Karp and Smale Receive National Medals of Science

In June of this year President Clinton announced the recipients of the National Medal of Science, the nation's highest honor in science and technology. Among the eight recipients are mathematician STEPHEN SMALE and theoretical computer scientist RICHARD KARP. Five National Medals of Technology were also awarded.

The National Medal of Science, established by Congress and administered by the National Science Foundation, honors individuals for contributions to the present state of knowledge in one of the following fields: physical, biological, mathematical, engineering, or social and behavioral sciences. The medal has now been awarded to 344 distinguished scientists and engineers.

Richard M. Karp

Richard M. Karp was awarded the National Medal of Science for "linking advances in theoretical computer science to real-world problems." Karp was born January 3, 1935, in Boston, Massachusetts. He received his bachelor's degree (1955), his master's degree (1956), and his Ph.D. in applied mathematics (1959) from Harvard University. He was a member of the research staff at the IBM T. J. Watson Research Center from 1959 until 1968, and he was on the faculty of industrial and management engineering at Columbia University during 1967–68. In 1968 he took a position as professor of computer science and operations research at the University of California, Berkeley, and starting in 1980 he also held a joint position in the department of mathematics. He also held a position as research scientist at the International Computer Science Institute in Berkeley. In 1995 he moved to the University of Washington in Seattle to take positions as professor of computer science and engineering and adjunct professor of molecular biotechnology.

The Contributions of Richard M. Karp to Computer Science

David B. Shmoys

Richard M. Karp has made fundamental contributions to the foundations of computer science and over the past four decades has dramatically extended our understanding of the nature of efficient computation. He has made seminal contributions to a wide variety of areas within the field of theoretical computer science and has had a profound influence on the directions in which this rapidly growing field has moved over this period, giving it both mathematical depth and practical relevance.

Karp's most significant contribution, presented in his landmark 1972 paper entitled "Reducibility among Combinatorial Problems", showed that twenty-one combinatorially defined computational problems are all $\mathcal{NP}$-complete. This provided concrete evidence that a plethora of well-studied optimization problems, such as the traveling salesman problem and the graph coloring problem, were hard to solve. This work focused attention on the "$\mathcal{P} = \mathcal{NP}$?" question as the central open problem in our under-

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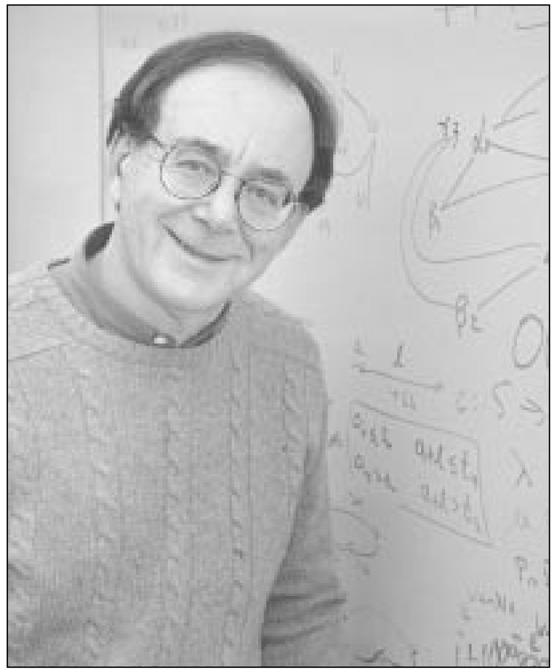
standing of efficient computation. For any set L, the membership problem for L is to decide, given an input x, whether $x \in L$. For some sets, it is trivial to decide if a given x is in L, whereas for others no efficient algorithm is known. The set L is in the complexity class $\mathcal{P}$ if there exists an algorithm that solves the membership problem for L and runs in time bounded by a polynomial in the length of the input; this is the leading theoretical characterization of an efficient algorithm, and $\mathcal{P}$ is the set of all efficiently solvable computational problems. The class $\mathcal{NP}$ is the set of problems L for which there exists $L' \in \mathcal{P}$ such that $x \in L$ if and only if there exists a polynomial-length y such that $(x, y) \in L'$; intuitively, $\mathcal{NP}$ is the class of problems L with the property that each $x \in L$ has a succinct, efficiently verifiable proof y of its membership. In 1971 Cook obtained a pivotal result: he showed that the satisfiability problem, i.e., deciding whether a given Boolean formula (in conjunctive normal form) has an assignment that makes the formula "true", is complete for $\mathcal{NP}$, in the sense that any problem in $\mathcal{NP}$ could be solved by a polynomial number of calls to a subroutine for the satisfiability problem; furthermore, he showed that this result might have implications for other combinatorial problems by showing, in essence, that the clique problem in graphs was also $\mathcal{NP}$-complete. Karp's paper refined this approach and showed that many of the most notoriously intractable computational problems were all $\mathcal{NP}$-complete. As Karp noted at the time, this suggests "that these problems, as well as many others, will remain intractable perpetually." The theory of $\mathcal{NP}$-completeness, as developed by Cook and Karp, and independently by Levin, has had an impact well beyond computer science, since $\mathcal{NP}$-complete computational problems arise in virtually every discipline of engineering and the physical and social sciences.

Karp also made important contributions to the design of efficient algorithms for a number of combinatorial problems. In particular, his work with Edmonds gave polynomial-time algorithms for the maximum flow and minimum-cost flow problems, two of the most fundamental network optimization problems and a subject of intense investigation in operations research since the mid-50s. These algorithms introduced basic techniques, such as data scaling, that were later used by others in a wide variety of settings. Furthermore, if one considers the work that has followed in the design of algorithms for network problems, there is scarcely a paper that does not build on this work in some substantial way. These results also highlighted what would become a central issue in this area: the difference between polynomial and strongly polynomial algorithms, where in the latter case the running-time bound is a function of the size of the underlying combinatorial structure, but not the magnitude of the numbers that are part of the input. In joint work with Hopcroft, Karp gave the most efficient algorithm known for another basic problem in combinatorial optimization, the bipartite matching problem; the algorithm is quite natural, and their running-time analysis provides deep insight into the structure of this well-studied problem.



Richard M. Karp

The traveling salesman problem, another central problem in operations research, was a recurring subject of Karp's investigations; in this problem the aim is to find a shortest tour that visits a collection of points, given the distances between points. In 1962, in joint work with Held, he gave an elegant dynamic programming algorithm for the traveling salesman problem. Although this algorithm takes exponential time, it was the most efficient approach known then to solve this problem. Nearly ten years later, in two further papers with Held, he gave an important way to quickly compute a lower bound on the length of the optimal tour. The Held-Karp lower bound remains one of the most effective ways of efficiently estimating the optimal tour length; years of testing have led to the general belief that it typically produces bounds within 1–2 percent of optimal. Even more importantly, the bound is based on a "Lagrangian relaxation" technique; due to its success in this context, this approach was immediately applied to a wide range of combinatorial problems. For well more than a decade, Lagrangian relaxation was the method of choice for obtaining strong lower bounds for optimization problems.

The traveling salesman problem was also the starting point for Karp's investigation into algorithms that work well with respect to inputs drawn from specified probability distributions. Suppose that the input to the traveling salesman problem consists of points that are selected independently and uniformly at random in the unit square $[0,1]^2$ and the distance between them is measured in the ℓ_2 metric. Karp proposed a natural divide-and-conquer strategy which partitions the points into clusters of nearby points, solves the induced problem for each cluster (using dynamic programming), and then patches these subtours together. He then showed that this algorithm produces a tour that asymptotically tends to the optimum with probability approaching 1. Many results on the probabilistic analysis of heuristics for $\mathcal{NP}$-hard problems followed, and this continues to be a thriving area of research. Karp made many significant contributions to this area, giving ingenious analyses of algorithms for such problems as linear programming, bin-packing, and set partitioning.

A recurring theme in Karp's research in the past two decades is the use of randomization in the design of algorithms. In this setting an algorithm is allowed to "toss coins" as one of its basic operations. Unlike the results on probabilistic analysis mentioned above, in this setting one still takes a worst-case view: one wishes to show that for *any* input, the algorithm's output, which is a random variable, has the desired property, e.g., is optimal with high probability. In many settings, a problem can be solved using a randomized algorithm and yet no deterministic analogue is known. In joint work with Aleliunas, Lipton, Lovász, and Rackoff in 1979, he gave an algorithm to determine if two given vertices are in the same connected component of an undirected graph that, surprisingly, uses space bounded by a logarithm of the size of the graph. Roughly speaking, such an algorithm is limited to auxiliary storage consisting of a constant number of node labels, and it is widely believed, but still unproven, that no deterministic analogue exists. The randomized linear-time pattern-matching algorithm of Karp and Rabin is one of the most appealing results of the area: it combines mathematical elegance with practical efficiency and generality. Luby and Karp considered the problem of computing the reliability of planar networks, which is $\#\mathcal{P}$-complete, and hence believed to be substantially harder than even the $\mathcal{NP}$-complete problems discussed above. They gave a randomized approximation scheme for this problem, which was the first known for any $\#\mathcal{P}$-complete problem; once again, the subsequent study of randomized approximation schemes for $\#\mathcal{P}$-complete problems has blossomed into one of the most exciting areas of algorithm design for combinatorial problems.

In the mid-80s Karp turned his attention to the design of parallel algorithms. This is another setting in which randomization has turned out to be an important tool for algorithm design. One theoretical notion of efficiency for parallel computation is the class $\mathcal{NC}$; this is the set of problems solvable with a polynomial number of processors in time bounded by a polynomial in the logarithm of the input size, with a shared memory for interprocessor communication. In a quite surprising result, Karp and Wigderson showed that the maximal independent set problem is in $\mathcal{NC}$; prior to their work, this problem had been widely suspected to be one for which parallelism could not be used effectively. Their algorithm was based on first designing a randomized algorithm and then "derandomizing" it by showing that the randomization could be restricted to uniform sampling from a polynomial-size sample space and then making parallel runs, one for each sample. This subsequently became one of the most useful paradigms of parallel algorithm design. His $\mathcal{RNC}$ algorithm for the matching problem, obtained jointly with Upfal and Wigderson, was another breakthrough result; this has prompted interesting research on techniques that might lead to a deterministic $\mathcal{NC}$ algorithm, but no such matching algorithm is currently known.

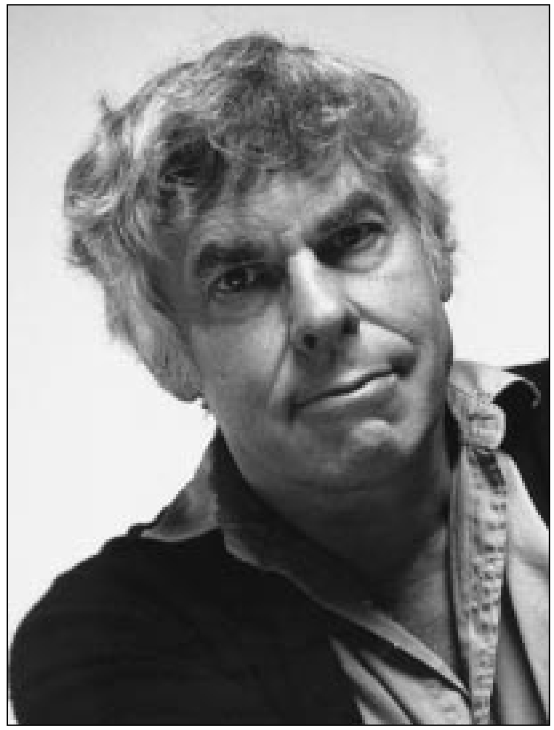
Among the issues prevalent in his research in the past decade, Karp has made leading contributions to the study of online algorithms and has worked towards bridging the gap between theoretical models of parallel computation and the realities of parallel machines. Most recently, Karp has focused on computational issues in molecular biology. Although the motivation for these problems is different from those that he studied previously, this research nonetheless is a natural extension of his investigations into efficient algorithm design for combinatorial problems.

Karp has been awarded virtually every major prize in computer science and operations research, including the ACM Turing Award, the ORSA/TIMS von Neumann Theory Prize, the Lanchester Prize, and the Fulkerson Prize; he is a member of the National Academy of Sciences and the National Academy of Engineering. In addition to his gifts as a researcher, Karp has a phenomenal talent for teaching. Having spent most of his career at the University of California, Berkeley, and now at the University of Washington, he has inspired several generations of students with his lectures of laser-sharp clarity. He has the uncanny ability to take the most opaque just-proven result, to understand its essence, and then to present it in a perfectly transparent way.

Throughout his career, Dick Karp has repeatedly initiated an important path of research, obtained elegant and significant results to start the area, and stimulated the research community to continue in his footsteps. It is difficult to imagine any individual having a more profound impact on any field.

Stephen Smale

Stephen Smale received the National Medal of Science for "four decades of pioneering work on basic research questions which have led to major advances in pure and applied mathematics." Smale was born on July 15, 1930, in Flint, Michigan. He received his bachelor's degree (1952), his master's degree (1953), and his Ph.D. in mathematics (1956) from the University of Michigan, Ann Arbor. He was an instructor at the University of Chicago from 1956 until 1958, when he became a member of the Institute for Advanced Study. In 1961 he went to Columbia University, and in 1964 to the University of California, Berkeley. He retired from Berkeley in 1995 and is currently a professor of mathematics at the City University of Hong Kong. His many honors include the Fields Medal, awarded in 1966.



Stephen Smale

The Mathematical Work of Stephen Smale

Steve Batterson

Steve Smale has made profoundly original contributions to a stunning array of mathematical specialities. In 1990 the Smalefest conference celebrated Steve's sixtieth birthday with a program entitled "From Topology to Computation: Unity and Diversity in the Mathematical Sciences". The proceedings of this conference (edited by M. W. Hirsch et al., Springer-Verlag, 1993) include individual survey articles on Smale's work in differential topology, economic theory, dynamical systems, computation, nonlinear analysis, and mechanics. See this volume for a more substantive review of the remarkable depth and breadth of Smale's mathematics.

Smale completed his Ph.D. thesis in 1956 under the direction of Raoul Bott at the University of Michigan. Two curves are regularly homotopic provided that there exists a homotopy through regular curves for which the tangent vector varies continuously as a function of the curve and homotopy parameters. The 1937 Whitney-Graustein Theorem classified regular closed curves in the plane, up to regular homotopy, by their winding numbers. In his thesis Smale generalized the result to regular closed curves living on an n-manifold. The classification is provided by a bijection to the fundamental group of the unit tangent bundle to the manifold. To establish the link, Smale skillfully employed algebraic topology, analysis, and especially the theory of fiber spaces. The thesis did not attract a great deal of attention.

A few months into his postdoctoral career at the University of Chicago, Smale obtained his first famous result. He succeeded in pushing the fiber space techniques to immersions of spheres. Smale classified C^2 immersions of S^2 into R^n, up to regular homotopy, by the second homotopy group of the Stiefel manifold, $V_{n,2}$. Since $\pi_2(V_{3,2})$ is trivial, an immediate corollary is the regular homotopy equivalence of all immersions of S^2 into R^3. In particular, there exists a regular homotopy of the inclusion to the antipodal immersion, in effect turning the sphere inside out. The result was counter to the prevailing intuition that placed these immersions in different classes. The fact that Smale's proof offered little insight into a comprehensible realization of the homotopy added to its mystique.

In the summer of 1958 Mauricio Peixoto introduced Steve to the Andronov-Pontryagin concept of structurally stable vector fields (the trajectory dynamics are preserved under small perturbations to the vector field). Peixoto sought a characterization of structural stability that extended beyond the surface setting. Smale quickly saw the relevance of transversality and topology to the higher-dimensional situation. He conjec-

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tured that a class of vector fields, now known as Morse-Smale, were exactly the structurally stable ones. Additionally, he asked if the Morse-Smale systems were dense in the C^1 topology. This was the first approximation in Smale's bold vision for dynamical systems, a qualitative study of differential equations that transcended the algebraic form of the equation.

Smale began a two-year National Science Foundation Postdoctoral Fellowship in 1958. Following 1-1/2 years at the Institute for Advanced Study, Steve moved to Rio de Janeiro to complete the final six months at the Instituto de Mathematica Pura e Aplicada. During this period in early 1960 he obtained two sensational results. Shortly after his arrival in Brazil, Smale received a letter from Norman Levinson asserting that there were structurally stable systems that were not Morse-Smale. This led to Smale's construction of the horseshoe map and his early study of chaotic phenomena.

Next, Steve resumed his work on the Poincaré Conjecture. The problem, a compact n-manifold with the algebraic topology of the n-sphere is homeomorphic to the n-sphere, had attracted Smale since his graduate student days. The situations $n = 3$ and $n \geq 3$ are known respectively as the Poincaré Conjecture and Generalized Poincaré Conjecture. At the time there was an overwhelming conventional wisdom that difficulty increased with dimension. The three-dimensional problem was so daunting that larger n appeared out of reach, at least until dimension three was resolved. Undeterred, Smale conceived a Morse Theory approach to the Generalized Poincaré Conjecture. As he developed these ideas, an extraordinary element emerged. His proof was valid for all dimensions $n \geq 5$, but failed in dimensions three and four. Smale had proved the Higher Dimensional Poincaré Conjecture. In the eighties Michael Freedman established the four-dimensional theorem, but $n = 3$ remains unsolved.

In 1961 Smale proved the h-Cobordism Theorem. This seminal result provides algebraic topological criteria for establishing that higher-dimensional manifolds are diffeomorphic. Having set a new agenda for differential topology, Smale then abruptly shifted his attention back to dynamical systems. Steve was renewing his quest for a generic structurally stable collection of dynamical systems. His original Morse-Smale candidate had been disqualified by the horseshoe. As Smale sought a second approximation he unearthed other essential elements of his developing program for dynamical systems. The project was put on hold from 1962 to 1964 as Steve's interests detoured into infinite-dimensional analysis. There he, independently with Richard Palais, extended Morse Theory to those nonlinear maps on infinite-dimensional manifolds that satisfy what is now known as the Palais-Smale condition. Next he obtained an infinite-dimensional generalization of the Morse-Sard theorem.

Smale returned to dynamical systems in 1965, showing that structural stability was not dense. Smale's original vision for dynamical systems could not be realized, but his approximations were converging to something exciting. In his landmark 1967 *Bulletin* survey article, Smale presented his program for hyperbolic systems and stability, complete with a superb collection of problems. The major theorem of the paper was the Ω-Stability Theorem, whose proof was a tour de force in the new methods.

By the late sixties Smale had moved into applications. He modeled physical processes by dynamical systems, opening new lines of inquiry. The n-body problem and electric circuit theory were among the applications that Smale framed in the language of dynamical systems. For much of the seventies Steve focused on economics, injecting topology and dynamics into the study of general economic equilibria. Having established the nature of the equilibria, Smale began to think about algorithms for their computation. While traditional approaches to the convergence theory of algorithms were local, Smale introduced a global perspective to the problems. Was the algorithm reasonably reliable, and how many iterations were to be expected? Newton's method and the simplex algorithm gained new meaning from this perspective.

In recent years Smale, in collaboration with Lenore Blum and Mike Shub, has sought to unify the fields of theoretical computer science and numerical analysis. Practical numerical algorithms involve the computation of real numbers, while the classical Turing machines manipulate discrete sets. The Blum-Shub-Smale model for computation operates on a ring, thus encompassing the 0–1 world of Turing machines and the real-complex number setting required for numerical analysis. The result is a generalization of the classical theory that provides a theoretical foundation for numerical analysis.

So many brilliant threads pervade Smale's mathematics that providing a summary is partly a matter of personal taste. Throughout his career Smale has approached mathematical problems with the scholarship to learn from others, the audacity to be unconstrained by conventional wisdom, and the power and vision to employ new methods and construct original frameworks. After the fact, a Smale development seems so natural, yet no one else thought of it.