Letters to the Editor

On Two Points in the Article by Wu

Although I do not consider myself to be a "true believer" of either the traditionalist or reform approaches to mathematics education, I would like to address two points in the "Forum" article by H. Wu ["The Mathematician and the Mathematics Education Reform", December 1996]. One clear misrepresentation is the statement that "the text [by Hughes-Hallett, Gleason, et al.] is written to be accessible to students with weak algebraic backgrounds." This would seem to indicate that algebraic skills are not required or expected by the book. In fact, anyone who has taught from the book would know that this is completely untrue. The explanations and exercises in this book require a proficiency with algebra equal to (and in many cases beyond) that expected by "standard" texts. The statement by Wu was probably a misinterpretation of the "Rule of Three" which the authors of the text attempt to use. This rule (which is not applied dogmatically) is that new ideas in the book are introduced through graphical and numerical descriptions as well as through algebraic statements. It is useful for anyone to hear a new concept described in several different ways, but I believe some of the authors may have been quoted as saying that the Rule of Three is particularly useful for students who think geometrically rather than algebraically. Note that many mathematicians prefer to think of things geometrically rather than algebraically (or vice versa) and that this is not a sign of deficiency in either area.

I would also like to comment on Wu's discussion of the "discovery" of the derivative of the sine function in the text by Davis, Porta, and Uhl. After students have graphically observed the difference quotient for sine converge to the cosine function, Wu's objection is that "students are explicitly asked to believe that…they have witnessed mathematics at work." Of course they have! Would Wu have us believe that any discovery which is not accompanied by a formal proof is not part of mathematics? In addition to being a good didactic tool, this sort of discovery is a very real part of much mathematical research (which for mathematicians is expected to be followed by a conjecture and a proof, but would be sufficient in itself in many applications).

On a separate note, the editorial "Differing Viewpoints on the Teaching of Mathematics" by A. Tucker was a welcome addition to the current discussion of proposed curriculum changes. Like similar debates in political, social, and religious arenas, it is often the extremists on all sides who are heard most frequently. This creates the mistaken impression that one must "choose sides". I have not yet seen a perfect approach—one which can be proved to have all of the benefits and none of the disadvantages of the other methods of teaching mathematics. Until one is developed, I believe that both "reform" and "traditional" approaches have their advantages and would be appropriate for certain student populations. The goal then is not to determine which approach is "right", but to become familiar with the successes and failures of each.

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About the Cover

The cover image shows Lyapunov exponents from a codimension two investigation of bifurcations in a neural network model. It was created by Mike Casey, with Arnold Mandrell, of UCSD's Department of Mathematics at the San Diego Supercomputer Center.

An Open Letter to Donald Babbitt

Together with colleagues John Jones, Rob Kirby, and Brian Sanderson, I have just launched a new international mathematics journal: *Geometry and Topology*. The journal has many new features, which we hope will become standard in future journals. Perhaps it is significant that we disagree almost totally with your prognosis for the future of mathematics journals.

I'll start with a critique of your article and then describe some of the features of *Geometry and Topology* which contradict much of your analysis; finally, I draw some conclusions.

I think your attitude toward the future of electronic publishing could be fairly summed up as "starry-eyed". You seem to have been seduced by the hype that the World Wide Web has generated. Thus you talk about "real mathematics on-screen" (by which you mean HTML files containing mathematics and links) which you contrast with files "viewable on screen in TeX code and downloadable in a variety of formats." But it is now the case that all electronic mathematics journals supply articles as files in DVI or PostScript or PDF formats (usually two or more of these). If I click on a DVI or PostScript or PDF file in my Web browser, I automatically get a window in which the file appears as fully formatted mathematics. Why is this not "real mathematics on-screen"? There is of course a delay before the mathematics appears on the screen, but there is a delay associated with everything on the Web (was the verb "surfing" ever given a less appropriate use?). As computers get more powerful and links more capacious, these delays will decrease—perhaps. A page does not have to be littered with buttons which zoom the (very patient) reader all over the world before it can be called "real mathematics on-screen".

Later in your article you describe an ideal future in which all electronic journals have "a standard structure (SGML?)" forming "the universal database of the mathematical research literature." You do not specify exactly what format this standard structure will use but you clearly have in mind that there will be some such format and that it will be very complicated and need expert help to facilitate. This forecast is important for your later predictions. Thus the need for "highly structured files required for the upcoming electronic milieu" is the primary reason you give for your forecasted disappearance of free electronic journals: because they will be too expensive in terms of editorial time. Well, you may be right, but it's a dangerous game forecasting the type of format which will eventually succeed in the computer world. It seems to me far more sensible to extrapolate from the past. Thus computers are likely to get cheaper and more powerful. File handling (including any new "highly structured files") is likely to get easier and quicker. Storage capacity is likely to get cheaper and easier to access; thus it is already the case that electronic storage is far, far cheaper than paper. These trends will drive down the cost of the "substantial subsidy by way of equipment, system support, etc." which you cite (and which is another reason for your forecasted disappearance of free EMJ's). Indeed, I challenge now your description of this as "substantial". For example, Warwick Mathematics Department has computing power and storage capacity between three and four orders of magnitude greater than that needed to run *Geometry and Topology*, the cost of which is therefore negligible in the total budget. Moreover the journal makes no special demands on system support, but merely uses the system as already set up for general departmental use.

The other trend from the past which seems likely to continue is the inertia in operating system and file formats. This is because of the real investment which we have all made in mastering the present systems and formats. Thus I cannot reasonably see that TeX will be superseded in the next ten years, nor can I see that PostScript will stop being (possibly one of) the standard languages for page description. The evidence of the past is clearly that a new system takes hold only if it includes a fully compatible copy of the old. So here is another dangerous forecast, but one based on past trends: I forecast that if a page description language with full-fledged Web links built in becomes a new standard, then it will be based on PostScript with the links invisible to a current PostScript printer[1]

However, the main fallacy in your description of the future is that no mathematician I have ever met actually does "real mathematics on-screen", nor are we likely to start. We all (or nearly all) use computers, of course. But computers are as incidental to the real process of mathematics (which takes place often elsewhere—in the bath or falling asleep) as paper or pencils or conventional libraries. We don't even read other mathematicians' articles on-screen. We browse them on-screen. If we seriously want to study them, then we print them out and carry them around, read them in the bath or in the garden or wherever. We treat electronically sourced articles and conventional journal articles exactly the same in this respect. We make our own paper copy for serious study. (Thus, for example, I am using a dog-eared photocopy of your article while thinking about this letter. It's been with me in the bath and is covered in scribbles.)[2]

I agree that it would be nice while browsing to go straight to cited articles and reviews. But I can already have a Web browser, tuned to MathSciNet, open in a separate window and feed in the data using the mouse, which is almost as convenient. The real added value to working mathematicians of these links is quite small and not worth a great deal of effort to provide.

And this brings me to the second part of my letter. It was not for the purpose of providing links of these kinds (or for other computer gizmos such as Java scripts or real-time movies—great fun though they are) that we have launched *Geometry and Topology*. We launched it because of economic factors. Warwick (in common with many other universities) went through a crisis in library funding about two years ago. It was caused by the excessive price inflation in commercial journal prices—the learned societies, which by and large charge a reasonable price, are not included here; we can all name the culprits. Now the real value of journals—the

[1] *Indeed it is quite easy to design such a hyper-PostScript system: the links are embedded in the PostScript file by dvips \special's. They appear as strings including particular keywords which are immediately removed from the PostScript stack and which are interpreted by an extended version of Ghostview, which passes the links to a Web browser. (HyperTeX, similarly based on DVI files, is not a satisfactory answer, because DVI cannot handle graphics properly.) Update: This forecast, made in all innocence a few weeks ago in the first version of this letter, appears already to have been overtaken by events—the latest version of Ghostscript now supports hyper-text links!*

[2] *Incidentally, Larry Siebenmann makes a good point here: Electronic articles are likely to be a better-quality product when used in this way, since they are usually laser-printed compared with a photocopy of a bulky journal printed with small fonts.*

mathematics and the refereeing, and now, because of TeX, the typesetting—is provided free by mathematicians. Moreover the Internet also now provides a perfect vehicle for free publication. Journals ought to be cheap, not absurdly expensive. We decided that the only way to make progress was to prove, by example, that a first-class journal could be run free or almost free.

We decided that the most important consideration was quality. Mathematicians are notoriously conservative and they will only send their best articles to reputable established journals. So we concentrated our efforts on the mathematical quality of the journal and on assembling a large and truly international top-quality academic editorial board. Thus *Geometry and Topology* is primarily a first-class fully refereed international journal, and only secondarily is it an electronic journal. Indeed, we think of the electronic aspect as simply the way in which the main publication takes place. We intend to publish a paper copy as well for individuals and libraries that want it and also a CD-Rom version. Obviously these versions will not be free, but will be priced according to demand. By contrast, electronic access to the journal is free, and we are determined to keep it so.

The other main consideration was making the journal simple to administer in order to minimize time management. Thus the academic board members are responsible for the refereeing procedure and make all the publication decisions, with conference taking place by e-mail. We have academic editors in countries all over the world—Australia, the U.S., Germany, France, and the U.K.—and it is perhaps in this way that the journal most radically exploits the now nearly universal access to electronic networks. It would not be possible to have a real working editorial board so widely spread without e-mail.[3]

[3]*For details of the procedure we have adopted to make the academic editorial board the real decision-making body, see the notes on procedure detailed on our Web and ftp sites:* `http://www.maths.warwick.ac.uk/gt/` *and* `ftp.maths.warwick.ac.uk`, *respectively.*

For the same reason we decided to be highly conservative in terms of computer technology. If other electronic journals are at the cutting edge of technology, then we are somewhere safely inside the handle! If new formats arrive and become widespread then we shall adopt them (provided it is easy to do so), but we do not intend to innovate new formats. Thus we have chosen the well-established and easily handled format of PostScript as the primary medium for publication; there were two further reasons. First, PostScript can handle graphics properly, and articles in geometry or topology are likely to contain a good deal of graphical material. Second, by dealing with PostScript files we can handle material sourced from a variety of TeX packages or even other word processors. We apply the journal running heads as a PostScript preamble (which also removes any spare logos or page numbers), and this eliminates the need for authors to use a standard package. Thus the "great virtuosity" shown by authors in the use of TeX which you clearly regard as a serious problem, because it entails large-scale editing, is for us a virtue. We encourage authors to submit their articles using whatever macro packages, including their own, that suit them best. In order to prevent the journal looking a complete mess, we insist on certain minimal standards of uniformity, which are easy to obtain using standard TeX packages. Initially, until the journal is established, we shall be a bit fussy about format and aim for a rather uniform appearance; however, once the journal settles down, we shall accept any reasonably uncluttered format. This relaxed attitude has two obvious advantages. First, it allows authors to choose a format which suits their material and expository styles, and second, it means that the amount of final editing by us will be minimal. The journal will always have a uniform style in running heads and footers, and title pages will always be formatted in a uniform style by us. My feeling is that the standard of uniformity we will obtain will greatly exceed a "camera-ready" journal and will probably be indistinguishable from a conventional printed journal.

Well, it remains to be seen how well this works out. We have only just started publishing, and one article has appeared. The editorial procedures have worked well so far, and the first article was nearly perfect in format as submitted—it took me only half an hour of final editing. Experience will tell whether it is really possible to run a first-class journal with minimal editorial effort, but the experience thus far is very encouraging.

So now to a few conclusions. I do not see the real nature of mathematics journals changing very much at all over the next ten years. The conservatism of the system of publication, job application, and promotion will dictate that good journals in mathematics will need to continue to be carefully refereed. I cannot see an equivalent of the Ginsparg LANL experiment in theoretical physics having the same kind of success in mathematics. We use publication far more as a final record and validation process and far less as a method of communication of hot ideas. Moreover I think that the basic item will continue to be the hand held, annotatable reprint of articles.

What I do see changing dramatically is the method by which this reprint is produced. At the moment we have a highly unstable situation whereby commercial journal publishers charge exorbitantly for printing and distribution of what, in many cases, is freely supplied camera-ready copy produced by the authors. At the same time, the technology exists, is freely available, and is easy to use to produce first-class printed copy. If *Geometry and Topology*, and other new journals produced in a similar way, succeed, then we shall see a very rapid transformation, with nearly all journals publishing primarily in the form of a free or cheap electronic format, with paper copy available as a sort of luxury item. There will of course be a luxury charge for this luxury item, but I forecast the emergence of a new sort of publisher who just produces paper copy of preformatted electronic sources for a variety of different electronic journals and makes a small but fair profit on the deal. The high-status (and high-charging) commercial academic publishers will have to adapt very fast if they want to stay competitive.

This process may happen far sooner than any of us expect. The crisis in library funding continues unabated. Some of the world's most important mathematics departments are now actively considering permanently cancelling top mainstream paper journals, previously considered indispensable, on the grounds of cost, which in some cases reaches a dollar a page and continues to inflate at 15 to 20 percent per annum (i.e., a real inflation of more than double figures). It does not need much imagination to see the consequences of the scale of cancellation which this implies. The learned societies which have much more reasonable charges per page will not collapse so fast, but I think that they too will have to change fairly rapidly and adopt electronic publication as their primary method. The AMS with its expertise in handling a variety of formats both paper and electronic should be very well placed to adapt. It would be ideal as the kind of paper publisher for a variety of primarily electronic journals which I predict. So, Donald, would the AMS be interested in providing this kind of subsidiary paper publishing service?

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We Call Them Lessons Because They Lessen

I think Paul Nevai in his letter to the March 1997 issue of the AMS *Notices* misunderstood Gian-Carlo Rota. "Publish the Same Result Several Times" was surely not intended to apply to trivial results. I understood it to mean "Don't hide your major results in technical journals where they will only be read by specialists. Do a less technical article for the *Monthly*, too." Then "Do Not Worry about Your Mistakes" means just what it says: don't worry. That is, don't fret, don't tear your hair out and quit just because you make mistakes. It certainly did not mean don't be careful. Finally, "Give Lavish Acknowledgments" did not mean give effusive acknowledgments; it only meant acknowledge credit whenever you can. It takes nothing away from you, and people feel so good when they see their efforts mentioned in print.

Rick Norwood
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On van der Waerden and Intuitionism

Mac Lane's article on van der Waerden's *Modern Algebra* in the *Notices* for March 1997 reminded me of something that happened when I was on sabbatical in Zurich in late 1958 or early 1959. I was talking to van der Waerden at the social gathering preceding a colloquium lecture at the University of Zurich. Thinking of the fact that in the second edition of *Algebra* he had dropped the sections on the Axiom of Choice and transfinite induction and then in a later edition put them back in, I asked him whether he was an intuitionist or not. I cannot quote his answer word for word, but the meaning was quite clear. It was that if a theorem had an intuitionistic proof, he believed it was really true.

John Dyer-Bennet
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On the Interactive Mathematics Program

David Klein comments briefly on the Interactive Mathematics Program in his letter to the *Notices* ("Withdraw Endorsement of NCTM Standards", March 1997). His sole specific observation about the program is that it "delays presenting the quadratic formula until the twelfth grade."

The Interactive Mathematics Program (IMP) is an innovative, four-year high school curriculum whose development has been funded primarily by the National Science Foundation. Kleins's comment might suggest that this program covers less mathematics than the traditional high school sequence. In fact, the opposite is true. As one of the directors of this program, I would like to mention several ways in which I believe the IMP curriculum provides a far more challenging course of study than the traditional program.

1. In the IMP classroom, the concept of similarity and a proof that the sum of the angles of a triangle is 180 degrees are both presented in the ninth grade, rather than "delayed" until tenth grade as in the traditional high school program.

2. In the IMP classroom, right-triangle trigonometry is presented in the ninth grade, rather than "delayed" until eleventh grade as in the traditional high school program.

3. In the IMP classroom, the concept of expected value is presented in the ninth grade, rather than omitted entirely as in the traditional high school program.

4. In the IMP classroom, the normal distribution and standard deviation are presented in the ninth grade, the chi-square distribution is presented in the tenth grade, and the binomial distribution is presented in the eleventh grade. These topics are all omitted entirely in the traditional high school program.

5. In the IMP classroom, the concept of a derivative is presented in the eleventh grade, rather than omitted entirely as in the traditional high school program. (Although some high schools offer calculus courses, calculus is not part of the standard high school program.)

The IMP curriculum also includes substantial work with matrix algebra, including the conceptual and computational development of matrix operations and the use of matrix inverses to solve systems of linear equations (in the eleventh grade) and the use of matrices to represent geometric transformations—both translations and rotations in two and three dimensions (in the twelfth grade).

Certainly, people may differ in their assessment of the relative importance of different topics within the scope of a four-year high school program. We have chosen to "delay" the quadratic formula in order to allow IMP students access to a variety of subject areas within mathematics that most students never see.

Finally, with regard to the quadratic formula, I would add that although this topic does not occur until the twelfth grade in the current draft of the IMP curriculum, evaluation studies have shown that a significantly larger percentage of IMP students complete four years of high school mathematics than do students in a traditional program.

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Restrictions on Export of Encryption Technology

Over the past few years, efforts to control and restrict the export of encryption technology have been a continuing theme in United States Congressional legislation. Aside from the legal and moral issues, efforts to legislate control over mathematical concepts are untenable.

Historically, attempts to control the spread of technology in advanced weaponry and computer hardware have been successful. However, this has been accomplished by restricting access to materials, manufacturing equipment, and the tools needed to integrate complex systems, leaving a potential adversary in the position of knowing how something works, but being unable to realize its development or implementation. Frankly, in mathematics knowing how something works is everything!

There seems to be a fundamental misconception on the part of government decisionmakers as to the nature of mathematics and the role it plays in the development of scientific thought and technological progress. One can only assume that the mathematical community has failed to connect on this issue. Either mathematicians aren't yelling loudly enough, or the government isn't listening.

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