Kiiti Morita 1915–1995

A. V. Arhangel′skii, K. R. Goodearl, and B. Huisgen-Zimmermann

Kiiti Morita, a pioneer in both algebra and topology, died in Tokyo on August 4, 1995, at the age of eighty. His last journal publication had appeared only six years earlier.

Born in Hamamatsu, Japan, on February 11, 1915, Morita received his Ph.D. degree from the University of Osaka in 1950 for a doctoral thesis in topology. His basic university education had been focused on algebra—as a topologist, Morita was largely self-taught. In 1939 he was appointed assistant at the Tokyo University of Science and Literature, and after an interlude as lecturer/professor at Tokyo Higher Normal School he was appointed professor at the former university in 1951, where he taught for twenty-seven years (a period during which the two institutions were combined and later relocated from Tokyo to Tsukuba). Finally, as emeritus from the University of Tsukuba, he continued his activities at Sophia University.

Morita's focus in teaching was again split between topology and algebra—throughout his research career he successfully bounced back and forth between the two subjects. His algebraic contributions, however, while sparser in number (about 75 percent of his eighty-seven journal publications were on topological subjects), were the ones that made his name a household word within the mathematical community at large. We will start with a brief account of his most influential work in this area and of its continuation and ramifications within current research. Following that, we will sketch his impact on topology.

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Morita's Contributions to Algebra

Unlike most of his prominent Japanese contemporaries in mathematics, Morita had not supplemented his education through an academic excursion abroad by the time he started publishing groundbreaking concepts and results. According to one of his Ph.D. students in algebra, H. Tachikawa, Morita was not connected with the Nagoya research group, the most active Japanese group in his field of algebraic specialty, homological algebra. As a result, significant developments in the field reached him only with considerable delay. For example, while draft copies of H. Cartan and S. Eilenberg's modern cast of the subject, *Homological Algebra* [10], were already in circulation in the early 1950s among the young mathematicians surrounding T. Nakayama, this text did not reach Morita until its official publication in 1956.

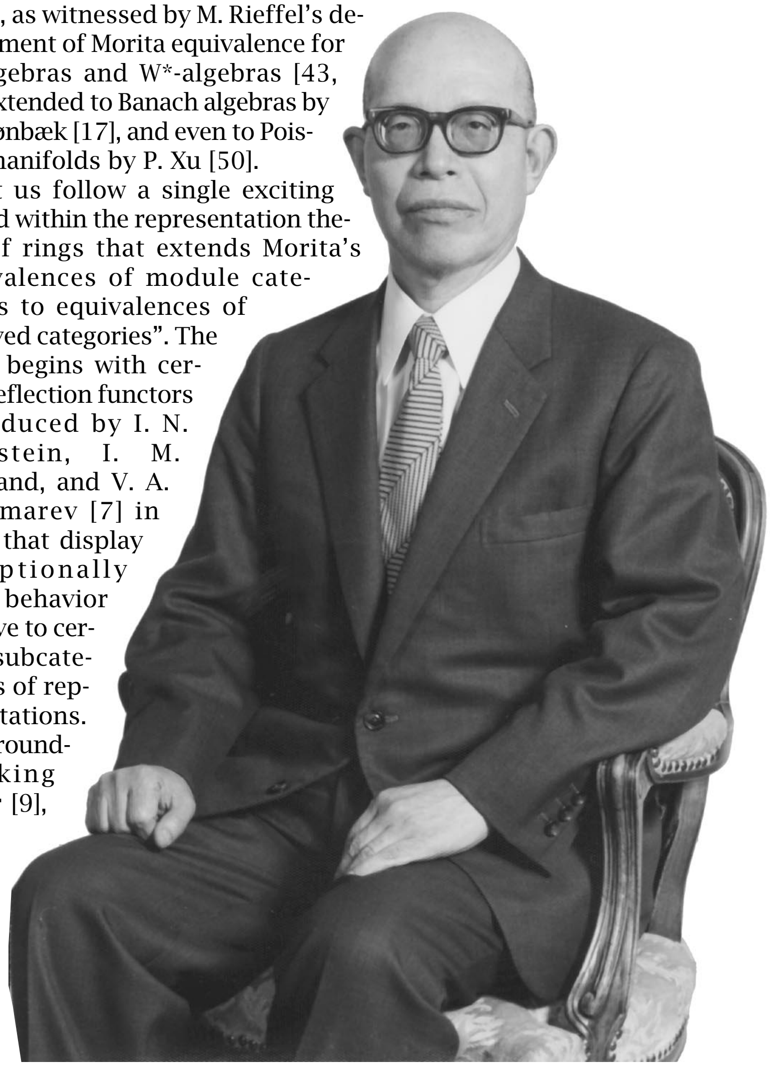
Morita's primary source of algebraic inspiration was Pontryagin's duality theory for locally compact abelian groups. In that case, duality is effected by the contravariant Hom-functor $D = \mathrm{Hom}_{\mathbb{Z}}(-, \mathbb{R}/\mathbb{Z})$. In experimentally placing injective modules over more general rings into the second argument of Hom-functors (injective modules take over the role of divisible abelian groups in the wider context), Morita noticed that correlations reminiscent of those established by Pontryagin were preserved for certain pairs of objects $(M, D(M))$. In particular, this naturally led him to new characterizations of quasi-Frobenius rings, namely, rings for which all free modules are injective. Jointly with his students Tachikawa and Y. Kawada, Morita exploited the findings in two articles in 1956–57 [38, 39] and assigned further exploration of this line to Tachikawa. The latter continued the study of annihilator relations for pairs $(M, D(M))$, obtaining, in particular, sharp results for the special case of rings satisfying the descending chain condition for one-sided ideals [47]. At that point, Morita succeeded in pinning down dualities between "reasonable" subcategories of module categories in full generality, proving that they are always of the form $\mathrm{Hom}_A(-, Q) : A\text{-Mod} \to \text{Mod-}B$, where B is the endomorphism ring of the A-module Q, and he completely determined the eligible objects Q inducing such dualities. (We mention that for the case of rings with the descending chain condition, G. Azumaya independently obtained results of the same strength, which were published a year later than Morita's in [4].)

It appears that Morita's treatment of equivalences of module categories came as an afterthought to his work on duality. For one thing, the title of his famous 1958 paper "Duality for modules and its applications to the theory of rings with minimum condition" [27] made no mention of equivalences. For another, Morita reportedly balked at an editor's suggestion to publish the material in two separate treatises in order to give the topics of duality and equivalence independent emphasis.

Morita's theorem on equivalence [27, Section 3] is probably one of the most frequently used single results in modern algebra. It states that, given two rings A and B, the categories A-Mod and B-Mod are equivalent if and only if B arises as the endomorphism ring of an A-module P which is a direct summand of a power A^n of the regular module A, with the property that A in turn is a direct summand of a direct power P^m of copies of P. Further, each equivalence between A-Mod and B-Mod is induced by a covariant Hom-functor $\mathrm{Hom}_A(P, -)$ with P as above. In this situation, the rings A and B are now referred to as being "Morita equivalent". (The symmetry of this relation is an easy exercise.) The identification of Morita equivalent pairs of rings—such as any ring A coupled with the $n \times n$ matrix ring $M_n(A)$—at once allowed casting a great deal of technical ballast overboard: there was no longer any need to verify computationally that the corresponding module categories display identical behavior. Indeed, Morita's motivation and immediate usage was of this ilk: He provided a general reduction principle by showing that any finite-dimensional algebra is Morita equivalent to a "basic" algebra—one which, modulo its Jacobson radical, is a finite direct product of division algebras (rather than, as in the general case, of matrix algebras over division algebras); in [27, Section 7] he actually proved this result for rings with minimum condition.

One of the main promoters of Morita's new and fundamental viewpoint was H. Bass, who had been alerted to Morita's ideas by S. Schanuel at Columbia. Bass's mimeographed notes on Morita's theorems [6], based on lectures he gave at a National Science Foundation summer institute at the University of Oregon in the early 1960s, were copied, recopied, and rapidly distributed throughout the U.S. and to many European mathematics departments. Another publication sped the dissemination of Morita's ideas in Western Europe, namely, P. Gabriel's dissertation of 1962 in which Morita's equivalence theorem is re-proved [16]. It is hardly surprising that the benefits of transporting desirable features from one category to another by means of well-behaved functors, as advertised by Morita's work, triggered the construction of a network of such bridges linking and unifying extensive parts of algebra. Furthermore, these benefits have not gone unnoticed in other fields, as witnessed by M. Rieffel's development of Morita equivalence for C*-algebras and W*-algebras [43, 44], extended to Banach algebras by N. Grønbæk [17], and even to Poisson manifolds by P. Xu [50].

Let us follow a single exciting thread within the representation theory of rings that extends Morita's equivalences of module categories to equivalences of "derived categories". The story begins with certain reflection functors introduced by I. N. Bernstein, I. M. Gel′fand, and V. A. Ponomarev [7] in 1973 that display exceptionally good behavior relative to certain subcategories of representations. In a groundbreaking paper [9],



S. Brenner and M. C. R. Butler proved that this striking behavior is shared by a much larger family of functors dubbed "tilting functors". The theory of such functors was clarified and pushed further by D. Happel, C. M. Ringel [19], K. Bongartz [8], and Y. Miyashita [23]. In their present incarnation, tilting functors are covariant Hom-functors $\mathrm{Hom}_A(T, -) : A\text{-Mod} \to B\text{-Mod}$, where T replaces the modules P inducing Morita equivalences by much more general objects; at this point, T is required only to have finite projective dimension, not to admit any nontrivial higher self-extensions, and to allow for a nice resolution of A in direct summands of finite direct powers of T. It was only at this stage of the development that a deeper reason for the remarkable preservation properties of tilting functors surfaced. In [18] Happel recognized, for special classes of rings, that they induce equivalences on the level of the derived categories of the two module categories involved. Such derived categories, introduced by J.-L. Verdier [49] after a suggestion of A. Grothendieck, can be thought of as devices to bare the homological skeleton of the original abelian categories. To be slightly more precise: they are homotopy categories of complexes of objects, localized at those morphisms which induce isomorphisms on the level of homology, part of the information they carry being encoded in certain distinguished triangles of objects and morphisms. Subsequent work of E. Cline, B. Parshall, and L. Scott [12] represents a push toward completing the picture along Morita's original model by establishing a first installment of necessary and sufficient conditions for the occurrence of certain types of equivalences between derived categories of module categories. Finally, this line of insights was topped off by J. Rickard [42] in the late 1980s. On the level of the original module categories, Rickard completely characterized those equivalences of their derived categories which take distinguished triangles to distinguished triangles. He thus established—in full generality—a Morita theory for derived categories of module categories.

Throughout his career, Morita kept returning to the interplay between category theory and algebra. In fact, one of the cornerstones of the theory of derived categories, namely, the technique of forming quotient categories modulo localizing subcategories first systematically explored by Gabriel [16], was thoroughly pursued by Morita throughout the 1970s [30, 31, 32, 36].

In view of this development and others of similar impact, respect for Morita grows. His work emerges as not only supplying immensely useful results, but as strongly contributing to our present mode of thinking about algebraic and geometric structures within categorical settings.

Morita's Contributions to Topology

Morita belongs to the third generation of general topologists, with Hausdorff, Vietoris, Alexandroff, Urysohn, Kuratowski, Sierpinski, and R. L. Moore making up the first, and Tychonoff, Čech, Hurewicz, and Tumarkin belonging to the second. Morita's first topological paper appeared in 1940 [24], which means that he started his research in general topology at roughly the same time as E. Hewitt, R. Bing, M. G. Katetov, H. Dowker, and A. H. Stone.

He contributed to all major directions within general topology, some of his most important topological work being on normality, paracompactness, classification of spaces by mappings, dimension theory, homotopy theory, and shape theory. However, behind the great variety of topics Morita considered, there was also in his topological work the idea of a conceptually unifying approach via category theory, an idea that was still new at the time. This is reflected, for example, in his interest in the stability of properties under formation of direct products. On the other hand, next to his fascination with general schemes, Morita was also a brilliant problem solver; this is witnessed by his numerous deep concrete results in dimension theory, for instance.

Let us mention explicitly some of Morita's achievements in topology. His most significant contributions to the subject can be grouped under the following three captions: paracompactness and normality, dimension theory, and shape theory. We will provide a few highlights for each of these.

Under the first heading he proved, in particular, that every (regular) Lindelöf space is paracompact, generalizing an earlier result of J. Dieudonné, who introduced the fundamental notion of paracompactness in 1944 and proved that every separable metrizable space is paracompact [13]. Moreover, Morita discovered an unexpected link between paracompactness and normality in terms of the product operation: A space X is paracompact if and only if the product of X with any (with some) compactification of X is normal (this was also proved by H. Tamano [48]). While in general paracompactness is not preserved in products, Morita established that the product of a paracompact Hausdorff space and a σ-locally compact, paracompact Hausdorff space is paracompact [28].

The following three problems, posed by Morita in [35] and later circulated as *Morita's Conjectures*, strongly influenced the development of the field.

1. *If $X \times Y$ is normal for any normal space Y, is X necessarily discrete?*

2. *If $X \times Y$ is normal for every normal P-space Y, must X be metrizable?*

3. *Given that $X \times Y$ is normal for every normal countably paracompact space Y, does it follow that X is metrizable and σ-locally compact?*

In fact, it was Morita who introduced the concept of a "normal P-space" X and characterized it by the condition that the product of X with every metrizable space is normal [29]. This topic greatly gained in importance and popularity after M. E. Rudin constructed her famous example of a normal space, the product of which with the closed unit interval fails to be normal [45]. A positive answer to Morita's first problem follows from results of M. Atsuji [3] and M. E. Rudin [46], while Z. Balogh [5] recently confirmed the third conjecture. The second conjecture finally has been answered in the affirmative under the additional set-theoretic assumption that $V = L$ (Chiba-Przymusinski-Rudin [11]).

Another important notion that was introduced by Morita in the context of normality of product spaces strongly influenced the subject, namely, that of an "M-space". Within the class of paracompact spaces, the M-spaces coincide with the p spaces introduced independently by Arhangel′skii [2]; they are characterized by the existence of perfect mappings onto metrizable spaces. Among the many excellent papers dealing with this topic, one finds J. Nagata's beautiful characterization: X is a paracompact M-space if and only if X is homeomorphic to a closed subspace of the product of a metric space and a compact Hausdorff space [41].

Morita's contributions to dimension theory constitute one of the most important parts of his scientific inheritance. It is well known that there are three basic topological definitions of dimension, based on different structures: small inductive (ind), large inductive (Ind), and covering (dim) dimensions. One of the central problems in the theory is that of establishing relationships among these three invariants. It is well known that they coincide for separable metrizable spaces (Tumarkin and Hurewicz), while the two inductive dimensions need not coincide for nonseparable metrizable spaces. On the other hand, Morita [25] and M. Katetov [20] independently established that $\dim(X) = \mathrm{Ind}(X)$ for arbitrary metrizable spaces X. Moreover, Morita obtained the fundamental inequality $\dim(X) \leq \mathrm{ind}(X)$ in case X is a Lindelöf space (see [14] for a discussion).

In addition to this comparison of different dimensions of a given space, Morita proved the following two mapping results for Ind which play a pivotal role in dimension theory (see [1] and [40] for references):

1. *If f is a closed continuous mapping of a metrizable space X onto a metrizable space Y and $|f^{-1}(y)| \leq k+1$ for each y in Y, then* $\mathrm{Ind}(Y) \leq \mathrm{Ind}(X) + k$.

2. *If f is a closed continuous mapping of a nonempty metrizable space X into a metrizable space Y and $\mathrm{Ind}(f^{-1}(y)) \leq k$ for each $y \in Y$, then* $\mathrm{Ind}(X) \leq \mathrm{Ind}(Y) + k$ [26]. (This latter implication was independently obtained by K. Nagami; see [40].)

Under the heading of shape theory, let us briefly touch on the background of this field. Shape theory for compact metric spaces was founded by K. Borsuk in 1968 (see [21] for the history). Subsequently, the theory was extended to compact Hausdorff spaces and arbitrary metric spaces by S. Mardešich and J. Segal [22] and R. H. Fox [15], respectively. Finally, Mardešich [21] and Morita [34, 37] independently pushed it to the level of general spaces. Morita's approach is based on P. S. Alexandroff's idea of the nerve of a locally finite covering of a space. He assembled the nerves of all locally finite normal coverings of a space and formed an inverse system in the homotopy category of CW-complexes. This approach exhibits a fruitful interaction among methods of set theory, category theory, and algebraic topology. In particular, this approach led Morita to a shape-theoretic version of the Whitehead theorem within the homotopy theory of CW-complexes [33].

Probably more than half of the contemporary generation of Japanese topologists consists of indirect students of Morita, in the sense that they are using methods he developed and solving problems he posed. But his influence goes far beyond Japan, being particularly strong in the U.S. and in the former Soviet Union, where major schools in general topology can be found. Undoubtedly, he is now considered worldwide as one of the great founders of modern general topology, and his work serves as a continuing source of inspiration.

One of the authors of this article (A.V.A.) had the honour to be a guest of Professor Morita in his home in Japan. He remembers him as a very warm, hospitable person, with a lot of humour and benevolence, surrounded by several members of his family—his wife, Tomiko, and son, Jusahiro (a top manager in the Hitachi company). Even after the ten years that have passed, the memory of this visit remains fresh.

References

[1] P. S. Alexandrof and B. A. Pasynkov, *Introduction to dimension theory* (in Russian), Moscow, 1973.

[2] A. V. Arhangel′skii, *On a class of spaces containing all metric and all locally compact spaces*, Soviet Math. Dokl. **4** (1963), 1051–1055.

[3] M. Atsuji, *On normality of the product of two spaces*, General Topology and Its Relations to Modern Analysis and Algebra (Proc. Fourth Prague

Topology Sympos., Prague, 1976), Part B, 1977, pp. 25–27.
[4] G. AZUMAYA, *A duality theory for injective modules*, Amer. J. Math. **81** (1959), 249–278.
[5] Z. T. BALOGH, *Nonshrinking covers and K. Morita's third conjecture* (abstract), 1995 Annual Spring Topology Conference, 1995, p. 4.
[6] H. BASS, *The Morita theorems*, Mimeographed notes, University of Oregon, 1962.
[7] I. N. BERNSTEIN, I. M. GEL'FAND, and V. A. PONOMAREV, *Coxeter functors and Gabriel's theorem*, Russian Math. Surveys **28:2** (1973), 17–32.
[8] K. BONGARTZ, *Tilted algebras*, Representations of Algebras (Puebla, 1980) (M. Auslander and E. Lluis, eds.), Lecture Notes in Math., vol. 903, Springer-Verlag, Berlin, 1981, pp. 26–38.
[9] S. BRENNER and M. C. R. BUTLER, *Generalizations of the Bernstein-Gelfand-Ponomarev reflection functors*, Representation Theory II (Ottawa, 1979) (V. Dlab and P. Gabriel, eds.), Lecture Notes in Math., vol. 832, Springer-Verlag, Berlin, 1980, pp. 103–169.
[10] H. CARTAN and S. EILENBERG, *Homological algebra*, Princeton Univ. Press, Princeton, NJ, 1956.
[11] K. CHIBA, J. C. PRZYMUSINSKI, and M. E. RUDIN, *Normality of product spaces and Morita's conjectures*, Topology Appl. **22** (1986), 19–32.
[12] E. CLINE, B. PARSHALL, and L. SCOTT, *Derived categories and Morita theory*, J. Algebra **104** (1986), 397–409.
[13] J. DIEUDONNÉ, *Une generalisation des espaces compacts*, J. Math. Pures Appl. **23** (1944), 65–76.
[14] R. ENGELKING, *General topology*, Polish Scientific Publishers, Warsaw, 1977.
[15] R. H. FOX, *On shape*, Fund. Math. **74** (1972), 47–71; **75** (1972), 85.
[16] P. GABRIEL, *Des categories abeliennes*, Bull. Soc. Math. France **90** (1962), 323–448.
[17] N. GRØNBÆK, *Morita equivalence for Banach algebras*, J. Pure Appl. Algebra **99** (1995), 183–219.
[18] D. HAPPEL, *Triangulated categories in the representation theory of finite dimensional algebras*, London Math. Soc. Lecture Notes, no. 119, Cambridge Univ. Press, Cambridge, 1988.
[19] D. HAPPEL and C. M. RINGEL, *Tilted algebras*, Trans. Amer. Math. Soc. **274** (1982), 399–443.
[20] M. G. KATETOV, *On the dimension of non-separable spaces, 1*, Czech. Math. J. **2** (1952), 333–368.
[21] S. MARDEŠICH, *Shapes for topological spaces*, Gen. Topology Appl. **3** (1973), 265–282.
[22] S. MARDEŠICH and J. SEGAL, *Shapes of compacta and ANR-systems*, Fund. Math. **72** (1971), 41–59.
[23] Y. MIYASHITA, *Tilting modules of finite projective dimension*, Math. Zeitschr. **193** (1986), 113–146.
[24] K. MORITA, *On uniform spaces and the dimension of compact spaces*, Proc. Phys.-Math. Soc. Japan **22** (1940), 969–977.
[25] ____, *Normal families and dimension theory for metric spaces*, Math. Ann. **128** (1954), 350–362.
[26] ____, *On closed mappings and dimension*, Proc. Japan Acad. **32** (1956), 161–165.
[27] ____, *Duality for modules and its applications to the theory of rings with minimum condition*, Sci. Rep. Tokyo Kyoiku Daigaku, Sec. A **6** (1958), 83–142.
[28] ____, *On the product of a normal space with a metric space*, Proc. Japan Acad. **39** (1963), 148–150.
[29] ____, *Products of normal spaces with metric spaces*, Math. Ann. **154** (1964), 365–382.
[30] ____, *Localizations in categories of modules. I*, Math. Zeitschr. **114** (1970), 121–144.
[31] ____, *Localizations in categories of modules. II*, J. Reine Angew. Math. **242** (1970), 163–169.
[32] ____, *Localizations in categories of modules. III*, Math. Zeitschr. **119** (1971), 313–320.
[33] ____, *The Whitehead theorem in shape theory*, Proc. Japan Acad. **50** (1974), 458–461.
[34] ____, *On shapes of topological spaces*, Fund. Math. **86** (1975), 251–259.
[35] ____, *Some problems on normality of product spaces*, General Topology and Its Relation to Modern Analysis and Algebra (Proc. Fourth Prague Topological Sympos., Prague, 1976), Part B, Soc. Czechoslovak Mathematicians and Physicists, Prague, 1977, 296–297.
[36] ____, *Localizations in categories of modules. IV*, Sci. Rep. Tokyo Kyoiku Daigaku, Sec. A **13** (1977), 153–164.
[37] ____, *Resolutions of spaces and proper inverse systems in shape theory*, Fund. Math. **124** (1984), 263–270.
[38] K. MORITA, Y. KAWADA, and H. TACHIKAWA, *On injective modules*, Math. Zeitschr. **68** (1957), 217–226.
[39] K. MORITA and H. TACHIKAWA, *Character modules, submodules of a free module, and quasi-Frobenius rings*, Math. Zeitschr. **65** (1956), 414–428.
[40] J. NAGATA, *Modern dimension theory*, Groningen, 1965.
[41] ____, *A note on M-spaces and topologically complete spaces*, Proc. Japan Acad. **45** (1969), 541–543.
[42] J. RICKARD, *Morita theory for derived categories*, J. London Math. Soc. (2) **39** (1989), 436–456.
[43] M. A. RIEFFEL, *Morita equivalence for C^*-algebras and W^*-algebras*, J. Pure Appl. Algebra **5** (1974), 51–96.
[44] ____, *Morita equivalence for operator algebras*, Proc. Sympos. Pure Math. **38** (1982), 285–298.
[45] M. E. RUDIN, *A normal space X for which $X \times I$ is not normal*, Fund. Math. **73** (1971), 179–186.
[46] ____, *κ-Dowker spaces*, Czech. Math. J. **28(103)** (1978), 324–326.
[47] H. TACHIKAWA, *Duality theorem of character modules for rings with minimum condition*, Math. Zeitschr. **68** (1958), 479–487.
[48] H. TAMANO, *On compactifications*, J. Math. Kyoto Univ. **1** (1962), 161–193.
[49] J.-L. VERDIER, *Categories derivees, quelques resultats (Etat 0)*, Inst. Hautes Études Sci. Publ. Math. (1963); publ. in Seminaire de Geometrie Algebrique du Bois-Marie SGA $4\frac{1}{2}$, Cohomologie Etale, Lecture Notes in Math., vol. 569, Springer-Verlag, Berlin, 1977, pp. 262–311.
[50] P. XU, *Morita equivalence of Poisson manifolds*, Comm. Math. Phys. **142** (1991), 493–509.