualization*. Obviously this is an important transition, especially for the cockroach. It also is important for

the theoretical structure of the book. The statement is the following Rule 5: *Physical reality is unique. It evolves in time in such a way that, when actual facts arise from identical antecedents, they occur randomly and their probabilities are those given by the theory.*

This statement by itself does not give a clear picture of the mathematical formulation of actualization. The intent may be that the notion of "fact" is external to the theory, so that the rule of actualization is merely a license to use consistent logic to reason from present brute experience. This is supported by the assertion: "The existence of actual facts can be added to the theory from outside as a supplementary condition issued from empirical observation." A dead cockroach is a fact; there is no more to it. This is a long way from the ambitious goal of basing everything on Hilbert space.

However, in another passage Omnès describes a change in the state given by what appears to be continuous reduction on the classical level. "Let $\{F_k(t_k)\}$ denote the quasi-projectors representing all the facts having occurred everywhere in the 'universe' at some time t_k earlier than the time t . One can then consider that the state of the universe at time t is the result of all these facts combined with the assumed knowledge of the initial state of the 'universe' at some initial time." The state is expressed in terms of an initial state and an operator $G(t)$ that "recapitulates all the facts occurring between an initial time t_0 ... and the time t . More explicitly, one has

$$G(t) = T \left\{ \prod_{t_k=t_0}^t F_k(t_k) \right\}.$$

This is a time-ordered product [later times to left of earlier times], as indicated by the symbol T ." In this new state certain classical properties have probability one and thus can be said to be factual or true. There are obvious questions. Does Rule 5 prescribing actualization require the new dynamics that recapitulates facts? What is the relation between the unitary dynamics given by Rule 2 and the new dynamics? Are Rule 1, Rule 2, and Rule 5 (however interpreted) consistent?

The actualization postulate thoroughly undermines any program of basing the theory on orthodox quantum dynamics alone. Omnès nevertheless wants to think of actualization in a positive way; he bravely remarks that "the inability of quantum mechanics to offer an explanation, a mechanism, or a cause for actualization is in some sense a mark of its achievement. This is because it would otherwise reduce reality to bare mathematics and would correspondingly suppress the existence of time."

The postulate of actualization is designed to forestall an interpretation of quantum mechanics in which the various histories that are predicted with nonzero probability all have a claim to current physical reality. This variant would “rely on Everett’s approach and to consider ‘our’ actual present as a branch of the histories of the universe separated from all the other ones.” Omnès finds this alternative “difficult to accept.”

Perhaps one could interpret the Rule 5 of actualization in another way, as introducing a particular history that is a new element of reality. Is this consistent with Rule 1? Perhaps not, since Rule 1 seems to require that the state vector and its relation to the various quantum properties provide a complete description of the current physical situation. If Rule 1 is relaxed, then certainly something like actualization can take place without a corresponding change in the state vector. This is the usual situation with a stochastic process, where, as we have seen, the outcome of the experiment is not predicted by the probability model. It is possible that the outcome can be explained by a more elaborate model, but the search for such a model is a new scientific endeavor, perhaps quite difficult. (It might be a considerable challenge to find a mechanical explanation for the sorry results of the evening of roulette.) In any case, if the outcome has been observed up to a certain time, then this provides an account of the current situation. There is no need to discard the original probabilities, though it is quite natural to consider the conditional probabilities given the outcome up to the present as a prediction of the future. In the context of a stochastic process given by a consistent family of histories, the state vector determines the probabilities of various histories, but the outcome would be a particular history. The particular history that occurs is extra information not specified by the state vector, so such an interpretation admits data outside the Hilbert space framework as part of the current physical situation.

Measurement

Up to this point this version of quantum theory seems to be a theory of properties of macroscopic objects. How about the properties of objects on the atomic scale; are they ever definitely true or false? The answer to this is to be given by the theory of measurement. For this purpose we decompose our system in a somewhat different way, into an atomic system and a larger system that is to act as a measuring device. The classical properties of the measuring device will be called *data*, while the corresponding properties of the atomic system will be called *results*. The results precede the data. In appropriate cir-

cumstances results and data fit into consistent histories, and furthermore the results and data are equivalent. This constitutes a measurement. We can again think of the decay of the radioactive atom and the resulting condition of the cockroach. Suppose that the only immediate menace to the cockroach is the decay of the radioactive atom and the subsequent volcanic eruption. The health of the cockroach may be thought of as an experimental datum. The corresponding result is a statement of what happened to the atom.

A factual property is always regarded as being true. According to Omnès, in the context of a measurement, if the property expressing the datum is a fact, then the property expressing the result should also be considered as true. Thus one is allowed to speak, at least in special circumstances, of the truth of properties of systems on the atomic level. The death of the cockroach is equivalent to the decay of the atom. If in fact the cockroach is dead, then the atom must have decayed. The measurement situation is exceptional; in most other cases a radioactive decay cannot be said to have happened or not to have happened.

The following more detailed sketch of the measurement process illustrates how one works with these concepts. The picture presented by Omnès follows the general pattern of the classic von Neumann account. The Hilbert space is a tensor product $\mathcal{H} = \mathcal{H}_C \otimes \mathcal{H}_E$ describing the combination of the collective system that describes the data with the atomic system describing the radioactive emission for which one wants to obtain results. There are orthonormal state vectors χ_i of the atomic system \mathcal{H}_E with corresponding projection operators E_i . These operators represent the results. The general pure state of the atomic system is a superposition $\sum_a c_a \chi_a$.

There are also corresponding orthonormal state vectors ϕ_i of the collective system \mathcal{H}_C with projection operators F_i . Their role is to represent the data. The initial state of the collective system is taken to be some ϕ_0 . The initial pure state of the combined system is a tensor product state, and the two systems are independent:

$$\Psi = \phi_0 \otimes \sum c_a \chi_a.$$

A measurement is a special kind of dynamical process given by a unitary operator U . In the Schrödinger picture U acts so that

$$U(\phi_0 \otimes \chi_i) = \phi_i \otimes \psi_i,$$

where the ψ_i are some other unit vectors whose nature is irrelevant. The important part is that U relates the state vector χ_i of the atomic sys-

tem at the beginning of the process (the eventual result of the measurement) to the state vector ϕ_i of the collective system describing the data at the end of the process. The resulting state vector for the combined system is a superposition that specifies a strong dependence between the constituent systems:

$$U\Psi = \sum_a c_a \phi_a \otimes \psi_a.$$

Let \hat{E}_i and \hat{F}_j be the projections for the combined system corresponding to the results and the data at time zero. In the Heisenberg picture the state vector remains the initial Ψ , while the projections for the data at the later time of the measurement are $\hat{F}_j^U = U^{-1}\hat{F}_jU$. In the state determined by Ψ we can use the fact that E_i projects on χ_i and the form of the unitary time evolution operator U to compute that

$$\begin{aligned} p(\hat{E}_i\hat{F}_j^U\hat{E}_k) &= \bar{c}_i c_k \langle \phi_0 \otimes \chi_i, U^{-1}\hat{F}_jU(\phi_0 \otimes \chi_k) \rangle \\ &= \bar{c}_i c_k \langle \phi_i \otimes \psi_i, F_j \phi_k \otimes \psi_k \rangle. \end{aligned}$$

However, F_j projects on ϕ_j , and the ϕ_j are orthonormal. So this is just

$$\begin{aligned} p(\hat{E}_i\hat{F}_j^U\hat{E}_k) &= \bar{c}_i c_k \langle \phi_i \otimes \psi_i, \phi_j \otimes \psi_j \rangle \delta_{jk} \\ &= |c_i|^2 \delta_{ij} \delta_{jk}. \end{aligned}$$

Since this is zero for $i \neq k$, the logic is consistent. In particular the probability for the ordered conjunction of result and datum is

$$p(\hat{E}_i\hat{F}_j^U\hat{E}_i) = |c_i|^2 \delta_{ij}.$$

This says that each result is equivalent to the corresponding datum. In particular they have the same probability $p(\hat{E}_i) = p(\hat{F}_i^U) = |c_i|^2$. As Omnès remarks, "When one thinks of how complicated a measuring apparatus can be and how different two experimental devices purporting to measure the same quantity may be, it is remarkable that there exists such a simple universal correspondence between them."

Experiments

Some of the peculiar features of quantum mechanics may be artifacts of theoretical interpretation, but there are experiments that more or less directly test the fundamental principles. Omnès briefly describes Leggett's experiment with superconducting quantum interference devices. The magnetic flux through a superconducting ring plays a role analogous to that of the position of a particle undergoing radioactive decay. By this means one can observe the analog of a radioactive decay, but with a macroscopic device. According to Omnès, this means that there is "no Heisenberg frontier between the

microscopic and macroscopic domains, nor is Bohr's point of view useful because one cannot tell here what is a phenomenon and what is not."

The relation of laboratory experiments to the mathematical apparatus of quantum mechanics is subtle. For instance, it is often held that the uncertainty principle relating position and momentum is a fundamental principle of quantum mechanics. On the other hand, it can be argued that momentum is measured in practice by measuring particle location in a scattering experiment. There are only position measurements, and so there is no independent empirical content to the momentum properties that play such a fundamental role in the mathematical formulation. This point of view is vigorously defended by some proponents of such alternative theories as Bohmian mechanics and stochastic mechanics.

One remarkable effect has a rather direct experimental test. Omnès discusses the famous example of a system prepared in a state in which two particles are widely separated in space but intimately related in their behavior. This manifests itself in strong correlations between experimental results at the two locations. The analysis of the implications of such experiments involves concepts of *locality*. Although the treatment in the book is not explicit on this point, it is helpful to distinguish two types of locality. The condition of *active locality* is that there is no instantaneous influence over long distances. This seems to be satisfied in nature. The condition of *passive locality* is more subtle; it says that simultaneous random events at widely separated locations that are correlated must be correlated only through events in their common past. Bell showed that any deterministic or probabilistic theory that purports to be an alternative to quantum mechanics and that satisfies both the active locality and passive locality conditions must have correlations that satisfy certain inequalities [6]. The experimental results agree with quantum mechanics; the correlations are so strong that Bell's inequalities are violated. From this, one can conclude that in any alternative theory one of the locality conditions must be violated. However, this is no worse than the situation in quantum mechanics, which also seems far from satisfying any condition like passive locality. There is a more complete discussion of this issue in the appendix to reference [7]. The conclusion seems to be that we live in a world generously furnished with unexplained and unexplainable coincidences.

Interpretations

In his final summary, Omnès compares three possible outlooks on quantum mechanics. There is the radical view that the theory is not yet in final form and that there should be a deeper de-

scription of reality at the atomic level than that given by the quantum state. There is the Everett many-world interpretation and such theories as that of Gell-Mann and Hartle; these maintain conventional quantum dynamics. Finally, there are views such as the Copenhagen interpretation, and the view of Omnès himself, where the quantum state is to provide the complete description of the physical situation; these require either an external interpretation or a new kind of quantum dynamics to make reality unique.

The radical view is provoked by the puzzle over whether we understand a world in which properties on the atomic scale are typically neither true nor false. Various people have attempted to provide a more satisfying picture; Wick's recent book [7] has a lively discussion of this history. Omnès describes the Ghirardi, Rimini, and Weber [8] proposal of spontaneous random wave packet reduction. He also mentions Bohm's theory [6, 9, 10, 11] and stochastic mechanics [12, 13]. In both these theories particles have trajectories, and statements about their positions are perfectly meaningful at all times. The particle trajectory is extra information not given by the state vector. However, the probability predictions for position at fixed time agree with those of quantum mechanics. The theories are rather similar, except that in Bohm's theory the particle moves deterministically, while in stochastic mechanics there is an extra diffusive component. Neither of the theories is as well developed in the relativistic domain as orthodox quantum mechanics. Furthermore, Bohm's theory and stochastic mechanics both violate the active locality condition on the level of the particle trajectories [14], and this is troubling. On the other hand, a study of these theories has a liberating effect that may point the way to new directions.

The Everett many-world interpretation accepts the fact that the conventional quantum dynamics applies in all cases and interprets it as a theory of multiple reality. However, conventional quantum dynamics is compatible with a single reality. A specified consistent family of histories defines a stochastic process; the experimental outcome can be a particular history. More recently Gell-Mann and Hartle have presented another attempt to maintain quantum dynamics. They use many of the same technical ingredients as in the Omnès theory, including decoherence and consistent histories. According to Omnès, they "attribute completely to decoherence the dynamical origin of phenomena...as well as the selection of the significant collective observables and...the occurrence of the histories making physical sense." These authors want a consistent family of histories to represent a "quasiclassical domain" of familiar experience.

The specification of the family is an important issue. They say in one of their articles [15] that, "We have posed the question as to whether there could be various kinds of essentially inequivalent quasiclassical domains or whether any quasiclassical domain is more or less equivalent to any other. The former case poses some challenging intellectual puzzles, especially if we imagine [information gathering and utilizing systems] evolving in relation to each of the essentially inequivalent classical worlds."

Finally, there is the version offered by Omnès. In the end this new synthesis of quantum mechanics turns out to be fairly close to the old Copenhagen account. There are differences; in the Copenhagen version the classical world is complementary to the quantum world, while in the Omnès picture it emerges from the quantum world. According to Omnès, quantum mechanics is divided into parts concerned with dynamics and with logic, and "contrary to dynamics, the logical structure of quantum mechanics must select a definite direction of time." The underlying problem is the same in the two interpretations. In the Copenhagen interpretation there is the external notion of measurement, and in some versions there is a "reduction of the wave packet" that takes place in the quantum world as a consequence of measurement. In the Omnès account the corresponding process is the spontaneous classical actualization of facts. This concept is never clearly explained. However, it appears that in both accounts one has to rely on external elements or violate quantum dynamics in order to salvage the interpretation.

The most important ingredient in the current attempts to maintain orthodox quantum mechanics is the notion of decoherence. This idea has been around for some time; Omnès gives several references. (The reviewer first encountered it in mathematically rigorous form in papers of Hepp and Lieb [16] in the mid-seventies.) It explains to some extent why it is so difficult to make the paradoxical-seeming statements of quantum mechanics expose their own weakness in some crucial experiment. The fact that this is a dynamical effect makes it a tempting subject for further research by mathematicians, whatever their views on the underlying philosophy.

Decoherence makes possible the existence of a family of consistent histories, at least on the level of classical properties. Such a family defines a stochastic process. If this process can be specified in some natural and precise way, then an experimental outcome is a history in some huge space of classical properties. If this outcome is regarded as part of the description of the actual physical situation, then this gives a strange role

for the atomic world: to define a reality that exists only on the level of the classical world. Furthermore, such a description puts the theory on the same ground as theories that define a stochastic process on the atomic level, where again the state vector is not the complete description of reality. This direction could lead far from quantum orthodoxy.

What can we conclude from such a book? Some physicists regard quantum mechanics as a totally successful theoretical framework; they consider any attempt to raise questions about its foundations as an irritating distraction. Omnès, to his credit, recognizes that the puzzles are profound and that the traditional resolutions do not achieve the level of clarity appropriate to a completed science. Furthermore, he embarks on the task of providing a resolution within the framework of orthodox quantum mechanics. If he or anyone else succeeds in this task, then the questions should be settled, once and for all. In the present case the book is energetic and lively and full of examples and ideas. But the core resolution is merely a desperate bluff. The resolution is to “add to the whole logical construction an assumption according to which present phenomena are unique (and therefore facts).” The bluff lies in such statements as: “the actuality of facts is something that need not be explained by a theory,” and “when one finds a gap between theory and reality only at their common extremities, this is not a failure but the mark of an unprecedented success for quantum mechanics, as compared with all the theories before it.” The fact that an obviously competent physicist is driven to such assertions is evidence that the quantum theory remains in conceptual murk. The challenge remains: interpret quantum mechanics on its own terms, without appeal to authority, in a way that makes sense to reasonable people. This challenge has not yet met an adequate response.

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