The Math Wars
California Battles It Out over Mathematics Education Reform (Part I)

Margaret DeArmond is a high school mathematics teacher in Bakersfield, California. She has taught for twenty-seven years, with many of the same frustrations occurring year after year: "Here they are, seniors in high school, and they're still saying that the square root of $a^2 + b^2$ is $a + b$! This kind of frustration makes you stop and ask, really deeply, Is there a reason kids don't get it?" Six years ago DeArmond started teaching with materials from the Interactive Mathematics Program (IMP). IMP units are centered not on mathematical topics but on complex, open-ended problems. When mathematics grows out of problems students find interesting, the thinking goes, the meaning of the concepts and the motivation to master them come naturally. IMP students work in groups, use calculators routinely, and write extensively about what they are working on.

At first DeArmond worried about the lack of drill in IMP, but "once you do a lot of work on setting the concept with a kid, then the amount of drill and practice can be reduced," she says. That the tradeoff is a good one could be seen one day in her class this year, when her students worked in groups with blocks of different shapes to derive formulas for surface area and volume. "I kept saying to them, if you forget this formula later on, can you develop it on your own? It was, 'Yeah, we know where it came from! Yeah, lady—leave us alone!'" she recalled with a laugh.

Envision DeArmond's classroom: students sitting in groups, discussing ideas, doing experiments, making diagrams, using concrete objects to test their conjectures, following blind alleys, and now and then experiencing the satisfaction of discovering something they did not know before. And now the traditional classroom: the teacher writes $V = l \times w \times h$ on the blackboard, students locate the formula in a shaded box in their textbooks, copy down the problems the teacher solves, and that night do homework consisting of the same problems the teacher did, only with different numbers. Could anybody disagree that the reform is an improvement?

You bet they could. Not only are they disagreeing, they are disagreeing loudly and in growing numbers. DeArmond, teaching with a program radically different from the traditional, is in the vanguard of a war over mathematics education reform in California. And she is not alone: teachers across California have been teaching reform-style since the state adopted new mathematics curricular materials in 1994. Many believe that the reform programs give students a much stronger conceptual understanding of mathematics, make them more independent learners, and allow them to see how mathematics is used to solve a wide variety of problems. Critics contend that the reform has spawned watered-down courses filled with cute but mathematically pointless activities and devoid of practice with skills that students need to go on in mathematics. There is also concern that teachers' backgrounds in mathematics are too weak to allow them to use the reform materials effectively. The most ferocious battles have taken place at the elementary and middle school levels, though some of the high school programs have come under fire as well.

The Warring Factions

Bill Evers is a political science researcher at the Hoover Institution at Stanford University. He heads a Palo Alto-based anti-reform group called HOLD, Honest Open Logical Debate on math reform. Evers believes the reformers have swung

The Backdrop for Reform

Views on how to teach mathematics are sufficiently diverse that disagreement over reform might have been inevitable. But what was it in California that caused the heat of disagreement to burst into a flame of controversy? At least two factors having little to do with mathematics were operating in the background.

The first is California's move in the late 1980s to incorporate "whole language" methods into the teaching of reading. Whole language is a literature-based approach that tries to get students to recognize words in context and focus on the meaning of whole sentences, paragraphs, and stories. Another component of reading instruction is work with phonics, in which students learn to sound out words. While the general consensus is that the best reading instruction uses both whole language and phonics, the two have come to be seen as polarized opposites, with critics of whole language saying it lacked rigor. When California had a poor showing in the reading portion of the National Assessment of Educational Progress, whole language took the blame. The state is now trying to insure that teachers use a balance of different approaches in reading instruction and has appropriated $377 million in 1996–1997 for professional development in reading for K–3 teachers. The brouhaha over whole language made many parents wary of any kind of educational reform, and this wariness spilled over to mathematics. While the connection to mathematics is not entirely clear, some see phonics as analogous to drill in mathematics. Take out the drill and you have what some have dubbed "whole math".

The second factor affecting the reception of mathematics education reform in California is the ill-fated California Learning Assessment System (CLAS), which was instituted in 1992 and, as one observer put it, "went down in flames" just two years later. CLAS was developed by the State Department of Education to try to better align state testing with the reforms it was introducing. Rather than being a straightforward multiple-choice exam, CLAS included open-ended and essay questions. Some parents charged that by asking students to respond to literary passages the test improperly pried into their children's privacy. Once the language arts questions came under fire, critics began scrutinizing the mathematics questions and found that students could get downgraded if they did not explain their answers sufficiently, even when the answers were numerically correct. In addition, parents complained that CLAS did not allow for individual scores and had too little emphasis on basic skills. Feelings ran so strong that some districts were threatened with lawsuits if they gave the test, while at the same time they were mandated by the state to give the test. Some real estate companies complained that CLAS was lowering property values in areas where the students had gotten high standardized test scores but did poorly on CLAS. Like the "whole language" movement, CLAS increased skepticism about the state's efforts at educational innovation. Governor Wilson killed funding for CLAS in 1994. Today California is without a state-wide testing system, though a new one is currently under development.

There is another aspect of the educational picture that inevitably affected mathematics education reform: Generally, education in California is in poor shape. Low per-pupil spending, large classes, and a sizable population of students whose first language is not English are among the most difficult problems facing the state. (see data box on facing page.)

too far in the direction of "discovery learning", in which students discover mathematical ideas on their own rather than the teacher telling them. Discovery-learning proponents, for example, "are willing to spend two weeks on the concept of the slope and drag it out of the students," Evers says. "People on my side, in contrast, tend to think that math is more of an architectonic system, a logical system that you want to take people through step by step." His side values applications and word problems, just as the reformers do, "but we think that a lot of time ends up being spent on the self-discovery that might not be the most efficient way of learning it," he remarks. "And it really shouldn't take that long to grasp the concept of the slope."

Evers got interested in mathematics education reform by attending a meeting of a group of Palo Alto parents who were concerned about the issue. While strong in mathematics, the group had little strength in political organizing and dealing with the media, which was something Evers could contribute. One of his more colorful stunts was to lead a demonstration in front of the Palo Alto Unified School District building in which he performed a ceremonial flushing down the toilet of California's 1992 mathematics curriculum framework, accompanied by demonstrators chanting "Flush it" and amplified sounds of a toilet flushing. Evers's more substantive contributions can be seen in newspaper articles describing HOLD's views and in HOLD members providing testimony in key legislative hearings and gaining appointments on state bodies making decisions about mathematics education in California. Evers himself was appointed to a commission charged with developing new standards for the state and was made chair of the commission's subcommittee on mathematics (in February Evers was replaced as chair, though he remains on the subcommittee).

HOLD is only one of many such groups that have sprung up around the state. There is PACE (Parents Advocating for Children's Education, in Davis), QED Advocates for Excellence in the Public Schools in Santa Barbara, BOLD (Brea Open Logical Debate, in Brea), Concerned Parents in Torrance, and a host of others. The granddaddy of them all is Mathematically Correct, whose Web site[1] is impressively up to date and informative, if entirely one-sided. Mathematically Correct is headed by Michael McKeown, a

biologist at the Salk Institute who also holds an adjunct professorship at the University of California, San Diego. The groups rallying against reform can count on their side a number of mathematicians, among them Gunnar Carlsson and Ralph Cohen of Stanford University, Henry Alder and Abigail Thompson of UC Davis, and H. Wu of UC Berkeley.

On the pro-reform side one finds many teachers like Margaret DeArmond, who is the current president of the 12,000-member California Mathematics Council (CMC), the California affiliate of the National Council of Teachers of Mathematics (NCTM). With its major focus on professional development for mathematics teachers, the CMC has had a fairly low political profile. But it became more sophisticated as the battle over reform escalated and now has its own part-time public relations specialist. DeArmond points out that the CMC is an organization for all teachers, those who like the reform and those who don't. Nevertheless, the CMC has the reputation as a pro-reform advocate, in part because of its ties to NCTM: after all, it was NCTM that issued what has become the Bible of the reform movement, *Curriculum and Evaluation Standards for School Mathematics* (usually called the NCTM Standards). Also on the reform side are those who have developed reform materials; among these are a number of mathematicians, including Tom Sallee at UC Davis and Diane Resek and Dan Fendel at San Francisco State University, as well as educational professionals working at the state level.

A Statistical Snapshot of Public Schools in California

Enrollment, 1995–1996:	5.5 million	
Children living in poverty, 1996:	24%	
Single-parent families, 1996:	26%	
Limited-English-proficient students, 1995–1996:	24%	
Number of full-time teachers, 1996:	219,000	
Average teacher salary, 1994–1995:	$41,000	
Per-pupil spending, 1995:	$4,753	
Ranking among 50 states in per-pupil spending, adjusted for regional costs:	No. 48	
Constant-dollar change in per-pupil spending, 1990–1995:	-$323	
Schools with at least one building in major disrepair:	43%	
Average class size, 1994:	30	
Percentage of high school teachers who taught one or more mathematics classes without at least a minor in the field, 1990–1991:	51%	
National Assessment of Educational Progress, 1996 Fourth-Grade Mathematics Achievement:		
Advanced	Calif: 1%	Nation: 2%
At or above proficient	Calif: 11%	Nation: 21%
At or above basic	Calif: 46%	Nation: 64%
Below basic	Calif: 54%	Nation: 36%
Eighth-Grade Mathematics Achievement:		
Advanced	Calif: 3%	Nation: 4%
At or above proficient	Calif: 17%	Nation: 24%
At or above basic	Calif: 51%	Nation: 62%
Below basic	Calif: 49%	Nation: 38%

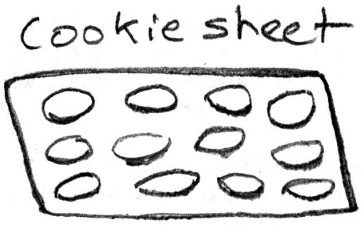
Data sources:
"Quality Counts: A Report Card on the Condition of Public Education in the 50 States", published in February 1997 by *Education Week*.
National Center for Educational Statistics, Department of Education.
National Assessment of Educational Progress 1996, National Center for Education Statistics
California Department of Education

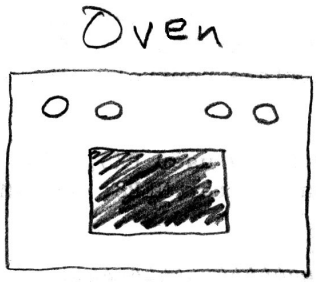
The Battle Lines

While some veterans of the math wars have been fighting ever since the NCTM Standards appeared in 1989—and some even earlier—the battle escalated sharply in 1994, when California moved strongly toward a Standards-based approach to mathematics instruction. The new approach focuses primarily on pedagogy and teaching methods; most of the content is the same as in traditional courses. The hallmarks of the reform are group work in class, calculator use, open-ended problems, communicating about mathematics, and deemphasis on drill and memorization, with an accompanying emphasis on conceptual understanding. While no one argues that any one of these pedagogical approaches is inherently bad, there are deep divisions over how they are carried out.

Take, for example, communication about mathematics. Too often students get by in mathematics by memorizing and mimicking what the teacher does. If students are required to explain their reasoning to other students and to the teacher, they are forced to think much harder about what they are doing. That sounds fine in theory, say the critics of reform, but what happens in practice is less than ideal. "The idea that students should be understanding what they are doing and not simply regurgitating facts and doing things by rote certainly is a very good idea; I agree with it entirely," says Richard Schoen, a mathematics professor at Stanford University. "But that does not imply that you therefore remove the technical side of the sub-

[1] *The URL for the Mathematically Correct Web site is* `http://ourworld.compuserve.com/homepages/mathman/`. Links can be found there to other web sites about the reform.





ject so that it becomes essentially a descriptive subject," with the students writing about mathematics more often than doing it. The overemphasis on writing sometimes results in "students being encouraged to write flowery essays about how they feel about a problem," he says. Indeed, some of the writing tasks do not involve any mathematics. To give an extreme example, one reform program assigns students to write about their "personal growth" during one of the course units.

There are some areas on which the two sides agree, but even these are not immune to debate. Both sides say that kids must know their multiplication tables, but they part ways over when and how. Some traditionalists suggest that multiplication tables should be memorized in the third grade, while many reformers say that is too early, that third grade should be devoted to giving kids experiences with the idea of multiplication. Contrasting with such relatively mild disagreements are the pitched battles, where the two sides seem unable to fathom the other's viewpoint. To the horror of many, some reformers say that in this age of cheap calculators students no longer need to become proficient at long division.

Some of the most intense skirmishes come over drill. Parents of elementary school students contend that the reform programs are so lacking in drill that kids must learn their "math facts" at home with flash cards. At the high school level, the reform materials have been criticized for not giving students enough drill in technical skills like factoring, which they will need in college. In reply, the reformers point to the sad fact that school mathematics has consisted largely of drill, and it is clear that many kids get little out of it. Alan Schoenfeld, a professor of education at UC Berkeley, has written papers about the fact that even when students can successfully perform mathematical procedures, their understanding is often woefully superficial. For example, in one paper he points to an item in the 1983 National Assessment of Educational Progress: "An army bus holds 36 soldiers. If 1,128 soldiers are being bused to their training site, how many buses are needed?" While 70% of the test-takers performed the long division correctly, only 23% gave the correct answer of 32. Worse, 29% answered "31 remainder 12." As Elaine Rosenfield, an elementary school teacher in San Luis Obispo, puts it, "We have spent too much time on algorithms and haven't balanced that with, Do the kids have a sense of them?"

Indeed, one unassailable argument the reformers have on their side is that mathematics education in this country has largely failed. The 1996 NAEP scores, released in February, show that nationwide 36% of fourth-graders and 38% of eighth-graders are at the "below basic" level in mathematics; for California the analogous figures are 54% and 49%. The Third International Mathematics and Science Study (TIMSS), which came out last November, once again found U.S. students trailing their international counterparts. One of the TIMSS reports comparing mathematics curricula in different nations concluded that U.S. textbooks are overstuffed and unfocused, revisiting the same topics year after year and rarely leaving any behind. Given these problems, one observer said of those angry about the reform: "I don't know why they weren't angry before."

From the Command Post: California's Education Bureaucracy

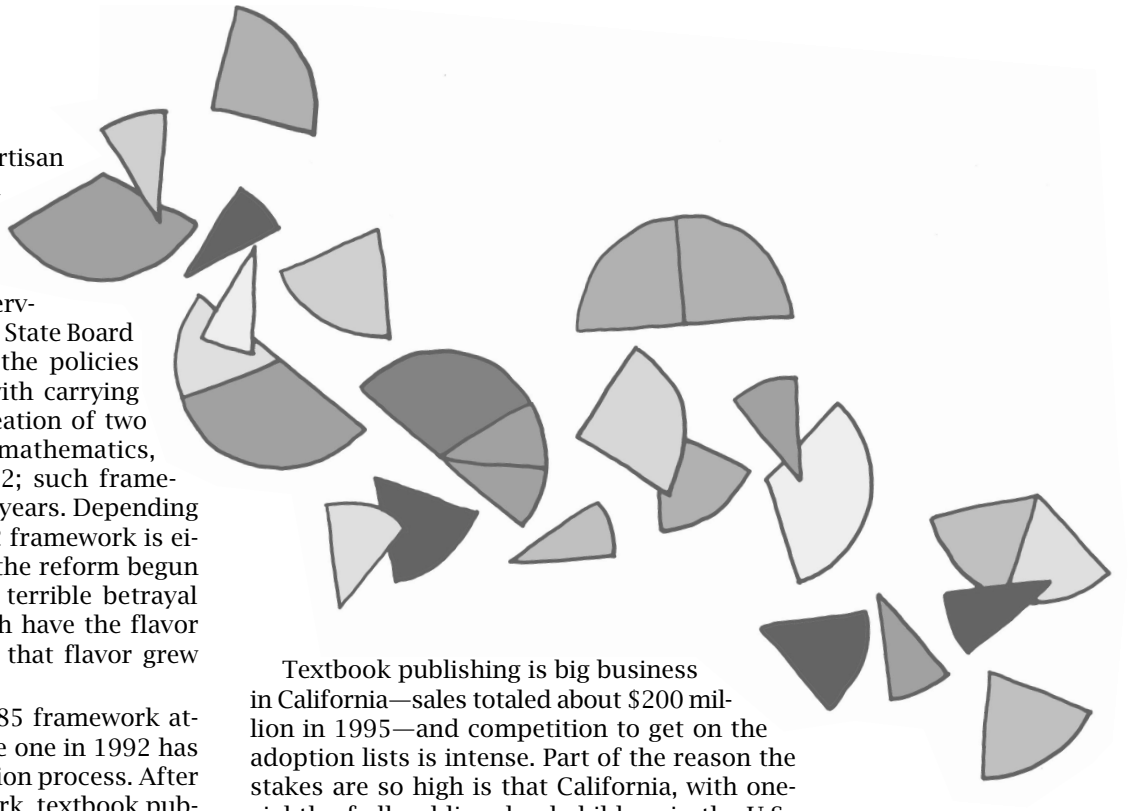
Walter Denham became director of the mathematics education division of the California State Department of Education in 1983 and was one of the main architects of the reform movement in the state. He has a doctorate in applied mathematics from Harvard University and worked in industry before he went into education. Denham is a strong believer in the "constructivist" approach to teaching mathematics, which holds that learning takes place when students construct their own understanding, rather than just following instructions. "I don't believe kids should passively listen and try to follow the teacher's thinking rather than think for themselves," Denham says. "I believe school should be dedicated to getting kids to think and be self-reliant. And if you take that position, you could be accused of undermining the authority of schools and parents." Those against reform have made much of the fact that there are no data showing the effectiveness of the new programs, and Denham concedes that there are no data on large-scale implementations. "The amount of instruction based on the reform vision is too small to do a controlled study," he says. "We have no statistical proof that reform works. All we have is proof that traditional programs fail."

Denham is a casualty of the math wars: in 1995, he was relieved of all responsibility in the area of mathematics and reassigned to a different post in the department by Delaine Eastin, the state superintendent of public instruction. Denham's views were well known, and Eastin was try-

ing hard not to be seen as a partisan for or against reform. Eastin, a Democrat, holds an elected office as head of the State Department of Education. Governor Pete Wilson, a conservative Republican, appoints the State Board of Education, which creates the policies the department is charged with carrying out. Denham oversaw the creation of two curriculum frameworks for mathematics, one in 1985 and one in 1992; such frameworks are issued every seven years. Depending on who one talks to, the 1992 framework is either the next natural step in the reform begun in the 1985 framework or a terrible betrayal and dumbing-down of it. Both have the flavor of the NCTM Standards, and that flavor grew more pronounced in 1992.

One of the reasons the 1985 framework attracted less attention than the one in 1992 has to do with the textbook adoption process. After California releases a framework, textbook publishers have a certain amount of time in which to submit materials for consideration by the textbook adoption committee. The committee examines the materials to see which adhere to the criteria of the framework and on that basis submits a recommended list to the Board. Fourteen programs were submitted in response to the 1985 framework, and the committee rejected all of them, saying that they did not make the changes the framework had called for. The Department of Education scrambled to assemble a group to work with publishers to edit their materials so that they were marginally in keeping with the framework. For this reason the reform elements of the 1985 framework did not have a lot of impact on the materials students ended up using. The publishers were put on notice that they had better take the framework more seriously next time around.

And they did. Among the twenty-three programs submitted to the textbook adoption committee, the committee recommended nine. The State Board then added another three programs to the adoption list. While it is entirely within the Board's purview to amend the list, some say that the Board added the programs under pressure by public relations firms hired by publishing companies to plead their cases to the Board. Some publishers grumbled that their programs had received higher scores from the adoption committee than had the three that were added, but they lost out because they could not afford to hire lobbyists. (One Board member told the *San Francisco Chronicle* that the lobbying was not an influence. One company's pleas were rejected, she said, though they were made "by a very fine public relations firm.")



Textbook publishing is big business in California—sales totaled about $200 million in 1995—and competition to get on the adoption lists is intense. Part of the reason the stakes are so high is that California, with one-eighth of all public school children in the U.S., buys a lot of textbooks. Only California and Texas command a large enough share of the textbook market to get publishers to create the materials they ask for. In this way the changes in California may have a profound effect on how mathematics is taught in the rest of the country.

Through the Field Glasses: A Closer Look at Two Reform Programs

Susan Addington, a mathematics professor at California State University at San Bernardino, has worked extensively with teachers and counts herself as pro-reform. "Most people, even most educated and mathematically educated people, think of math as a bunch of procedures," she remarks. The reform materials try to give a different view of mathematics, "that it's something that you can do, that you can explore, that is fun, that goes off in different directions." However, Addington has some reservations. In preparation for conducting a workshop for teachers, she examined some reform materials called *Interactive Mathematics*, published by Glencoe. "They were quite disquieting," she says. The books are very different from the traditional—each chapter starts not with an exposition but with a dialogue that students are supposed to act out. They also lacked the structure that would allow students and teachers to pull the mathematical threads together. "The books are graphically very busy and very colorful, and I found them kind of nervous-making," she remarks. "I think there's some good material in there, but a lot of the teachers couldn't pick out where the math was." In fact, when she asked teachers at the workshop

Math Homework of the '70s

Word Phrases and Number Phrases

…In a product involving a variable, you will recall that the multiplication sign is often omitted.

Example 1: Translate into a number phrase:

The product of 7 and n.

Solution:

$7n$. **Answer**

You will recall, also, that parentheses are sometimes used in place of the multiplication sign. Thus you may write

$(-2)(7) = -14$ or simply (-2)7=-14.

Example 2: Translate into a word phrase.

- a. $x + 5$;
- b. $y - 2$;
- c. $8(z + 1)$;
- d. $w \div 9$

Solution:

a. The sum of x and 5.

b. The difference when 2 is subtracted from y.

c. The product when the sum of z and 1 is multiplied by 8.

d. The quotient when w is divided by 9. **Answer**

Written Exercises

Write an open number phrase.

1. Three greater than n.
2. Seven less than a.
3. The product of x and $\frac{1}{2}$.
4. The quotient when n is divided by -6.

[Exercises 5-14 omitted.]

Write word phrases for each of the following.

15. $-3y$
16. $2a - 5$
17. $\frac{11-n}{4}$
18. $\frac{1}{2}rs$

— from Modern School Mathematics: Structure and Method, by Mary P. Dolciani, William Wooton, Edwin F. Beckenbach, and Walter Markert, pages 250-251.

whether a certain activity in the book had some mathematical content, only one teacher was able to find it.

Addington's reaction is not unusual. Part of the opposition to reform materials is due to the simple fact that they look and feel very different from traditional textbooks. However, as Addington found, there are more substantive differences. A closer look at two particular reform programs—MathLand and the Interactive Mathematics Program—can give a flavor of what advocates for and against reform hold dear. MathLand, produced by Creative Publications for kindergarten through sixth grade, is one of the most heavily criticized of the reform programs. Part of the reason for the amount of criticism is simply that MathLand has been very visible, commanding 60 percent of the market share. But there are other reasons as well. The focus of mathematics teaching in the early grades has traditionally been mastery of pencil-and-paper arithmetic computations, and this is what parents expect their kids to be doing. MathLand takes off in the opposite direction, emphasizing student exploration and understanding with little drill.

For example, the sixth-grade MathLand segment on computing with fractions has students using plastic pie-shaped segments, fitting them together in different ways to represent addition and subtraction, as well as drawing pictures and working with collections of cubes. At one point the teacher writes on the board some addition problems involving fractions, and some of the answers are actually wrong; students are supposed to figure out which are true and which are false. The teacher questions students without giving away the "right answers" in an attempt to get students to think things through on their own. The MathLand *Teachers' Guide* has fairly detailed instructions on what teachers should do in the classroom, in some cases presenting actual wording they can use. At a certain point in the week, the teacher has each student assemble a list of his or her own "Do's and Don't's" for adding and subtracting fractions. At the end of the week, the class discusses their lists of "Do's and Don't's", and the teacher has each student hold up a certain number of fingers indicating how confident they are in the correctness of their lists. Then the teacher gives the students a harder addition problem than they have done before, one involving fractions that they cannot represent with their plastic pie-wedges.

This is the culmination of the lesson, and it exemplifies the nub of some of the criticisms of MathLand and other reform programs: the teacher does not explicitly present to students the usual algorithm for adding and subtracting fractions. The question of if and when teachers should bring mathematical "closure" to exploratory activities has been hotly debated. While traditional materials have been criticized for emphasizing closure at the expense of exploration, the reform materials can be just as imbalanced, albeit in the opposite way. Another common concern is exemplified in this MathLand lesson: Amid all the activities with plastic pie-wedges and drawings and students' explanations written in their best sixth-grade prose, will the teacher be able to pick out the mathematical threads of the students' explorations and make sure they reach a mathematical understanding that is correct? This concern is magnified by the fact that elementary school teachers often have weak mathematics backgrounds.

Jan Powers, a sixth-grade teacher at Santa Rosa Elementary School in Atascadero, Califor-

nia, is in her third year teaching from MathLand. She points out that in the segment on adding and subtracting fractions, students essentially arrive at the standard algorithm through their lists of "Do's and Don't's". On the other hand, she feels that the one week MathLand sets aside for this topic is insufficient, so she makes sure she covers the standard algorithm and gives her students additional drill. (At the request of teachers, the publisher of MathLand has now developed a new component for computation practice and skill development.) On the whole, Powers believes MathLand is an "absolutely wonderful" program because "it builds a thinking base for kids so that they are always questioning why math works." But it is not easy to use: it requires more time and more mathematical discernment on the part of teachers. "You can't just put an activity on the table and expect the students to draw the mathematics out of it," Powers notes. The teacher must "help the students connect the activity to the mathematics." Teachers need more training in using activity-based programs as well as in mathematics, she says. "My concern is that we will throw out a good program because we don't train teachers to use it."

Many MathLand critics agree that the exploratory aspect of MathLand is good but take issue with its approach to skill development. In an editorial in the *Davis Enterprise* newspaper, Abigail Thompson, a mathematics professor at UC Davis, proposed that "every child finishing sixth grade should be able, without a calculator, to compute any percentage of anything, to add, subtract, multiply, and divide positive and negative whole numbers of any size, any fractions and decimals." Her point is that, although this goal sounds fairly uncontroversial, MathLand does not aim for it. Thompson points out, for example, that in MathLand there are very few multiplication problems with numbers with more than two digits. The assumption is that students will use estimation skills and calculators for "harder" or larger numbers. Thompson also notes with dismay the absence in MathLand of the traditional algorithm for long division. This lack of attention to what many believe are basic building blocks of mathematics is what has caused the most outcry over MathLand.

MathLand was developed by a commercial publisher specifically in response to the 1992 California mathematics framework. By contrast, the Interactive Mathematics Program (IMP) was developed for high school students through a grant from the National Science Foundation to a group of four people: Lynne Alper and Sherry Fraser, high school teachers who have worked on curricular projects, and Dan Fendel and Diane Resek, mathematics professors at San Francisco State University. IMP has been in development since 1989, and most of the units in the program are in their fifth round of rewriting. Having started out as an experimental program in the Bay Area, IMP was in use last year in about seventy high schools in California and in about the same number nationwide. IMP is a four-year program intended to replace the traditional sequence of Algebra, Geometry, Algebra 2 (or Trigonometry), and Precalculus. The University of California system accepts three years of IMP as fulfilling its entrance requirement for mathematics.

Like MathLand, IMP represents a radical departure from the traditional ways of teaching mathematics. IMP consists of a set of workbook units that take students several weeks to go through. The units are not organized around a mathematical topic, but around a problem to which mathematics can be applied. For example, one unit, called "Baker's Choice", presents students with the problem of how to maximize profits at a bakery given certain constraints such as the cost of ingredients, the capacity of the ovens, the amount one can charge for different types of cookies, and so on. The students then go on to develop the mathematics—for example, how to graph and interpret linear inequalities—which they will need to solve the problem. Even the development of specific pieces of mathematics is usually done through word problems, which may or may not be part of the cookie problem. In "Baker's Choice" students get practice with inequalities in a problem about deciding which kind of pet food to buy based on the minimum amount of fat and protein the pet needs and how much it should eat each day.

Math Homework of the '90s

Homework 3: Variables of Your Own

1. Make up a set of between 5 and 10 variables for a situation, similar to the list [made up in the previous homework called High School Letters]. For instance, you might call your situation Marching Band Letters, Baseball Game Letters, Party Letters, or Clothing Store Letters. If you prefer, you can make up a situation of your own.
2. On the front side of your homework paper, write three [algebraic] expressions using your variables for which someone can write a summary phrase. [The term "summary phrase" was defined earlier as a phrase that describes in words what an algebraic expression says.] On the back side, write the summary phrase.
3. On the front side of your homework paper, write three summary phrases for which someone can write an algebraic expression using your variables. On the back side, write the expression.

In the next class, you will exchange papers with other students and see if you can figure out the summary phrases and algebraic expressions for each other's situations.

—from "Baker's Choice," Interactive Mathematics Program, page 7. Copyright 1996 by Key Curriculum Press, Inc.

Calculators are used whenever needed in class and on the homework. And as with most reform materials, students do a great deal of writing, including explanations of solutions as well as more free-form writings about their thoughts and feelings about the work.

In this way students spend a great deal of time working on problems involving inequalities. But never are they drilled in the traditional way, with a bare list of inequalities which they are asked to simplify or graph. Is this lack of drill a problem? Alex Reisbord, who has taught high school mathematics for thirty years and now teaches at the California Academy of Mathematics and Science, does not think so. He admits that by the time his IMP students are in his calculus class, he has to spend time reviewing algebra. "But I more than make up for it because I don't have to do some other things that took me a lot longer to do in the past, like problem solving," he says. Also, the real-life problems seem to make the mathematics stick. "Because it had meaning and a problem, they can remember the mathematics they learned." Does IMP prepare students well for calculus? Again, Reisbord expresses optimism. He believes his students "are going to be prepared to walk into a situation and succeed because they know how to study, they know how to read the book, and they have certain skills." In addition, he pointed out that college calculus is changing all over the country, and it is often difficult to know what kind of preparation is needed. It used to be that by October of their freshman year, "they're doing limits, no matter what textbook they're using, no matter where they're going to school," he says. "Today that's not true."

Some have raised concerns about the structure of IMP. Rather than the centerpiece being mathematics and how it can be used to solve problems, IMP puts the problems at the center and then brings in the mathematics needed to solve them. In "Baker's Choice" the focus is on solving a problem about profits in a bakery, not the mathematical tool of linear programming, remarks Dick Stanley, a mathematics education specialist at UC Berkeley. "Is this a good idea?" he asks. "I'm not sure. But for students to see mathematics as the resilient and robust subject that it is may be difficult if the focus is on the specifics of cookie factories. Mathematics is a remarkable subject worthy of learning because it has powerful tools, and ultimately you have to focus on those tools."

Reform programs like MathLand and IMP take a decidedly "constructivist" approach, relying almost entirely on students discovering things for themselves rather than having the teacher tell them what to do. While the traditional approach was often extreme in leaving out student discovery, some reform materials can be just as extreme in avoiding any "teaching by telling". "If you're serious about learning something, there will be an active balance" between trying to figure it out on your own and being told or reading about it, says Stanley. Looking at many of the reform materials, one finds "there is no longer any mathematical exposition at all. Now there's just problems. That's literally true if you look at some of the curricula. You open up the chapter, and it's problem 1 to problem 106. This approach will not survive," he predicts.

—Allyn Jackson

Editor's Note: *Part II of "The Math Wars" will appear in the August issue of the* Notices.