Book Review

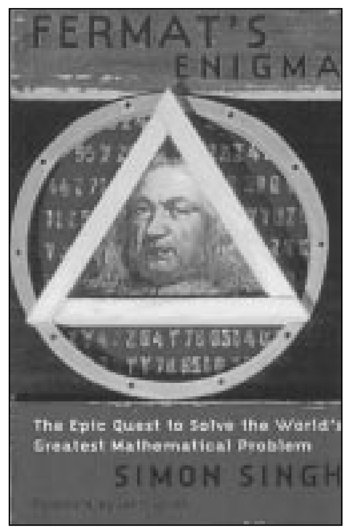
Fermat's Enigma

Reviewed by Allyn Jackson

Fermat's Enigma: The Epic Quest to Solve the World's Greatest Mathematical Problem
Simon Singh
Walker and Company, New York
$22.00 hardcover
288 pages

Despite the increased interest in Fermat's Last Theorem since Andrew Wiles announced his proof in 1993, there have been few popular books on the subject. In the months immediately following his announcement, one book capitalized on the moment: *The World's Most Famous Math Problem*, by newspaper columnist Marilyn vos Savant. That book suffered from many problems, the worst being a woefully wrong-headed attempt to discredit Wiles's proof. The only other popular book to appear in the U.S. was fortunately much more serious. *Fermat's Last Theorem: Unlocking the Secret of an Ancient Mathematical Problem* by Amir D. Aczel, an associate professor of statistics at Bentley College, was published in 1996 by Four Walls Eight Windows. The book received favorable reviews in the popular press (for example, see the *New York Times* review, available at `http://www.nytimes.com/books/home/`) and was for a short time distributed by the AMS. However, complaints about some mathematical inaccuracies in the book led the Society to stop selling it. (A review of this book will appear in a future issue of the *Notices*.)

Another popular book about Fermat will appear in bookstores this month. On October 28 Walker Books will publish *Fermat's Enigma: The Epic Quest to Solve the World's Greatest Mathematical Problem* by Simon Singh. The publication is timed to coincide with the broadcast in the United States of *The Proof*, a BBC documentary



about Fermat's Last Theorem directed by Singh. (Those interested in seeing the program should consult their local public television stations for broadcast times or check the Public Broadcasting System Web site, `http://www.pbs.org/`. The program was reviewed in the *Notices* by Andrew Granville, January 1997, pages 26–28.)

Wiles's proof, which makes use of some of the deepest and most technically difficult mathematics of the twentieth century, presents a formidable challenge to any nonexpert who would write about it. Singh, who has a Ph.D. in particle physics from Cambridge University, has done an admirable job with an extremely difficult subject. He has also done mathematics a great service by conveying the passion and drama that have carried Fermat's Last Theorem aloft as the most celebrated mathematics problem of the last four centuries. The book landed in the #1 spot on the bestseller list of the *The Times* of London, proving that "useless" mathematics can have a primal fascination for people.

The book begins with a brief look at that historic day, June 23, 1993, when Wiles delivered the last of his three lectures about the proof at the Isaac

Newton Institute in Cambridge, England. He concluded by writing Fermat's famous statement on the blackboard and saying, "I think I'll stop here"; those words provide the title for the first chapter. Singh then largely leaves Wiles behind and goes into five chapters' worth of history about Fermat's Last Theorem. His discussion of the life and work of Pythagoras makes for absorbing reading and offers the opportunity to fix in the reader's mind the idea of proof, using simple examples such as the Pythagorean theorem. Singh has a knack for ferreting out interesting historical tidbits and portraying colorful personalities. However, at times the book is unclear about what is fact and what is not. For example, Singh writes that there are no first-hand accounts of Pythagoras's life and work, and yet a few pages later one finds what is purported to be a direct quotation of Pythagoras. A sentence about the origin of the quotation would have been helpful.

There are other problematic moments in the book. In discussing the deep connection that the Shimura-Taniyama conjecture proposed between the world of elliptic curves and that of modular forms, Singh comments on the way that such "bridges" in mathematics help mathematicians in different areas to share insights. "Mathematics consists of islands of knowledge in a sea of ignorance," as he eloquently puts it. Caught up in the imagery, he carries it too far: "The language of geometry is quite different from the language of probability, and the slang of calculus is meaningless to those who speak only statistics." Not only does this statement err in implying that statisticians do not know calculus, it also implies that calculus and geometry are distinct fields. Under some interpretations, geometry could be said to encompass calculus.

Such small problems occur throughout the book, but Singh is such an enthusiastic guide that it is easy to forgive them. The best parts of the book intertwine history, personalities, and mathematical ideas. Especially effective is the discussion of Euler's work with complex numbers, which brings a sense of naturalness and inevitability to an idea that can seem strange and arbitrary. Singh is equally effective in getting across the difficult notion of different "sizes" of infinity, especially in his appeal to the device known as "Hilbert's Hotel". Some of the digressions—such as the description of public-key cryptography and Gödel's work on undecidable statements—demonstrate the way that mathematics has influenced nearly every aspect of human endeavor.

The book handles elementary mathematical ideas well, but becomes increasingly vague as the material becomes more sophisticated. When it comes to elliptic curves, Singh does a good job getting at the idea of the L-series (which is defined in terms of the number of solutions to the elliptic equation mod p, for every prime p), though I think it is not especially helpful that he decides to call it the E-series instead. (One needs to keep in mind that the series discussed in the book are power series, although the book treats them essentially as numerical sequences consisting of the coefficients of the power series.) He has found a wonderful way to communicate the importance of this series: "In the same way that biological DNA carries all the information required to construct a living organism, the E-series carries the essence of the elliptic equation."

Not surprisingly, such compelling imagery gets progressively rare as the book wades into the deeper waters of Galois theory, modular forms, Iwasawa theory, and the other ingredients of Wiles's proof. Despite a spectacular description of the life of Galois and an attempt to describe his work, the book leaves the reader with little understanding of the power and elegance of Galois theory as it blossomed after its creator's death, and there is little indication of the role it played in Wiles's work. Modular forms are a struggle: Singh talks about their "inordinate level of symmetry," but never gets sufficiently specific and vivid to give the reader something to carry in mind for the rest of the book. He creates more of his own terminology for modular forms: the series that defines a modular form, which mathematicians might call the Dirichlet series or the L-series of the form, Singh dubs the M-series. The book does not say much about this series, except to say that it is the list of "ingredients" of the modular form and to liken it also to DNA.

Despite the difficulty of explaining these technical points, the discussion of the roots of the Shimura-Taniyama conjecture is the best part of the book. Yutaka Taniyama's assertion of a connection between modular forms and elliptic curves perplexed mathematicians because it was so far ahead of its time. After Taniyama's tragic death in 1958, Goro Shimura took Taniyama's brilliant but unfinished ideas and built them into what is now generally known as the Shimura-Taniyama conjecture, which says that the L-series of an elliptic curve can be paired with a modular form. This story has more poignant drama than many of the other historical tales in the book: Singh clearly benefited from having Shimura as a primary source. Indeed, from that point on many present-day characters—Ken Ribet, Barry Mazur, John H. Conway, John Coates, and of course Wiles himself—feature prominently.

As the book explains it, Wiles's strategy for proving the Shimura-Taniyama conjecture is to use induction on both the set of elliptic curves and on the set of modular forms—a sort of "double" induction. There is a clever discussion of proof by induction, which is likened to toppling dominoes by knocking over the first one and then proving that if the nth one goes over, all the rest will follow. According to the book, Wiles matched the first elements of each of these series and then later was able to show by induction that the rest of the "dominoes" would fall. From what I understand of the proof, this is not quite how it goes. Perhaps Singh has presented a reasonable rendition of the proof for a popular work such as this; it would take a reader more expert than I to say for sure. The image of toppling dominoes is useful for explaining how Wiles juggled his infinite sets, but I thought it was repeated a few too many times. Eventually it became a substitute for directly addressing complicated and technical mathematical ideas.

The back cover of the review copy states that Singh had "more access to Andrew Wiles than any other journalist," so I was curious about what Singh discovered about the personality of this very private man. The portrait of Wiles that emerges is admiring but not altogether positive. While he was working in secrecy, the book says, Wiles devised a "cunning ploy" of slowly publishing some earlier work in order to make it seem as if he were maintaining an ordinary level of productivity. "As long as he could maintain this charade," the book says, "Wiles could continue working on his true obsession without revealing any of his breakthroughs." Singh also attributes Wiles's secrecy in part to a "craving for glory." This craving must operate in anyone who attempts to prove Fermat, but it does seem at odds with the quotes from Wiles, which indicate that he shrouded his work in secrecy simply because he knew that isolating himself was the only way he could summon the concentration he needed. Wiles also speaks of his battle with Fermat as a very personal one and even expresses mixed emotions about finally letting the world see the fruit of his labor. "I got so wrapped up in the problem that I really felt I had it all to myself, but now I was letting go," he is quoted as saying. "There was a feeling that I was giving up a part of me."

Singh is an unpretentious writer with a true appreciation for the beauty of mathematics and for the passion mathematicians have for their work. His book has an awed enthusiasm about it that makes it an appealing read. He is writing all the way out to the very brink of his understanding of this extremely technical subject, and for this one cannot help but to admire the book. Nevertheless, there remains a place on bookstore shelves for a popular work that would more squarely face the technical thickets of the proof of Fermat's Last Theorem and provide lay readers with a deeper understanding of what it's all about.