Ennio De Giorgi (1928–1996)

Jacques-Louis Lions and François Murat

Ennio De Giorgi died in Pisa on October 25, 1996.

E. De Giorgi was born in 1928 in Lecce, a city of Puglia (southern Italy) with which he always maintained deep ties. He received his *laurea*, the diploma which marks the end of the Italian undergraduate curriculum, in Rome in 1950. He began his research work in 1951 at the Istituto per le Applicazioni del Calcolo (Institute for the Applications of Calculus) in Rome. The Institute was then directed by Mauro Picone, and E. De Giorgi became one of his assistants. Professor and student were indeed an odd match: the former, a classicist, dressed with rigor and elegance; the latter, unorthodox, already wearing his strange beret. But M. Picone, a seasoned observer of the development of science, knew how to spot talent: he soon acknowledged E. De Giorgi's exceptional abilities. The assistant was freed from all constraints and worked as he pleased at his own leisurely but, in the end, frightfully efficient rhythm.

His attention was first attracted to problems of the calculus of variations. He began with the problem of minimal surfaces, for which he obtained important results as early as 1954. His views of geometric measure theory were already original when he attended a lecture by Renato Caccioppoli in Rome. Unabashed by R. Caccioppoli's great fame, E. De Giorgi did not hesitate to speak up at the end of the lecture and to suggest alternative solutions. According to Eduardo Vesentini, who also was a young researcher at the time and who was attending the lecture, R. Caccioppoli instantly recognized the exceptional character of E. De Giorgi's proposal.

The word "exceptional" comes up again and again when talking with anyone who had the opportunity of meeting E. De Giorgi. Let his work speak for itself.

Minimal Surfaces and Geometric Measure Theory

From the very beginning of his research activity, E. De Giorgi was interested in geometric measure theory. He gave a rigorous definition of the perimeter of a Borel set and applied this concept to the study of minimal surfaces, generalizing in particular to n dimensions a classical theorem of Bernstein: if $n \leq 8$, the only complete minimal graphs in R^n are hyperplanes. In 1969, in collaboration with E. Bombieri and E. Giusti, he showed that this result is false for $n \geq 9$.

In the 1980s he returned to applications of geometric measure theory to the calculus of variations. He introduced the space SBV of "special" functions of bounded variation, i.e., functions the distributional derivatives of which are measures consisting only of a part which is absolutely continuous with respect to the Lebesgue measure and an $(n-1)$-dimensional measure concentrated on the jump set of the function. He used this space to study problems of the calculus of variations "with free discontinuities" (in the sense that the

Jacques-Louis Lions is professor at the Collège de France. François Murat is directeur de recherches at the CNRS, Université Paris VI.

Translation of an article in French published in Matapli, *No. 49, January 1997, pp. 15–17, and in* Gazette des Mathématiciens, *No. 71, January 1997, pp. 31–34.*

discontinuity set is not fixed a priori), such as problems of image segmentation. In 1989 he proved, in collaboration with M. Carriero and A. Leaci, the existence of a minimizer for the Mumford-Shah functional through the solving of a weak formulation of the problem in SBV.

More recently he developed a general theory of motion of a surface according to its mean curvature and formulated a collection of conjectures in the field. He also formulated a general Plateau problem in metric spaces of finite or infinite dimension.

Regularity of Solutions of Elliptic Equations

In 1956 E. De Giorgi proved that every solution of a scalar elliptic equation of second order in divergence form with bounded coefficients is Hölder continuous. This theorem, known as "De Giorgi's theorem", is the crucial step to solve Hilbert's nineteenth problem, which consists in showing that a function which minimizes a convex integral functional of the calculus of variations is analytic if the functional is analytic. This result and its proof have had a considerable influence on the study of the regularity of solutions of elliptic equations.

In 1968 E. De Giorgi gave an example that shows that this regularity does not extend to systems.

General Theory of Partial Differential Equations

In 1955 E. De Giorgi gave the first example of nonuniqueness for the Cauchy problem for linear partial differential equations of hyperbolic type with regular coefficients. In 1971 he proved, in collaboration with L. Cattabriga, the existence of solutions to linear elliptic equations with constant coefficients and analytic right-hand side, which are analytic in the whole plane.

In 1979 he proved, in collaboration with F. Colombini and S. Spagnolo, that the Cauchy problem for hyperbolic equations with coefficients which are not regular in the time variable is well-posed in Gevrey spaces.

Γ-Convergence

In 1973, in collaboration with S. Spagnolo, and later in 1975 in an article entitled "Sulla convergenza di alcune successioni di integrali del tipo dell'area" (On the convergence of some sequences of area-type integrals, Rend. Mat. (6) **8** (1975), 277–294). E. De Giorgi introduced a new notion of convergence for functionals. Γ-convergence, as he called it, provides an easy-to-use necessary and sufficient condition for the convergence of minimizers and values of the corresponding minimization problems. This concept has proved extremely fruitful, as witnessed by the hundreds of papers which make use of it.

Logic and the Foundations of Mathematics

Besides his work in analysis, E. De Giorgi became involved in the mid-1980s in logic and the foundations of mathematics. He developed a self-reference-oriented theory of the foundations of mathematics which tries to reconcile the hierarchical principles of the classical set theories with the wide self-referential power of natural languages.

The exceptional quality of the ideas and of the mathematical work of E. De Giorgi has been recognized through numerous Italian as well as international prizes and distinctions. He received the National Prize of the President of the Italian Republic in 1973, an Honoris Causa doctorate from the Universities of Paris in 1983, and the Wolf Prize in 1990. He was a member of the Accademia Nazionale dei Lincei, of the Pontifical Academy of Sciences, of the Accademia dei XL, of the Academy of Sciences of Turin, of the Lombard Institute of Sciences and Letters, and of the Ligurian Academy. More recently he was named a foreign member of the Academy of Sciences of Paris and of the National Academy of Sciences of the U.S.A.

In 1959, after a year as full professor in Messina, E. De Giorgi was appointed full professor at the Scuola Normale Superiore di Pisa at the age of thirty-one. For almost forty years, he lived there, taught there, and was a constant source of inspiration for the mathematical school that he founded. Always cheerful, always available, he enjoyed long debates with his students during which he would toss out original ideas and propose conjectures, often interrupting himself to read the newspaper before returning to the discussion, proposing new conjectures or sketching the lines of a proof. He attracted many students, young and not so young, not only from the Scuola Normale, but from all of Italy and from abroad. Well known throughout the world, his school has had a deep influence on mathematics.

Ennio De Giorgi was a mathematician of exceptional creativity. An original mind, an authentic believer in God, and a man with an innate sense of humanity, he enjoyed sharing his thinking on the connections between Mathematics and Wisdom (in the meaning of the Bible, which he often quoted). A passionate advocate of human rights, he was in particular an active member of Amnesty International. His memory will stay alive, not only because of his exceptional mathematical work, but also because of his exceptional human qualities, which have left their imprint on all those who have had the good fortune to know him.