Book Review

A Tour of the Calculus

Paul Zorn

A Tour of the Calculus
David Berlinski
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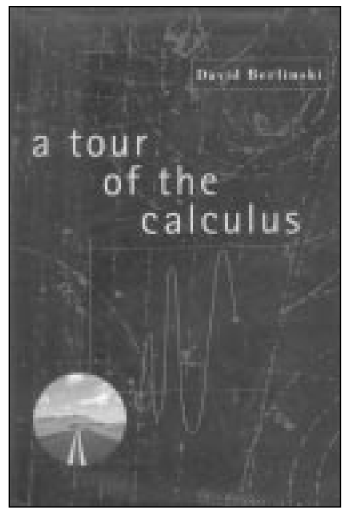
The book lives up to its name—it is indeed a tour of the subject. But there are tours and tours: package tours of Europe, walking tours of Spain, treks in Nepal, Caribbean cruises, a prospective college student's campus visit, a junior year abroad, Disney World, a time-shared condo in Cancun.

Berlinski's tour is none of these. The cover art—a lonely road in arid landscape—is misleading. What Berlinski's tour most resembles in the tour catalog is an air-conditioned "study cruise" up the Amazon River, with on-board naturalist, historian, and ethnographer—all of them chatty and slightly manic, willing to invent a bit when certain knowledge runs out. There is an occasional short, sweaty walk into the jungle, or even a run through the rapids. But qualified medical personnel are standing by, and there is good Chilean chardonnay chilling back on board.

Understood this way, Berlinski's *Tour* is a good value and an entertaining read. In about 300 pages, Berlinski visits—briefly—most of the "standard" destinations, from basics of the real number system through limits, continuity, and the mean value theorem to the fundamental theorem of calculus. More remarkable for mathematically versed readers, however, may be the variety and number of side trips and optional excursions. Among the attractions: Dekedind cuts; Zeno's paradoxes; historical interludes, partly invented, featuring Newton, Leibniz, Euler, Cauchy,



Rolle, Gauss, Riemann, and others; the author's Eastern European travel memoirs and classroom misadventures; frequent encouraging words for mathematical novices; proofs of such results as the irrationality of $\sqrt{2}$ (proved in Socratic-style dialogue—with a taxi driver), the mean value theorem, and the fundamental theorem of calculus; and some quite sharp and perceptive musings on the culture and nature of mathematical science.

The prose style is distinctive, to say the least. Here, for instance, is Berlinski setting the stage for Galileo's law of falling bodies (page 96):

> Now imagine a gorgeous tower, its parapet jewel-encrusted, the dreamy Perugian hills in the background, lacy clouds above. And on this tower an Italian dandy, dressed in silks puffed at the wrists and at his thighs, is fingering a large and lavish rose and rufous stone, a fabulous ruby or garnet, something luscious and lustrous. He dangles an elegant forearm over the parapet, holding the ruby in his upturned palm, and then slowly and with vast sensual deliberation rotates his wrist so that the precious stone, its cut facets catching the golden Tus-

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> can light, slides from his polished palm and winking colored fire slips off into space.

This style may be too rich for some tastes, like cruise-ship food. But here Berlinski has a good point to make: Galileo's law (and, by extension, the calculus) is universal; it applies regardless of the Tuscan light, the puffed silks, the cut facets, and every other accident.

But a little of this vivid, charged writing goes a long way. Here and there the style gets out of hand: the adjective burden can be taxing and the breathless tone annoying. About polynomials, for instance, we read (page 80, italics in original):

> The polynomial functions constitute an archipelago, and like the islands in an archipelago they are close to one another, the pale blue ocean waters bright, but not deep. The *transcendental functions*, by way of contrast, are *not* algebraic; they go beyond—they *transcend*—the algebraic operations …. They rise as isolated and volcanic atolls out in the open ocean where the waters are dark blue and where an object that is dropped would travel for years before hitting the ocean's sandy bottom.

By the time this tour ends, some passengers will welcome the respite from metaphor overload.

Sometimes Berlinski's imagery overpowers the mathematics. Distinguishing between lines and curves, for instance, Berlinski writes (page 178, italics in original):

> …But *curvature at a point* would seem to tremble on the same margin of incoherence as speed at an instant. The moving finger meanders over the rounded shoulder and stops, creating a pressure dimple in the molded flesh, but the hand having stopped, the sense of curvature conveyed by the caress disappears as well, the indented point being simply what it is, a way station along a sensuous arc; it is the *whole* of that shoulder that conveys to the voluptuary the conviction that he is getting anywhere at all.
>
> …The lesson of love is not to linger too long at a point, the same lesson, curiously enough, taught by the calculus.

What were we talking about again? This time, even the author appears distracted by the metaphor—in the next few pages he seems to conflate slope with curvature.

Such technical slips are unusual, however. Berlinski's mathematics is generally sound and clearly explained and sometimes goes surprisingly deep. For example, there is more here on the subtleties of completeness (not so-named, by the way) than in most college calculus texts; many a math major could learn something from the discussion. Here is a sample (page 96, italics in original):

> The line is in some sense richer than the numbers that are used to represent it, and this is an old, an inconvenient fact; but Dedekind's diagnosis goes beyond a revisiting of such facts in order to display the long-hidden source of the discrepancy between line and number. Every rational number produces a cut among the numbers; but some cuts answer to no rational number and in this respect— *this alone, no other*—the numbers and the line are different. Dedekind's calm but profound investigation succeeds as an act of intellectual liberation because it connects a particular fact—that some distances cannot be measured by any rational number—with the much larger, the more general, fact that some *cuts* cannot be made at any rational number.

For readers who want a tour itinerary similar to Berlinski's, but with a much more Spartan feel and a different balance between describing and doing, there still seems to me nothing that quite dethrones W. W. Sawyer's 1961 should-be-classic, *What Is Calculus About?*, which describes many of the same mathematical ideas in spare, crystalline prose. Sawyer's final chapter, which glances at some topics of "higher" mathematics, is still fresh after thirty-five years.

In the end, readers who do not already know calculus will not learn much here about actually *doing* calculus. (That is not a criticism—Berlinski's guidebook has different aims.) But even mathematically sophisticated tourists will learn something *about* calculus and about the author's view of its nature, development, and uses. Indeed, Berlinski's luxury cruise might well lure some tourists back, perhaps on foot, for another, longer, closer look.