Letters to the Editor

On Plato and Logic

I was particularly interested in fact with Huber-Dyson's comments regarding Plato and logic as mentioned on page 837 of the August *Notices*. It seems Socrates's warnings from section 275 of Hamilton's translation of the Phaedrus and Letters VII and VIII were overlooked. Therein does Socrates advise Plato, "Then it shows great folly—as well as ignorance of the pronouncement of Ammon—to suppose that one can transmit or acquire clear and certain knowledge of an art through the medium of writing." Any logician would say that Plato's diligence at committing Socrates's insights to writing and paper is thus completely suspect on the premises or more precisely a perfect specimen of the contradiction.

I am sorry Huber-Dyson was so evidently unaware of Socrates's Phaedric position and recommend a review of that literary vehicle.

C. Felicitas
New York Academy of Sciences
(Received August 7, 1996)

Follow Ad Policy Indiscriminately

I write to point out an apparent violation of your classified ad policy (September 1996, *Notices*, p. 1067). Your own "positions available" rules state that ads from institutions outside the U.S. cannot be published unless they are accompanied by a statement that the institution does not discriminate on the basis of color, age, sex, race, religion, or national origin. The Uppsala University ad does not include such a statement and appears to be blatantly discriminatory with respect to sex. Please assure me that this was merely an oversight on the part of the editor rather than a gross example of "politically correct" hypocrisy.

Eric M. Freden
Utah State University
(Received August 20, 1996)

Editor's note: The advertisement in question was reviewed and approved by an officer of the Society before publication.

Comments on the Harvard Consortium Calculus Text

Calculus from the Harvard Consortium (CHC below) recently called forth a lively exchange of letters between Jerry Rosen and several consortium members. As a one-time Harvard professor, my first reaction was: "What, no epsilon delta at Harvard?" But it is not that simple.

The central notion of calculus is that of a limit, say limit as h goes to zero. In the nineteenth century, mathematicians got it right: "For every positive epsilon there is a positive delta such that for all h less than delta in absolute value…" There it is, in full glory with three alternating nested quantifiers—and this before the logicians, in the person of Gottlob Frege, got around to talking clearly about quantifiers. But that is a tough story to offer to freshman calculus. So we had better be user friendly; on page 138 of CHC reads

> "We define $\lim_{g \to c} g(h) = L$ to mean that $g(h)$ can be made as close to the number L as we like by choosing h close enough to c."

The third quantifier is obscured! One might have used a traditional phrase "becomes and remains"; I did not find that. Will the students understand that? I do not know. But I can ask if the authors of the book understand their own definition. On the next page the discussion of the continuity of $f(x)$ reads

> "The closer x gets to a, the closer $f(x)$ gets to $f(a)$"

False, the authors may have forgotten that famous function $x\sin(1/x)$.

The notion of a function is central to calculus—and hard.

In a box on page 121, I find, "One quantity *H*, is a function of another, *t*, if each value of *t* has a unique value of *H* associated with *t*..."

Here "quantity" indicates that we consider just numerical functions of numbers. Fair enough. The word "associated" is not defined, fair enough, so I imagined a physical situation where one can measure two different quantities *H* and *t*. But then lower on the same page I find "Finding a function which represents a given situation is called making a **mathematical model**." What is going on here? Can I understand what a model is? Is there a function without a situation?

But the text has many merits. It is user friendly. After a careful three page discussion of instantaneous velocity as a limit, one finds this neatly formulated in a box (page 95), plus some reassurance "Be sure that you are not confused by the notation... you should realize that we have not introduced any new ideas in these definitions, we have simply found a compact way to write the ideas developed previously."

On page 121 (and later), there is a splendid statement and explanation of the Leibniz notation dy/dx. But then one reads at once that "many scientists and mathematicians really do think of *dy* and *dx* as separate entities representing infinitesimally small difference in *y* and *x*, even though it is difficult to say exactly how small 'infinitesimal' is." This leaves me mystified.

Despite this, the book has many attractive features. There are elegant figures (e.g. of absolute values), and persuasive descriptions of intuition,

> "If the graph of *f* looks more and more straight and nonvertical as the region near *x* is magnified." There is a persuasive pictorial description of the Riemann sums, left and right: "for any function you are ever likely to meet the limits of the left- and right-hand sum will exist and be equal."

The historical notes are great. You join Galileo at the leaning Tower of Pisa and learn why Aristotle was all wrong about motion. There are very clear statements of the standard rules of differentiation—although there is no proof that the limit of a sum is the sum of the limits ("can you see that it is true?" p. 189).

Many results are not proved, but "justified". This often means that an approximation is stated as $\approx$, the word "limit" is produced and the approximation becomes an equation. For example, in the "proof" of the Fundamental Theorem of the Calculus a $\approx$ on line -8 becomes = on line -4.

I venture to suggest that the book needs a clear statement that a justification is not a proof and that there are indeed rigorous proofs which need to be learned. There might well be some students who would like to know this.

The book has many good exercises—perhaps a bigger collection of real world exercises than anywhere else.

Exercises are user friendly. On one page you are instructed how to measure the increase of affection between Romeo and Juliet. This will prepare you for a subsequent exercise, inviting you to set up a pair of DEs which will measure why a sequence of successive dates will or will not lead to a real relationship.

Advice to students who want action: Do not give up; if you persist to p. 551, you can help Admiral Horatio Nelson win the battle of Trafalgar, all this without standing on the poop deck of HMS Victory exposed to French sharpshooters. But the data for the numbers of ships in this problem is all wrong (Ency. Britt., 11th ed., has 27 British ships, 33 allies). Maybe the authors counted one ship-of-the-line (three gun decks) as equal to one frigate (one gun deck). The problem also misses the essentials of Nelson's strategy. Perhaps the next edition can dispose of Themistokles at Salamis, Don John of Austria at Lepanto and Nimitz and Fletcher at the Battle of Midway, complete with math-broken Japanese codes.

Now we can have real world problems really using differential equations. The sky is the limit, and not just for aircraft carriers.

Hold on, my yacht is off course. Hard a-lee and strike the sky sails. Miscounting frigates does not harm the teaching of the calculus. This text represents much thought and work and presents many attractive features. They should be saved.

But when it comes to the central idea of the calculus, this text has

- Misstated the definition of the Limit.
- Confused the notion of a function.
- Left the notion of a proof obscured by a deep sea fog.

Sailors, like Horatio and Saunders, need warnings: Buoys and groaners.

May I urge Harvard Consortium to anchor those needed groaners: A clear supplementary statement correcting the major errors and the minor slips.

At first I was worried that the epsilon trouble happened at Harvard. But then I remembered that there is nothing new under the sun. When I listened to Gilbert A. Bliss lecture on his beloved Calculus of Variations at the University of Chicago, I noticed that he realized that most of his graduate students were not all that attentive—so he just dropped the epsilons from his existence proofs. He was user friendly in the best modern sense. Then I was brash, and bearded Bliss in his office only to see that he knew how to find the beloved delta for every errant epsilon. I learned something. Students can learn big ideas.

At Harvard in the old days (1910), William Fogg Osgood was a world authority on functions of one and of several complex variables (a subject exemplifying rigor). But Foggy knew well the intellectual limitations of the Harvard undergraduates, and wrote his texts on calculus accordingly. After his retirement we still used one of these texts for Math A. I recommended the appointment of one Leonidas Alaoglu, Ph.D. Chicago, whose merits were known to me. He came as a Benjamin Peirce instructor, and was of course set to teaching Math A. In that distant time, undergraduate women students did not at-tend classes in the Harvard Yard. So on one late October day Leon came to his class in Sever Hall,

> "Gentlemen, we now come to Chapter IV, differentials and infinitesimals. Take pages 138 to 184 between the thumb and fingers of the right hand. Tear them from the book!" He did so.

So weak students and the related troubles are not new to Harvard. In this present age, despite the elegant exposition which CHC present on localization, the current trouble can not be localized in any one chapter. So corrections are badly needed. Till they are at hand, any users are at risk.

P.S. August 16, 1996

I have just received from the publishers a new copy of the book. The definition of limit on page 138 now reads: "we define $\lim_{h \to c} g(h)$ to be a number L (if one exists) such that $g(h)$ can be made as close to L as we please whenever h is sufficiently close to c (but $h \neq c$)."

That lost third quantifier is now reborn as a sickly "whenever". But the application to continuity on the next page is unchanged.

Saunders Mac Lane
University of Chicago
(Received August 21, 1996)

Affirmative Action, Then and Now

In the July 1996 *Notices* (pp. 669–670), in a reminiscence of Professor Carl Herz, W. H. J. Fuchs wrote:

> [Herz] felt that it was a betrayal of the university's sacred mission to adopt criteria other than intellectual ability for the admission of minority students.

This is a most unfortunate characterization of the implementation of affirmative action programs in college and university admission programs. Were this merely the report of a political view from twenty-five years ago or the singular opinion of Professor Herz, I could let the mischaracterization pass. However, the reshaping and very continuation of affirmative action programs, particularly at colleges and universities, is an important contemporary issue which affects mathematics departments and mathematicians. Moreover, the almost syllogistic style of the quoted opinion, without regard to the historical setting of the question, is quite typical of the way some mathematicians (and others) deal with legal and societal issues.

The quoted view presupposes that until the adoption of affirmative action programs in the 1970s "intellectual ability" was the sole criterion for university admissions. However, most admission offices have always used criteria other than intellectual achievement to inform decisions. Admission to an individual institution has traditionally been based on a variety of factors, including prior academic success, athletic prowess, musical achievement, and family ties to the institution. Admission offices often consider a candidate's ability to overcome obstacles of a personal, family, or social nature. Selective institutions, for most of this century, have sought to assemble classes of students who are diverse in geography, interests, ethnicity, and, in recent decades, color.

Also presupposed is that intellectual activity is the sole mission of colleges and universities. Successful graduates achieve their success in various realms within and beyond the intellectual world, including business, government, the arts, and community leadership. College admission offices often recognize this diversity of personal trajectories and base admission decisions on a host of factors. Indeed, most institutions have broad missions to educate "the whole person", missions which include, but are not limited to, the support of predominantly intellectual pursuits such as scientific research. Thus, affirmative action programs in admission that explicitly recognize factors other than prior academic achievement are well within the tradition of most institutions' practices and in support of their missions.

We must never forget that for most of this country's history African-Americans were denied admission to higher education (indeed, were denied freedom) precisely because of their color; other ethnic groups and women have suffered serious discrimination as well. Affirmative action programs were adopted to remedy these historical circumstances and must be discussed in the context of history, not as a matter of pure logic.

The very complex affirmative action debate cannot be thoroughly explored in the pages of the *Notices*. Nevertheless, considering the challenges now facing those colleges and universities which consider the development of an inclusive society an important aspect of their mission, I did not want the mischaracterization of affirmative action cited in the Fuchs article to go unchallenged.

Donald Y. Goldberg
Occidental College
(Received August 29, 1996)

On the "Lost Generation" of Chinese Mathematicians

During the cultural revolution most Chinese mathematics students were forced to leave the universities to work in mines, factories, or farms. After ten years, some of them returned to continue to do mathematics, to teach, and to do research. Some of them have published in international refereed journals. Furthermore, since there was no mechanism to give degrees at that time, many of these Chinese mathematicians do not have a Ph.D., even though they have written over a dozen papers, more than many of our own Ph.D. graduates.

This spring I visited China for about four weeks, and I met some of these mathematicians. I am writing this letter for two reasons.

1. History: Some of our students are writing their Ph.D. thesis on the history of mathematics. The history of this lost generation of Chinese mathematicians should be told. It would be interesting to know the driving force that made many of them return to mathematics. They should be interviewed before their stories are lost. I am sure their stories would be an

("Teaching at the University Level", *Notices* **43**, p. 863) inspiration to future mathematicians. It would also be important to recognize their contributions to the mathematical community.

2. Official University Recognition: As noted above, many of these Chinese mathematicians do not have a Ph.D., and this now affects their career. One way that the American mathematical community can recognize their contributions is to invite them to visit our universities and, if possible, to give some official university title such as research professor or even, perhaps, some type of honorary degree.

I hope this letter has interested some people in the AMS to help recognize the contribution of our Chinese colleagues.

Seymour Lipschutz
Temple University
(Received September 3, 1996)

On Student Ratings of Instructors

There is something in my article that I feel needs clarification. I wrote, "Other things being equal, an easy course will rate higher than a demanding one." The "other things" include the instructor and the students. As such, the whole course evaluation process can be manipulated in a simple and predictable way.

It does *not* say, however, that a person who got high ratings in some course had to have taught the course at a low level. Rather, it only means that (in the usual range of possibilities) the ratings of a fixed professor in a calculus course would be a decreasing function of the demands (level of instruction, amount of material) he or she chose to place upon the students. By the way, I asked several upper-class science and engineering majors about this point, and they felt it was obvious.

The context of the article is clearly *freshmen* in *calculus courses*. I know that there have been studies done that show that, overall, undergraduates (all levels) tend to prefer a challenging course (all subjects) to an easy one. This does not imply anything about freshmen, nor anything about math courses, nor (evidently!) anything about freshmen in math courses. I might add that when I asked my department chair whether he knew of any studies on the last item, he replied, "You don't need a study to know that freshmen tend to want an easy calculus course! It's obvious from years of experience."

Steven Zucker
Johns Hopkins University
(Received September 15, 1996

Editor's note: The *Notices* regrets that authors William Yslas Velez and Evans M. Harrell II's names were misspelled in the October 1996 issue. The correct spellings are as above. Prof. Harrell's e-mail address (also incorrectly listed in the October issue) is `harrell@math.gatech.edu`.)

Due to editorial oversight, the review of Keith Devlin's book *Mathematics: The Science of Patterns* contained incorrect information regarding publisher and price. The book is published by W. H. Freeman in the Scientific American Library series. The correct price is $32.95, hardback version. A paperback version is scheduled to appear in January 1997, priced $19.95. The *Notices* apologizes for this error.