Calculus Reform—For the $Millions

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Professor Mumford's arguments in "Calculus Reform—For the Millions" (May 1997 *Notices*) are typical of those who support calculus reform. They create a "straw man"— a fictitious model of a calculus course which they claim has played a significant role in our math education problems—and then give the generic solution: wholesale elimination of proofs and definitions.

Professor Mumford, with perhaps intentional hyperbole, ends his article with a plea not to alienate the 99 percent of students who will not need to see rigorous proofs for their future careers. At our school, "business calculus" and "calculus for biology majors" have far more students than our calculus sequence for math, engineering, and physical science majors. Traditional texts for the former two courses have never emphasized rigor, and the word problems are tailored for the respective majors. From conversations with colleagues at other institutions, we surmise this is also the case nationally. Professor Mumford need not worry about overexposure to proofs for these students, with or without "reformed" calculus. Overexposure never existed.

The main battle is whether reform approaches—such as Harvard Calculus—that eliminate virtually all rigor should be used for math, engineering, and physical science majors, as is being done at many colleges and universities. Reform advocates typically justify this with unsupported characterizations of traditional texts as consisting of "drill and kill" and/or an incomprehensible chain of rigorous proofs. But, for this group of students, who ever said that their first collegiate exposure to calculus should be inundated with deltas and epsilons? When reform fails, as it often does, reformers claim that the reform pedagogies were not properly implemented or that teachers lack sufficient knowledge. But it is axiomatic that no such explanations may be given for traditional approaches. Multimillion-dollar grants would collapse without this axiom.

The calculus reform movement was a facile response to a real problem: the declining performance of American students in college calculus courses. Without any credible scientific study, many reformers put forth "solutions" such as eliminating theory, making more conceptual problems, decreasing the reliance on algebra, and increasing the use of computers and calculators. But what if the reformers were wrong about the causes of the problem?

The traditional calculus books we used as students and have taught out of as professors contained a fairly even mix of computation, conceptualization, and theory. Most math professors taught using similar mixtures. Useful topics such as related rates, partial fractions, L'Hopital's rule, infinite series, and many others are abridged or missing from the Harvard Calculus text. The "proofs" and "definitions" in the Harvard Calculus text are often incorrect. This means that Harvard Calculus is virtually useless as a reference for later courses. The traditional books have correct proofs and definitions. This does not mean that the courses using these books cover all the rigorous material. Every calculus teacher we know avoids trying to cover all material available in a traditional calculus book.

Traditional texts give the student the opportunity to delve more deeply into the subject if (s)he chooses and to "look up" formulas and their justifications later in life as the need arises. The Harvard Calculus text gives less opportunity for motivated students to look further on their own. Worse still, it mandates "knowing things" that are not even true. If there is lack of balance in this debate, it is clearly on the reform side of the fence.

We agree with Professor Mumford's explicit support for learning the rules of algebra and the multiplication tables as "the rules of a game." Many reform advocates do not. Even the memorization of the multiplication tables is a debatable issue in education circles these days [4]. Harvard Calculus ac-

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commodates these trends by minimizing the need for algebra. This phenomenon is not surprising and could even be predicted by simply observing the impact of reform math at the K–12 level. Widely used K–12 reform pedagogies do not systematically develop what used to be high school algebra. This creates powerful economic and cultural pressures to lower standards for new students.

Rather than succumb to these destructive trends, university math departments should maintain realistic standards, even if it means high failure rates for incoming students. Universities are the last line of defense against the reform-inspired corrosion of standards. This is the only real way to send the message to the public schools—and equally importantly, the nation's colleges of education—that high school algebra must be learned and cannot be supplanted by button pressing of calculators and computers.

Mathematics has the unique distinction of being the only subject which is expected to apologize for being what it is. Imagine a large proportion of history professors pointing out that very few people will ever need to know even approximate dates for important historical events. Imagine these history professors then calling for the elimination of *all* dates in history classes. It would be virtually impossible to have an intelligent historical discussion without such basic information. Professor Mumford and many others ask how we can justify teaching logical deduction and proofs in math classes when so few people will need that in future employment. Professor Mumford does concede that it is useful for future lawyers, and considering the huge number of attorneys in the U.S., this is a major concession. Abraham Lincoln, once an attorney, wrote of himself in his *Short Autobiography*:

> He studied and nearly mastered the six books of Euclid since he was a member of Congress.
>
> He began a course of rigid mental discipline with the intent to improve his faculties, especially his powers of logic and language. Hence his fondness for Euclid, which he carried with him on the circuit till he could demonstrate with ease all the propositions in the six books…. [3]

What about nonmathematicians and nonattorneys? Surely others such as electrical engineers, physicists, chemists, and many others benefit in subtle ways from judicious exposure to mathematical proofs. Even popular accounts of science designed for the layperson require limited exposure to mathematical rigor. For example, a short introduction to relativity theory written by Einstein, widely available now in popular bookstores for the nonscientific reader, requires previous exposure to the logic of Euclidean geometry [2].

It is especially important for prospective high school math teachers to see some theory in calculus and to learn most of the important topics omitted by the Harvard Calculus text. An ominous trend in reform math is the over-use of calculators. Calculus students are being pushed into numerical solutions when easily derived closed-form solutions are better. Part of the problem is that many high school math teachers are not even aware of traditional topics.

One way to improve the mathematics performance of college students is to set higher standards for future teachers. Jaime Escalante (immortalized in the movie *Stand and Deliver*) demonstrated that a well-educated high school teacher who loves the subject can teach mathematics to large numbers of students [1]. It is no secret that his students were Hispanic and working class. Many went on to universities and successful careers. These successes stem from well-taught traditional calculus courses.

We agree with E. D. Hirsch's thesis [5] that the failures in mathematics education have been caused partly by the very reform programs that were supposed to improve student performance. In addition, we suggest that by minimizing arithmetic and algebra in K–12, leaders in mathematics education have underestimated the innate abilities of the nation's youth. Students are meeting these low expectations. It will take years to undo the damage caused by the math reform movement, all the more so when prominent mathematicians maintain the fiction that the hard work required by traditional curricula can be successfully avoided.

We are sure that Professor Mumford and all reformers would not want to deny minority students and working-class white kids the type of math education that the best universities demand. Unfortunately, while the privileged few can find the means for genuine education, much of "reform" is closing the door on opportunities for those who depend the most on public education as a vehicle for success.

References

[1] *A math teacher's lessons in division*, The Washington Post, (May 21, 1997).

[2] ALBERT EINSTEIN, *Relativity: The special and general theory*, Crown Trade Paperbacks, New York, 1961.

[3] RICHARD L. FABER, *Foundations of Euclidean and non-Euclidean geometry*, Marcel Dekker, New York, 1983.

[4] *Formulas for Math Problems*, Los Angeles Times, Column One, (January 5, 1997). "One missionary in the Reform cause is consultant Ruth Parker, who rejects long division and multiplication tables as nonsensical leftovers from a pre-calculator age. She urges audiences to 'let kids play with numbers,' and they will figure out most any math concept. Parker has spoken before 20,000 people over the last six months at the behest of school districts."

[5] E. D. HIRSCH JR., *The schools we need; Why we don't have them*, DoubleDay, New York, 1996.