Views on High School Mathematics Education

This is the third in a series of articles composed from responses of mathematicians, mathematics educators, and teachers of mathematics to a set of questions on secondary education in mathematics. The present article contains responses primarily from high school mathematics teachers. The questions posed were:

1. When you consider what high school graduates should understand about mathematics, what do you care about most?
2. What do you think should be essential features of every high school mathematics curriculum?
3. How would we know when high school mathematics education is working well?
4. What do you think is most important in the mathematical background, attitude towards mathematics, and pedagogical approaches of high school mathematics teachers?
5. What are your expectations of higher education in mathematics (a) for your own students as they go on to colleges and universities and (b) for prospective mathematics teachers?

The articles are inspired by the efforts of Al Cuoco and Wayne Harvey of Education Development Center, Inc., to increase the involvement of mathematicians in high school mathematics education. Their work has resulted in a project funded by the National Science Foundation and entitled "Building on Strengths: Stimulating Cooperation among Mathematicians and Mathematics Educators". The highlight of the project is a national meeting to be held in the spring. For further information on the project, visit the Web site `http://www.edc.org/LTT/BOS/`.

The first two installments of the responses to the questions appeared in the *Notices* in August 1996, pages 866–873, and September 1996, pages 979–986. The full text of all the responses is posted on the education home page on e-MATH at the URL `http://www.ams.org/committee/education/`.

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— *Allyn Jackson and Hugo Rossi*

1. When you consider what high school graduates should understand about mathematics, what do you care about most?

DAN KENNEDY: I want my students to understand that mathematics is widely applicable, quite beautiful, and very much a work in progress. I want them to recognize that creativity plays a role and that the connections between models are as important as the models themselves. I want them to know how to use technology to explore, logic to verify, and language to explain their ideas. I also want them to believe that mathematics is accessible. I am much less concerned than I used to be about computational skills and the necessity of having them before doing anything significant.

JO ELLEN HILLYER: I care most that high school graduates understand that mathematics is accessible. I want students to know that they can "do" mathematics. Currently, such self-confidence is shared by a painfully small minority of our high school population. Large numbers of students dutifully go through the paces, earning quite acceptable grades. Yet when they are confronted with a problem that bears little resemblance to problems they have seen before, there is a nearly universal conviction that the solution is a mystery that cannot be cracked by ordinary mortals such as they….

JOHN R. LASSEN: Considering the range of students' abilities, interests, and plans for post-high school activities, I suppose it would have to be "problem solving". I think all students, whatever their level, should be able to solve problems. I am not referring to the traditional method of students mimicking a method for types of problems that a teacher has shown them "how to do". I'm talking about a systematic approach used to solve problems that they have not seen before.

JUDITH BROADWIN: High school graduates should be able to apply the mathematics that they have learned to the solution of problems. Even at this date, more than five years after the publication of the NCTM Standards, many students still think that algebra is mathematics. Students should be able to solve problems algebraically, graphically, numerically, and verbally. They should understand that a problem does not solve itself at a glance and that they must think through a solution, applying all the mathematics that they have learned. Most important, they should be encouraged to collaborate with other students in the solution of problems and the creation and completion of projects.

PHILIP R. MALLINSON: I would hope that having studied mathematics for twelve years, a student would face a completely unfamiliar problem with equanimity. He (my hypothetical high school graduate is male for purely literary reasons) would have developed a toolkit of mathematical and problem-solving strategies that would enable him to weigh the problem thoughtfully; to walk around the problem, surveying it, getting to understand it, prodding it, looking for possible entry points, trying out simpler versions, comparing it to similar problems solved in the past, asking friends, breaking it down into manageable chunks rather than tossing it back like a grenade. I hope that our hypothetical graduate would understand that mathematics is not a collection of tricks and formulas, but is a process that has been refined for thousands of years by some of the finest intellects of the age….

BOYD E. HEMPHILL: I care about what each kid is planning to do with what I teach.

For college-bound students I try to serve the needs of their majors. If I have a student who plans to be a doctor, I pull questions from biology. If I have students who will major in business, I pull questions from mathematical finance.

For the non-college-bound, I do essentially the same thing. If a kid plans to sell cars, we compute interest, tax, commission, etc. Depending on the kid we may even write a simple computer program to keep track of these things.

In short, what is most important is that they understand that real mathematics exists in their future and that the particular topic of the curriculum I am teaching at the time is connected to that mathematics in some way.

SUSAN S. ROOSENRAAD: (a) All students should be able to work with numbers and be reasonably adept at arithmetic. Many years in the classroom show me that students who have mastered the multiplication tables have a much easier time mastering algebra techniques because they "see" the arithmetic relationships present in the problem. Students without good arithmetic skills are forced to do algebra as a rigid set of rules applied in an arbitrary manner. These arithmetic skills of the students should include fractions, decimals, percents, and the ability to move between forms with ease and accuracy. I would want all students to have a well-developed number sense.

(b) High school work should reinforce these number skills while building algebra skills. There are certain skills (lists vary from teacher to teacher) that all students with a high school diploma should have. In addition to being able to perform various algebraic techniques, the student should understand what he/she is doing and why.

(c) The arithmetic and algebra skills listed in (a) and (b) are the minimum *all* high school students should have. Those students with the resources to continue their education should be able to *think* logically and, to some degree, abstractly. They should appreciate that mathematics is a tool that can be applied in a wide variety of areas (even some real-life situations).

MARGARET DeARMOND: The group of all high school graduates is very diverse and differs greatly from the group of the college-bound graduates. All will be the voting citizenry of our future. A quality mathematics education is a vital component in producing the next generation of leaders and citizens who understand our complex world. I mention this because many and maybe most high schools have a two-year graduation requirement for mathematics. These courses do not necessarily include any college preparatory mathematics and are often completed in the ninth- and tenth-grade years, leaving students void of any mathematical study thereafter. Many of these graduates become the growing population of people trying to reenter college or training programs after they have discovered the "hard way" that they are not qualified for the job market. I am greatly concerned about this large population of high school graduates, not to mention my concern for the growing number of drop-outs, and I strongly feel that the mathematics courses offered in high school are the "gatekeepers" for all of these students. For many students their first mathematics course in high school is another arithmetic review course or an algebra course based on memorizing procedures. Bored and not seeing any relevance, these students often quit their study of mathematics after the minimum requirement is met. To keep more students studying more mathematics, we—the entire mathematics education community—should take a long and hard look at our long-standing practices of what high school mathematics is and how it is taught.

With this in mind, I'll respond to the question. I want my students to be confident problem solvers; that is, they can solve complex problems that they have not seen before. Students should be able to use many strategies and approaches to start the problem-solving process—not just mimic some procedure they have memorized. Students should have a deep understanding of the concepts used in algebra, geometry, probability, statistics, and discrete mathematics and should understand the connections between these branches of mathematics. I want students to know when it is appropriate to use technology and how to use it to further investigations and discovery. I want all students to be competent doing mental mathematics when needed and able to estimate when necessary.

2. What do you think should be essential features of every high school mathematics curriculum?

LORING COES: A good high school must offer a diverse mathematics program that leads toward the fundamentals of calculus, discrete mathematics, and statistics. A high school should not be measured by how many of its students take Advanced Placement calculus. All schools must start with the assumption that all citizens—all future voters—need to understand more mathematics than in decades past. Therefore, all students need to see a higher standard of mathematics than we have had in the past. I am not advocating that all students be in the same courses, only that there be an appropriate, rich program for students at all levels of ability and background. In theory, tracking is a good idea because it can group people who can work at the same level. In practice, however, we have in too many instances a success track, taught by strong teachers to strong students, and a failure track, taught by weaker teachers to underachieving students. We need the strong courses obviously, but we also need to create programs that can spark the underachievers and that can position those students to take the strongest courses in their turn. We cannot have curricular deadends….

"High school mathematics will be considered successful when all partners in the education process collaborate to assure that students are prepared for a technical world fraught with unknown circumstances and unplanned events."

WENDY TOKUMINE: a) Know how to use a calculator (either scientific or graphing).

b) Have estimation skills for distances, weights, time. Many students cannot make comparisons between two numbers because of weak estimation skills. Examples: weight of a head of cabbage, length of a room, time to drive from town to another place.

c) Develop a willingness in students to attempt new problems, using problem-solving

skills learned and/or developed in the mathematics curriculum (guess/test, look for patterns, working backwards, etc.). Many students often give up too easily. They read a problem, decide it is too hard, and will not attempt it because they feel they will not succeed.

d) Develop decision-making skills based on data gathered and analyzed. Students should learn probability and statistics (on a very concrete level with models they can understand) and apply what they learn to make good decisions.

e) Develop a "joy of mathematics". So many students are turned off to math, feel they are dumb when it comes to math, and, even worse, feel that it is okay to feel this way. The curriculum should provide students opportunities to be successful in math and to enjoy doing math.

f) Develop mental math skills so that students can do simple calculations in their heads without the use of pencil and paper. Estimation is part of this area.

JOSHUA ABRAMS: The only way to become good at solving real problems (problems which are unfamiliar) is to have continual practice. The only way to get good at posing new and interesting problems is to be required to do so. The best way to learn the mechanical skills of mathematics is to use them repeatedly in the service of answering questions which you want to resolve. (Dewey said that you do not learn the basics by studying the basics, but by engaging in projects that require the basics.) My students are usually engaged in projects or problem sets of one to four weeks in duration. They invariably pose questions far more difficult than I ever would for them. They care about these creations and become far better at basic algebra and geometry in the process than they would if we focused on rote exercises.

JOHNNY W. LOTT: The mathematics that all students should know is not what many normally get in a traditional secondary mathematics program. That mathematics should include a basic understanding of probability and statistics, especially statistics at a level where graphs and charts in common newspapers—yes, even *USA Today*—can be understood. The probability should be at a level where students can understand odds and probabilities so that they can understand lotteries and probabilities of baseball players getting hits in at bats. These involve very different types of expectations than those usually discussed. Further, students should understand some basic geometry and trigonometry at least of the triangular variety. By basic geometry I do not mean traditional proofs. I do expect all students to be able to present logical arguments for why they do what they do mathematically. In addition, I feel that every high school graduate should be familiar with technological tools that they will use outside school. The technological tools are just that. They aren't a substitute for mathematics, but students should know that just as word processors made writing somewhat easier, the technology can make some mathematics easier.

ZALMAN USISKIN: Any answer must include more than high school, because what is done before grade 9 has great impact on what happens in grades 9 through 12. It also must be general, because every school has its own mix of students, teachers, and community, and what works in one place may not work in another.

Every high school mathematics curriculum should prepare students not only for calculus but for college-level statistics and discrete mathematics, which may be taken by students before or along with calculus. For this, there needs to be a balance of work with algebra, geometry, functions, trigonometry, probability, statistics, discrete mathematics, and introductory analysis. The curriculum should have a balance of work studying algorithms, mathematical theory, applications, and representations; it should give students exposure to large numbers of routine exercises and nonroutine problems; and it should balance questions with short answers with those with longer answers, and some opportunity to work on projects that take more than an evening or two to complete. It should ensure that all students are familiar with the latest in calculator and computer technology for doing mathematics and exploring mathematical ideas.

Fifty years ago, even without teaching any applied mathematics and with only a smattering of mathematical theory, the first course in college was typically analytic geometry, not calculus. The evidence is that, except for the very best and most interested students, this cannot be done if the study of algebra begins in ninth grade, because we expect much more of students today than we did then. Algebraic ideas should begin in the early elementary grades, and a concentrated study of algebra should begin at seventh grade for better students and at eighth grade for average students.

Even courses for the best high school students should not be designed to weed out students, nor should they be designed with mathematics contests in mind...

DAN KENNEDY: I think that every high school curriculum should follow the four-year precalculus sequence, but with less emphasis on computational subtleties and more emphasis on modeling, problem solving, connections, and applications of technology. The algebra and the geometry should probably be integrated, but not before teachers and textbooks are ready. A healthy majority of students should begin the se-

quence in eighth grade and take AP Calculus as seniors. There should be more emphasis on statistics, numerical data, and matrices, exploiting the abilities of new, inexpensive machines to handle these easily. Students should see computers often, but they themselves should use graphing calculators to do mathematics. (Schools cannot keep up with computer technology today, and it is naïve to expect that they ever will. Graphing calculators, on the other hand, are both affordable and effective. The computer distraction is effectively *slowing down* the incorporation of technology in many schools!)

3. How would we know when high school mathematics education is working well?

JO ELLEN HILLYER: Teachers would know when mathematics education is working well when they: a) can give a problem which does not resemble any that their students have seen before and not be met with blank stares or protests of unreasonableness; b) receive papers in which students are able to write about concepts and explain processes with comprehension and lucidity; c) see students taking mathematics courses because they want to, not because they have to; d) get "why" and "what if" questions rather than "just tell me what to do" statements; and e) have as many girls, young people with handicaps, and students from minority groups in higher-level mathematics classes as in the lower-level ones.

To some, this might seem like an unrealistic wish list. Yet I have seen first-, second-, and third-graders be excited about mathematics, be able to come up with solutions to interesting—and, often, open-ended—problems, and proudly explain their reasoning. In twenty years we will know if mathematics education has been working when we can go to a cocktail party, tell a stranger that we are mathematicians, and not get a response of veiled distaste ("That was my worst subject!") or muted awe ("Are you really?"). While I certainly enjoy the respect granted me as a practitioner of the mysterious, we can no longer afford to have only a small minority of this country privy to the workings as well as the beauty of mathematics.

ARNIE CUTLER: Perhaps the best answer is, When people quit asking the question. I do not believe that standardized test results are an adequate measure. I do not believe that the impressions of college mathematics professors are an adequate measure. High school mathematics will be considered successful when all partners in the education process collaborate to assure that students are prepared for a technical world fraught with unknown circumstances and unplanned events. This requires a commitment from society at large to support education in a variety of ways. Family support is the initial condition. Adequate resources to support a quality education is next. Job opportunities for successful students must be available. The work of mathematics and mathematics-related careers must be understood and valued by society. The role of mathematics in being a productive citizen and understanding things like voting and budgeting must be advanced. When these factors are present, students will have motivation to be successful mathematics students. In effect, high school mathematics will be as successful as the society which surrounds the high school.

MARGARET DeARMOND: We never will know unless we agree on what is important and how we should hold the system accountable. If I teach my students problem-solving skills, topics other than algebra and geometry, and I require the use of graphing calculators, and then these students take placement tests at college that are based on the "old" curriculum, we will never know if the programs are working. We could show increased numbers of students taking more high school mathematics courses, but there is always going to be a cry that we have only lowered the standards. We—again, the entire mathematics education community—need to agree on what is important to teach and how we are going to measure student performance.

STEPHANIE J. PETERSON: When students demonstrate success (i.e., tests, grades, taking advanced courses, etc.) and when we can compete on an equal plane with other countries.

ZALMAN USISKIN: This is a difficult question with no simple answer. One criterion for success with an individual student coming out of school is that the student would want to take more mathematics even if it is not required by the field of study. Without wanting to take more mathematics, virtually everything is lost.

A related criterion is that the student have a realistic view of what mathematics is needed for certain areas of study. For instance, a student who thinks that the study of business requires little mathematics beyond introductory algebra has been poorly educated. As another example, a student who does not realize that political scientists should know a great deal about statistics and sampling is similarly deficient.

The criterion many people would put first is that the student be competent in the mathematics needed for the student's field of study. This seems like an obvious criterion, but what is needed is not universally agreed upon, particularly in today's technology-rich environment. Is a student who uses a symbol manipulator such as Derive or Mathematica to factor polynomials worse off or better off than a student who relies on paper-and-pencil algorithms? Is a

student who uses a geometry-drawing program to explore the concurrency of bisectors of interior or exterior angles of a triangle spending time as productively as one who is proving that the bisector of the vertex angle in an isosceles triangle is also an altitude? One of the distinguishing features between high school and college mathematics is that there is almost always a time crunch, and the priorities may be more determined by philosophy than able to be determined by research studies….

JOHNNY W. LOTT: We would know that high school mathematics is working well if we have industry telling us that their new employees easily adapt to the mathematics that is necessary to do jobs. I would not base how well mathematics education is doing on how high school graduates do in college or university freshmen classes, because many of those are so antiquated that they are of no consequence in real mathematics. For example, many freshman classes are based upon what I call a junior high mathematics mentality of ten years ago. In the past, junior high mathematics was two more years of doing what you had done for the past six years with almost nothing new. That is the same type of mathematics in most freshman college classes. Also, the majority of college entrance tests and placement tests cannot be used as the judge and jury to evaluate high school programs. Virtually all of those are based upon algorithmic computation with little understanding of what should be in high school classes. A true evaluative instrument would be one that would pose a real problem that could be attacked with mathematics and technological tools where all assumptions have to be shown and where assessment takes into account those assumptions and the logical reasoning used to reach conclusions. Students should be able to write a summary of all the processes used to reach the solution.

JOSHUA ABRAMS: a) When all of our high school math teachers like math enough to study it independently, use some of their time at home to read about new math, play with mathematical problems, etc. No one would accept an English teacher who never read novels, but it is the norm for math teachers to have no connection to mathematics outside of the classroom.

b) When we can provide portfolios of our students' work that demonstrate their ability to use math productively.

JANICE A. BUSSEY: If all students of the high school population are completing the core mathematics sequence with passing grades, regardless of ethnicity, gender, or prior math experience, then the program is working well. If students are enjoying mathematics, seeing mathematics as important and relevant, and considering taking more mathematics than what is required, then the program is working even better. I am continually suspicious when I see mathematics programs which allow less able students to complete their high school math requirement by taking only general arithmetic courses and which never force them to get into the mainstream core mathematics sequence. I am suspicious when I see girls and minorities overwhelming some math classes, while white males are predominant in the higher math classes.

CYNTHIA LANIUS: Good question…when university professors say, "Wow, these students are brilliant; I can't believe how much they know.…" Just kidding…. An accumulation of evidence, I suppose: test scores, universities not doing remediation, employers no longer having to retrain the work force.

ELEANOR PALAIS: Truly, not until you have some comparative data, i.e., some sort of national exam á la Europe…. What my colleagues and I have noticed is that our students are less good than they were five years ago and much less good than ten years ago. We measure our feelings by looking at our expectations of what they could or couldn't do on tests and assignments we used to give. In addition there is more pressure on teachers to pass students (pressure from parents to administrators). In the end, although we are not able to assert our professional expertise as to how to teach students, we are being held accountable for the outcomes of methods which some of us do not believe will work well. In our school we want empowerment to teach students in the very best way. We used to have four levels of math; for example, we taught our Algebra I in levels 2 and 3—the same material, but taught in very different ways. Now we have all these students together. It becomes impossible to teach to the top (too many will fail), so both groups get the short end if we teach to the middle. Incidentally, in grading these students it still is very clear that they are made up of two distinct ability levels. We are also concerned with "algebra for all eighth-graders". We find many eighth-graders fail their first time through and aren't really ready to deal with the abstractions of algebra until they take it or retake it in ninth grade, when they often then end up with an A or a B.

CATHERINE LATHAM: I wish I knew the answer to this question. Traditional testing seems inadequate to measure this. I would think that a student's enthusiasm to elect mathematics courses when entering college would reflect a level of competence, comfort, and interest which would indicate high school mathematics is working well. When my daughter recently went to college, various tables were set up with members of academic departments available for ques-

tions. The mathematics table with faculty at the ready was empty. No one asked a question. For no one to inquire about mathematics is such a shame. We high school teachers seem to be discouraging our students. Of course, those very students have been, in some cases, subjected to eighteen years of their parents saying how much they hated mathematics and how poorly they did. That is one horrible obstacle which other subjects seem not to have to overcome.

4. What do you think is most important in the mathematical background, attitude towards mathematics, and pedagogical approaches of high school mathematics teachers?

DIANA HERRINGTON: I believe that high school mathematics teachers should have degrees in mathematics or equivalent degrees, with supplementary work in areas that utilize mathematics extensively: i.e., physics, biology, chemistry, computer programming, business, etc. As a district mathematics mentor teacher I am always amazed at how few of the mathematics teachers have actual degrees in mathematics and, of those with degrees in mathematics, how little they can apply mathematics. I know mathematics teachers with degrees in mathematics who have never seen a vector or a reason for teaching this concept. It surprises and bothers me that colleges/universities do not require courses to be taken that utilize mathematics even for the high school student. Science and mathematics teachers should work hand-in-hand to create the strongest students. Mathematics teachers should enjoy children as well as mathematics. With this attitude the pedagogical approach to teaching would become student centered and extremely interactive….

LORING COES: Mathematics teachers need to be experts in their field. They need mechanical skill and conceptual understanding. They need most of all to be mathematics enthusiasts, people who always show interest in mathematics problems and concepts. As teachers they need in some way to transfer their enthusiasm to students. I believe in the model of the teacher as a coach, on the sidelines with leadership, advice, strategies, and ideas, but where the students themselves really do the mathematics. On another level, teachers need to keep changing what they do and the courses they teach. There are too many teachers, I believe, who have been teaching the very same courses for a great many years. Doing so may be comfortable, but it must also be mind-deadening for both the teachers and for the students. Teachers must also have a sense of the culture and history of mathematics. I am bothered, for example, by teachers who are experts at simplifying radicals (say, changing the square root of 60 to 2 times the square root of 15) but who cannot explain why it was once important to do so. Teachers need perspective so they can better judge what is really important to know.

ZALMAN USISKIN: At Chicago we only train prospective mathematics teachers at the master's degree level, and we require a bachelor's degree or its equivalent. A broad mathematical background is essential...

Teachers must have a positive attitude toward their subject and toward what students can learn. They must also have a broad view of the subject; for example, teachers who think that geometry is worthless or that algebra is merely a bunch of rules or that everything is functions or that calculators should not be allowed in classrooms hurt students by distorting their education and narrowing their vistas. Teachers must also be tolerant of the differences among students that will affect student interest, student performance, and the amount that a student can work in the class.

We do not know what pedagogy or pedagogies work best for a given student, but we do know that students learn most when they are actively involved in their learning, when they are doing mathematics and talking about it rather than passively listening. Like adults, students need variety, so lecturing all the time or discussing all the time or putting students in groups all the time will not be as effective as variety. Furthermore, some mathematical topics lend themselves to lecture (e.g., definitions and summaries), others to group work (e.g., analyses of data), still others to discussion (e.g., alternate proofs), so it is not wise to think of only one approach as being paramount.

MAUREEN BURKHART: Teachers need to be able and willing to let go of their "control" in the classroom and let the students be in the spotlight. The learning has to be centered around the student rather than the teacher showing the way. Students need to learn how to enter a problem and figure out what the problem is asking, rather than be told. Then, the students have to figure out ways to solve the problem and also share them with the class. In this way, students (and the teacher!) will see the rich variety of problem-solving strategies available for arriving at the same answer. I feel a teacher's attitude toward mathematics has to have a sense of newness and wonder. Otherwise, how else will the students be able to show new ways of solving problems? In the past, teachers have been too willing to say, "This is the way to do the problem." There are too many other ways to solve [a problem] that oftentimes lead to bigger con-

cepts never realized before. So the teacher, as well as the students, has to have that sense that there is going to be something new for them to learn (I'm speaking from personal experience).

DAN KENNEDY: High school teachers need first of all to want to reach all their students. It may sound corny, but all sorts of problems arise when this prerequisite is either lacking to begin with or else sublimated to achieve other goals. Second, they need a broad mathematical background that will enable them to guide their students through wide seas rather than narrow channels. Third, they must appreciate, use, and do mathematics in their own lives so that they can inspire their students to do the same. Fourth, they must be willing to look beyond their own mathematical upbringing to find more effective ways of getting their students to learn mathematics (such as group learning, student discovery, and alternate assessments). Finally, for sanity's sake, they must understand children well enough to rise above their own emotions when dealing with those of their students.

JACKIE SULLIVAN: I'm a good one to answer this one, in my humble opinion! My math background consists of one year of calculus before I switched majors to accounting. I have a clear math credential because I passed the NTE in math. Despite my lack of college math courses, however, I feel I do a good job in my teaching, the reason being I have good *math sense*. I can often think through a problem better than some of my colleagues who have math degrees. I also have an intense interest in what math to use for solving which type of problem, and I try to develop that same interest in my students. A good math teacher should try to get his or her students excited about solving complex problems using the math they have learned. That will not happen unless the teacher is excited about it in the first place. The high school math teacher, however, also needs to realize that math is an extremely difficult subject for most kids, so there needs to be a lot of patience on the teacher's part and willingness to try other approaches in teaching a concept.

ELEANOR PALAIS: …[A] teacher must be perceived by society and, in particular, by school committees and administration as a strong and important member in a team and not a scapegoat or "low man on the totem pole". In addition, I would like high school math teachers to be mathematicians first and education students second…. I would want teachers all to have the equivalent of a master's course in pure mathematics. What many education schools now offer teachers for their credits toward postgraduate teaching are packaged quickie courses on discipline or cooperative learning, often a series of tapes, which offer little pedagogical enrichment in pure mathematics, which is necessary to pass on a love of or the ability to do good mathematics. Society should expect more of their teachers and in return should elevate them to positions of economic and professional power in their fields of expertise. Only then will mathematics in schools shine again.

WALTER R. DODGE: Number 1 is the total package of content to which our students are exposed. If there is no exposure, there is no possibility of the student acquiring the necessary mathematical knowledge. Beyond this, though, comes the very important ideas of attitude and pedagogy. All the content thrown at students is worthless if the students believe this content is not important for them to know. All the content is also worthless if students are not encouraged by teachers to reflect on the ideas, generalize them, use them in meaningful ways, and begin to see how they as students are capable of doing mathematics. Pedagogy is very important to the learning process and cannot be divorced from the content of mathematics. Again, though, pedagogy devoid of meaningful mathematics is also worthless. It is the interplay of a strong content base presented in an interesting and meaningful manner that creates the strongest curriculum possible.

MICHAEL SHERMAN: I have been teaching math in various settings for twenty-five years, and I think the most important quality of a math program, a math teacher, a math curriculum, and a math student is enthusiasm! If one's natural love for math shines through, then one will be eager to find the best new ways to teach math, the best applications of math in the real world that are relevant to one's students, the best explanations for math, and the best conveyance of a positive attitude that one's students can really learn this stuff! The three most important things in real estate are location, location, location, and in math teaching it's enthusiasm, enthusiasm, enthusiasm. When I hear stories of adults who look back on their math careers, inevitably they point not to a particular course and its content, but to a teacher who either turned them on or turned them off to math. Math is, in my opinion, a deeply teacher-dependent subject, so teachers have to be gifted, talented, able, knowledgeable, sensitive, caring, demanding, relevant, but above all enthusiastic. A teacher's influence is eternal, and let us hope that we are continuing the many centuries of fine work that the great minds of the ages have handed down to us.

CAROL CASTELLON: First, the teacher has to have a solid background in higher math—an analogy would be that a bookkeeper and an accountant can both do the same job, but the accountant has the "big picture". Second, teaching

is a skill that either comes naturally or must be learned—you have to know what questions to ask during class and when to ask them. You have to invent creative approaches in teaching or the students "tune out". Third, a teacher must go to class with excitement every day, no matter how he/she feels—excitement about math *is* contagious. Fourth, you have to be alert to students who are "lost" and know how to help them understand—this country is run by "C students".

5. What are your expectations of higher education in mathematics a) for your own students, as they go to colleges and universities and b) for prospective mathematics teachers?

BOYD E. HEMPHILL: a) I expect higher education to speak with one voice. One of my biggest frustrations as a teacher of college-bound students is that I have no basis to judge what pedagological challenges my students will face once they enter college. The best example of this is technology.

If I send a kid to Texas A&M, they will be expected to learn Maple along with the standard calculus. They will also be expected to be adept at the use of a graphing calculator. On the other hand, if I send a kid to Rice University, the calculator is all but banned from their possession in most cases. In many cases, different professors at the same institution have varying tolerances for technology. How am I to prepare a college-bound student who could go to a traditional calculus, a reform calculus, or some other variation? Higher education needs to set some basic guidelines.

b) I expect teachers coming from college to have a better command of mathematics than I do. I expect them to be able to communicate it effectively, and I expect colleges to raise their expectations for teachers in general.

I expect new teachers to be up to date on the latest teaching and assessment strategies and be well versed in the use of different forms of media and technology for communicating mathematics.

CAROL CASTELLON: I wish college teachers/professors had the same pressure that high school and elementary teachers have in thinking about how to teach. The image of college professors is totally "teacher-centered" (lecture, explain, prove, test on details), with sleepy students sitting quietly writing notes for an hour. No wonder prospective math teachers often have so much trouble—they are just teaching as they were taught!

JOSHUA ABRAMS: That professors stop lecturing. That they stop seeing techniques as the essence of mathematics. That they acknowledge that creativity plays a role in doing mathematics and believe that their students are capable (at some level) of exhibiting that creativity. That they emphasize learning skills of immediate utility at least as much as those that will be preparation for some other course of delayed gratifications.

I believe there are common cores of experience and content that would serve math majors and teachers equally well. As matters currently stand, neither group is comfortable exploring and posing their own questions. Traditionally taught math is too rote and soulless an activity for all concerned.

JUDITH BROADWIN: I expect that my students will continue to learn in college as they have been taught by me in high school and that they will be expected to solve problems graphically, numerically, and verbally as well as algebraically. As I write this, there are still freshman math courses in college that are taught in the same way that I learned. Assessment is still often composed of students performing rote algorithms, and cooperative learning and assessment are not there.

It is vital that prospective mathematics teachers be taught in the spirit of the NCTM Standards and calculus reform. Recently, when I was an adjunct at a university, I had a student in a graduate course finishing her last course for her master's degree in mathematics education. She had never touched a graphing calculator in any of her courses until she took my course. I do not know what is happening at that university in math education, but it seems to me that she should have been better prepared for her entry into the teaching profession.

"I would hope that all of my students might experience mathematics as an adventure, not an obstacle course. Given how wonderful the field is, it seems a reasonable wish."

PHILIP R. MALLINSON: When my students graduate, I hope they will be equipped with the tools to make sense of the mathematics they meet. I expect they will find classrooms with a very different flavor from their high school—much larger, more regimented classes. They will probably find it less easy to have access to their teachers, who may well be graduate students or

mathematicians for whom teaching is a lower priority than research. This is an observation and not a judgment. I anticipate that very few of them will become mathematics majors but that many of them will be forced to take some mathematics courses to complete their major. I expect that there will be a wide range in the support systems that will be available to them when they have difficulties.

If they go on to train to become mathematics teachers, I expect that their professors will be aware of the changes in thinking in mathematics education over the last ten years. From "A Nation at Risk" on, there has been nothing less than a revolution in mathematics education, particularly at the precollege level. I expect professors of education to train students to become teachers who have a strong background in mathematics, are confident in their mathematics, are excited by mathematics, and are prepared to take risks in their classrooms to produce a new generation of students who are not afraid of mathematics….

JO ELLEN HILLYER: To the extent that my students feel empowered as problem solvers and take pleasure in doing mathematics, I would hope that they can continue to grow in confidence and facility as they move on to institutions of higher learning. I would hope that they continue to experience: a) technology as an integral part of their learning, b) a variety of ways to learn about a new concept, c) the requirement that they clearly describe the process as well as the results of their investigations, d) opportunities to work in cooperation with their classmates, e) an emphasis on the larger picture of the mathematical process rather than a preoccupation with the details of a specific topic, and f) substantial problems and long term projects which have meaning for them.

Most important, I want my students to feel enlarged, not diminished (as has happened so very many times), by their experience with mathematics courses on the college level. For many of my students—especially the girls and students of color—the spark of interest in mathematics is precarious. I would hope that all of my students might experience mathematics as an adventure, not an obstacle course. Given how wonderful the field is, it seems a reasonable wish.