Presidential Views: Unity in the Mathematical Sciences Community

The mathematical sciences community today faces some of the toughest problems it has seen in decades. The combination of tight research funds, shrinking higher-education budgets, a dismal job market, and questions about the adequacy of graduate education has hit hard. And despite surprising discoveries of links between branches of mathematics once thought to be quite separate, the increasing specialization required to do research has often isolated the branches from one another. In this climate traditional divisions within the mathematical sciences—between, for example, pure and applied areas, or elite and nonelite schools—could harden. However, the presidents of the three major societies concerned with mathematical sciences research—the AMS, the Institute of Mathematical Statistics, and the Society for Industrial and Applied Mathematics—are optimistic about the community's ability to work together to solve these problems. The *Notices* presents below the presidents' responses to five questions about important issues facing the community today.

—Allyn Jackson

JAMES BERGER, past-president, Institute of Mathematical Statistics, and **NANCY REID**, president, Institute of Mathematical Statistics

Question 1: How has the increasing breadth of research in the mathematical sciences affected relations among researchers in different areas?

First, we need to make a few comments to clarify where we are coming from. The Institute of Mathematical Statistics represents, by and large, academic statisticians and probabilists with the closest ties to mathematics; statisticians who are mainly involved in applications in other subject areas would typically turn to one of the more applied statistical societies. As the IMS is at the mathematical end of statistics, we find ourselves in the curious position of looking very applied from the viewpoint of most mathematicians and yet looking very theoretical from the viewpoint of most statisticians. So one day we argue for the benefits of theory and the next for the benefits of interacting with applications.

We also should point out that the IMS represents both statisticians and probabilists. We happen to be statisticians, and our comments will thus naturally be more reflective of the statistical viewpoint; probabilists tend to relate more directly to the pure and applied mathematical communities than do statisticians.

But back to the original question.

Statistical research has always covered a broad range, from mathematics to economics and psychology through to medical applications, and so on. So we are accustomed to debates on the relative importance of various subareas. The increasing breadth of research in the mathematical sciences (or perhaps what is more relevant is the increasing *depth* of research) has changed the nature of the interactions between mathematics and statistics and within statistics. Statisticians, even mathematical statisticians, are less conversant with current work in probability and pure mathematics than they were even twenty years ago and vice versa. And, even within statistics, it is increasingly difficult to simultaneously work in several areas. We thus know less today about the work of the statistician in the office next door, let alone the work of the mathematician in the next building. Luckily, the reaction to this seems to be one of increasing tolerance. As we know less about the details of the work of others, we tend to interact and appreciate each other more on the basis of our common view of the centrality of the mathematical sciences.

The increasing breadth of research in the mathematical sciences has probably helped the relationship between statistics and mathematics. Statistics derives its motivation from application to practical problems. This is not to say that research in statistical theory is not important, but theoretical investigations are often performed with at least a long-range eye towards practical needs. This practical orientation of statistics was not always appreciated by the mathematical community. Today, however, the increasing breadth of mathematics seems to have greatly elevated the status of work concerned with potential applications, and this has eased strains between the mathematical and statistical communities.

Question 2: What are the major issues in graduate education?

As in other areas of the mathematical sciences, there is a tension between breadth of knowledge and specialization, and it is difficult to find the right compromise for graduate education. As the result of various pressures, statistics graduate students are now spending more time learning to do applied work and less time studying mathematics or mathematical statistics.

There is also a trend, as in many areas of science, to increasing emphasis on interdisciplinary work, and a graduating student with a subject area specialty is very employable. But to ask students to have a graduate-level education in, for example, statistics and sociology or statistics and geology further increases the tension between specialization and breadth. This tension will probably also be increasingly felt in other areas of the mathematical sciences; not only is interdisciplinary work a current "fashion", but with the job market for academic researchers shrinking, we need to educate our students to be conversant in more than one specialty. Of course, this means a major change from the way we've always done things, and we are all struggling to find the right course. It is interesting that the statistics community seems to have agreed upon the necessity of this change, even if it does not yet know how to go about it.

Question 3: Has the difficult job market produced strains?

We have started to feel the strain in the past few years; until recently the job market for academic statisticians was very strong, and we are only now facing problems that have long been familiar to mathematicians. For example, as recently as five years ago it was relatively unusual for students to do a postdoc before their first tenure-track appointment, but it is now becoming quite usual.

We are in the fortunate position of having many nonacademic employment opportunities for our students, although increasingly these jobs are requiring specialization in particular application areas of statistics, leaving even less time for general mathematical training. Interestingly, most statistics departments seem to be adapting to this situation without too much stress, eliminating courses on theory and replacing them with more applied or specialized courses. One, of course, continuously hears the "theory-versus-application" educational arguments, but the trend is clear. Perhaps since statistics has always been intimately connected with applications, we are rather amenable to shifting our educational programs in response to the perceived outside needs.

Question 4: How do the pure and applied mathematical communities relate to each other?

It makes more sense for us to answer a different question, namely, "How do the statistics and mathematics communities relate to each other?" The community represented by the IMS is, by and large, rather closer to mathematics than is the average statistician; most of our members are academic, many are working in mathematics departments, and most of the members began their academic training as mathematicians. Even so, the answers of our members to this question would vary enormously. Many would say that they have little to do with the mathematics community these days, while others would attest to frequent collaboration with mathematicians. The amount of such collaboration has probably declined over the years, but this could simply be due to the problem raised in Question 1.

There is a fairly widespread feeling among statisticians that their approach to scientific problems is not well appreciated by either pure or applied mathematicians, since statistical modelling requires the development of insights that cannot be reduced to mathematics alone. Conversely, because our research tends to be more directly driven by applications than is mathematics, we tend to be skeptical of the notion that research in the mathematical sciences can usefully proceed in isolation from the world.

On the whole, however, we feel that the statistical and mathematical communities are sympathetic to each other. We approach the world from a similar perspective and have similar long-range goals. And, if anything, statisticians have an even higher appreciation of the need for the mathematical sciences than do mathematicians in that we continually encounter the disasters that have occurred in other sciences and society because the need for careful mathematics and statistics is not appreciated.

Question 5: What signs do you see that the mathematical sciences community is coming together to try to tackle some of these difficult problems?

Well, clearly we are seeing much more discussion of all this. The increasing discussion is undoubtedly motivated by the increasing pressures on the mathematical sciences community, but that is okay; the community is really too large to forge a continuous mode of cooperation but does seem to get together in times of need. If anything, the community does not use statisticians enough in helping to respond to the outside pressures on the mathematical sciences. By the nature of what statisticians do, we can produce a virtually unending stream of success stories concerning the value to the outside world of research in the mathematical sciences. We will be pleased to work together with the rest of the mathematical sciences community to carry the message of the value of research in mathematics and statistics.

CATHLEEN MORAWETZ, president, American Mathematical Society

Question 1: How has the increasing breadth of research in the mathematical sciences affected relations among researchers in different areas?

The mathematical sciences are broader, and core areas are going deeper in their explorations. There have been many major breakthroughs. Mathematicians concentrating on research have to take a narrow view because one needs a lot of specialized knowledge to do research, so in this sense there is not so much breadth. But in the last fifteen years there have been many interactions between different parts of pure mathematics that never interacted before. Core mathematicians are very proud of this fact.

Certainly there are strains in all sciences between their individual branches; it's part of what keeps it going. I think in other sciences it's less spelled out, but it's there. On the other hand, progress depends not only on cooperation but also the natural competition that drives people forward. But in fact there are fewer strains in the mathematical sciences today than there were in the past and a much stronger sense of working together. I would like to add that I don't like either expression, "core" or "pure", but for lack of a better alternative, I have to use them.

Question 2: What are the major issues in graduate education?

One deep problem is the length of graduate education. A person embarks on a project of five years and at the end can't get a job. There ought to be more places to go if, say, after two years you want to change your direction. On the other hand, if you stay the course and are destined to be a researcher, there ought to be more places for postdoctoral training, so that if you take a broader program for your Ph.D., you can really specialize afterwards.

I believe in the long run there will be a need for people with the kind of skills a mathematical sciences education gives. But right now we're floundering because the era of technology has not yet called on the mathematical sciences community. We have a lot that is needed, and we must convey it in understandable terms. At the same time, it's always been the case that a number of new Ph.D.s go into industry or government labs, and their mathematical sciences training gives them a way of thinking, of attacking problems. That ability is extremely useful and much appreciated.

Another point of strain is the growing awareness that we can't "brain drain" the rest of the world forever. The fact that we can attract mathematicians from other countries has led us to neglect developing our own. In particular we need to bring more women and minorities into the mainstream of the mathematical sciences. It's a question of the future of the science. I take seriously the projections of what the character of the population will be in the future. At the present time we take the best from all the other countries. If some other country is richer and can provide more for them, then they won't come here, and we will not have produced our own. We will have lost on all counts. We have to insure that mathematics education is broad enough that it catches everyone in its net and is at the same time deep enough to prepare future mathematicians.

Question 3: Has the difficult job market produced strains?

Certainly, because we're producing more students than there are jobs for them. People in strongly applied areas like statistics or operations research are having much less trouble. And people from places outside the top tier are also having fewer problems because their advisors and departments have more connections for teaching jobs, especially in local community colleges. At the top schools the best students are getting good jobs, but the second- to third-best students are having a lot of trouble, in part because they have fewer connections to jobs involving mostly teaching and not so much research. This strain cuts the community in a different way from the research strains.

This is just a painful situation, and I don't see how we can remedy it. I think dramatic efforts to cure that situation might lead to overkill. In the 1970s, when the job situation was bad, though not as bad as it is now, there was a tremendous decline in the number of Ph.D.s produced and then a great shortage. So there's a cycle here, and it's a question of how to dampen the cycle.

Question 4: How do the pure and applied mathematical communities relate to each other?

There is no question that, in the U.S. fifty or sixty years ago, despite the fact that there had been very prominent Americans who did a great deal of applied mathematics—like Hill, or Gibbs, or Birkhoff—there was a historic reluctance toward applied mathematics. It was much less so in Europe. During the Second World War, with the refugees coming and the war needs, there was an enormous growth in applied mathematics. After the war there was a sense among many of them that they weren't being recognized. So it's a history of "on-again, off-again" relations between the two communities. I for one see it as a great loss when, for example, a mathematics department separates into two departments of pure and applied mathematics. But as I said before, I think that today there is much more unity than in the past. This can be seen in the trend toward having unified departments with both pure and applied mathematicians.

People in pure mathematics feel they are at the very basis of structure of a science. And they're building that structure, putting the pieces together, linking them, and many of them are not particularly anxious about the fruit of their labor entering into some practical problem. But in the end mathematics history shows that those things eventually percolate into applications or get applied. On the other hand, in digging in the structure of mathematics, it often turns out that input from the physical world gives some new insight. They feed on each other.

Question 5: What signs do you see that the mathematical sciences community is coming together to try to tackle some of these difficult problems?

Although there is naturally a tendency to grow apart, I can assure you that strong efforts are going on to counteract that tendency. Historically, the MAA split from the AMS; SIAM grew out of the alienation of applied mathematicians. I don't know the history of the IMS. So trying to draw our societies together is an old challenge that we are beginning to meet. We do a lot with MAA through joint meetings and joint committees. SIAM and AMS are jointly funded by the Sloan Foundation to work on employment opportunities. The Joint Policy Board for Mathematics is our triple front on government policy. In the area of public awareness of the role of and the need for mathematics, we are all working and working together. Among our joint committees with the IMS are two especially active ones, the Data Committee and the Committee on Joint Summer Research Conferences. I would like to see us also do a lot more with the IMS, and these committees provide a place to start from.

MARGARET WRIGHT, president, Society for Industrial and Applied Mathematics

Question 1: How has the increasing breadth of research in the mathematical sciences affected relations among researchers in different areas?

Let me mention two opposite effects of the increasing breadth of research.

First, the growing volume of new results in individual subject areas means that it is more and more difficult to keep up to date. Since expertise and depth are needed to survive as a credible researcher, some mathematicians may focus mostly on their specialty, go to conferences only in that area, read mainly papers in that area, and so on. When this happens, these researchers talk less often to mathematicians in other fields, and the overall mathematical sciences community becomes more fragmented.

There is, however, a second, countervailing trend: that once-separate topics have spread out so far that they have started to blend together. In several fields, intellectual structures and techniques from what was once "outside" are now securely in the mainstream. This phenomenon means that mathematical scientists need to understand what is going on in multiple areas, even those apparently far from their special interests, and thus formerly disparate areas are naturally brought together.

On balance, I like to think that increasing breadth in research will help the mathematical sciences to recognize our essential unity.

Question 2: What are the major issues in graduate education?

U.S. graduate education has been the subject of several recent reports—for example, from the National Research Council (NRC) and the National Science Foundation (NSF)—suggesting changes. It's impossible to summarize these in a few words, so let me make some remarks about only one aspect, the growing interest in interdisciplinary work.

Interdisciplinary research as a way of doing science is here to stay. Having said that, I absolutely do not think that it is necessary for mathematicians to look outside their own field for inspiration and nourishment. Nonetheless, I know that mathematicians can learn about problems that are extraordinarily rich *mathematically* by interacting with other disciplines. I tend to regard it as a sign of a good mathematician that he or she can find something mathematically interesting in essentially any problem. If we accept that future Ph.D.s will be engaged in interdisciplinary work for at least part of their careers, this argues for broadening exposure to other sciences as part of graduate education. It also suggests that training in written and oral communication to nonspecialists would be helpful. In addition, computation is fundamental in all of science today; therefore, I believe that someone with a Ph.D. in mathematics needs to have a substantive understanding of the intellectual issues in computation, even if that person never subsequently goes near a computer.

The prospect of changing graduate education naturally arouses concern, especially when the impetus comes from outside forces. In my view the suggestions made so far are not meant to change what is working, but rather to offer mechanisms that provide graduate students with more professional options and flexibility along with qualities (like good writing and speaking) that are useful in any context.

Question 3: Has the difficult job market produced strains?

This issue is closely tied with those mentioned in the answer to Question 2. The academic job market is extremely difficult, which means that many Ph.D.s are thinking, in some cases reluctantly, about nonacademic jobs. SIAM has many members who work in industry (including me), and of course we believe that nonacademic jobs can be extremely rewarding, but that's a different subject! SIAM's 1995 "Mathematics in Industry" (MII) report indicates that mathematicians are highly valued in nonacademic settings and that graduate education provides excellent preparation in perceiving, formulating, and analyzing the structure of complex problems. However, current graduate education in the mathematical sciences does less well in other domains, such as providing some breadth of scientific interests, an understanding of computation, and skills in communication. In fact, these skills are important for mathematicians regardless of where they end up working, so that including them in a graduate program does not, in my opinion, compromise quality or integrity. The MII report recommends that graduate departments consider ways to provide these qualities and makes numerous suggestions for how to do this.

Question 4: How do the pure and applied mathematical communities relate to each other?

The wording of this question touches on a point of great concern to me. In speaking about the mathematical sciences at a policy level, I try never to draw distinctions between pure and applied mathematics, or between "core" mathematics and the rest of mathematics (whatever the noncore is called). First, I am not aware of an unambiguous definition of these terms—and precise definitions are something we are supposed to be good at! For example, most people would assert that number theory is pure mathematics, but of course number theory is applied in a big way in cryptography. And many people working in fields usually called "applied mathematics", such as partial differential equations, develop mathematical structure and prove theorems that are completely divorced from any application. So we can't label fields as pure and applied, or even draw a clear distinction between research and applications. Some define by intent—that pure mathematics happens when the mathematician doing it has no interest in practical consequences—but this definition seems like theology, not mathematics.

Even more important than the lack of a precise definition is that dividing up mathematics into pure and applied, or core and noncore, seems to me to be unproductive. Mathematicians are not a homogeneous group, and no one wants them to be. But rather than stressing what divides us from each other, I hope that we can think about mathematics as a discipline that is *unified*, with common themes (such as rigor, formalism, analysis of underlying structure and patterns, etc.) that apply to all areas of mathematics. Our main priority should be to clarify why *mathematics* is important as a way of thinking. I would like to think that we can move forward together, that we can appreciate each other's work, and that the only distinction that really counts is between good and bad work.

Question 5: What signs do you see that the mathematical sciences community is coming together to try to tackle some of these difficult problems?

My perception is that many mathematicians recognize that we need to work together on all of these problems. Let me cite a few instances of this.

For several years the Joint Policy Board for Mathematics (JPBM) has been one means for the AMS, SIAM, and the Mathematical Association of America (MAA) to present a unified case for the mathematical sciences. For example, last year Arthur Jaffe testified for support for the NSF budget, and Bob Plemmons of Wake Forest University testified to two separate committees in support of Department of Defense funding for basic research.

There are four mathematicians on the Advisory Committee for the NSF Directorate of Mathematical and Physical Sciences (I am one of them). We are there not only to represent the mathematical sciences but also to participate in the broader issues that face science policy and funding within the NSF.

Cathleen and I have made an explicit effort during our terms as AMS and SIAM presidents for the two societies to work together whenever possible. For example, in February we met with NSF director Neal Lane and spoke strongly about the centrality and unity of the mathematical sciences. The AMS and SIAM are continuing a joint activity, funded by the Sloan Foundation, to provide information and resources about nonacademic careers.

Let me conclude by saying that the mathematical sciences have such a good story to tell—it's real, not hype—and we alone are responsible for making our case. No one else is going to do it for us. Try to picture yourself in a room filled with physicists, chemists, biologists, and so on—each of them can argue, correctly, that his or her individual area is important. But mathematics is uniquely fundamental and pervasive; we need to say so, both often and together.