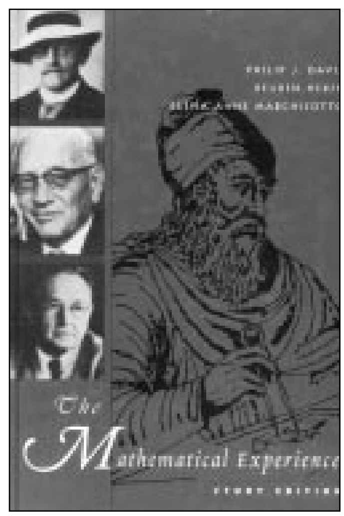
Book Review

The Mathematical Experience, Study Edition

Reviewed by Kenneth C. Millett

The Mathematical Experience, Study Edition
Philip J. Davis, Ruben Hersh, Elena Anne Marchisotto
Birkhäuser, Boston, 1995
487 pages, hardcover

In the fifteen years since the first edition of *The Mathematical Experience* there have been many important efforts to enlarge the public understanding of and support for contemporary mathematics. Despite these efforts little if any progress is apparent. Why? First, the effectiveness of many mathematics teachers, including college and university mathematicians, in advancing this goal of a wider understanding has been limited by an understanding and experience of mathematics that is far too narrow in scope. These limitations are passed on to their students, who quickly become the next generation of teachers, corporate and public leaders, and parents. One result is an educational system that is not preparing students for the mathematical challenges that they encounter. A second consequence is that even "the experts", mathematicians employed within and outside education, are unprepared for the wide range of educational and mathematical problems on which the public expects them to provide expert advice. For many people, including far too many teachers, mathematics is more a matter of memorization and mimicry than one of thinking, creating, or puzzling out quantitative meaning. *The Mathematical Experience* is an effort to provide a robust encounter with a wide range of mathematics from which a stronger understanding and appreciation can grow.



In this new edition Elena Marchisotto has joined Philip Davis and Ruben Hersh in, quoting from Gian-Carlo Rota's introduction, "a treacherous navigation between the Scylla of professional contempt and the Charybdis of public misunderstanding." They have tried to provide a book usable in a course for liberal arts students and for future secondary teachers. They have done much more! This course should be required of every undergraduate major employing the mathematical sciences. It differs from the "mathematics appreciation" courses—courses that are merely a collection of amusing puzzles and toy problems giving an illusion of a mathematical encounter—presently found in many institutions. Students of this course are introduced to the context in which mathematics exists and the incredible magnitude of words devoted to communicating mathematics (hundreds of thousands of theorems each year). How much mathematics can there be? they are asked. Instructors in a *Mathematical Experience* course must

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be prepared to respond to questions from students concerning the fundamental nature of the whole mathematical enterprise. Stimulated by their reading of the text, students will ask about the underlying logical and philosophical issues, the role of mathematical methods and their origins, the substance of contemporary mathematical advances, the meaning of rigor and proof in mathematics, the role of computational mathematics, and issues of teaching and learning. How real is the conflict between "pure" mathematics, as represented by G. H. Hardy's statements, and "applied" mathematics? they may ask. Are there other kinds of mathematics, neither pure nor applied? This edition of the book provides a source of problems, collateral readings, references, essay and project assignments, and discussion guides for the course. I believe that it is likely that this course would be a challenge to many teachers and students alike, especially those teachers and students who are willing to follow their curiosity beyond the confines of this book and follow up on the many references that are provided. For example, students meet the mathematics of number mysticism, hermetic geometry, astrology, and religion. Few of us, I suspect, are well prepared to respond to our students' questions about the recent statements of mathematicians concerning coded predictions hidden in biblical texts.

However, there is much more that will significantly enrich the subsequent study of mathematics than connections to current recent newspaper and magazine articles. Discussions of the role and power of symbols, abstraction, generalization, normalization, existence, and proof are reflected in the titles of a chapter devoted to inner issues. To make concrete the varieties of mathematics, there are sections dedicated to finite simple groups, the prime number theorem, non-Euclidean geometry, non-Cantorian set theory, Fourier, and nonstandard analysis. These are later followed by a discussion of the Riemann hypothesis and four dimensions. In other chapters students look at the nature of teaching and learning from several perspectives that encourage them to consider critically the nature of the mathematical experience in their own classes. To further enrich their reflections on the nature of mathematics, they are introduced to Platonism, formalism, constructivism, and other "isms" relating to its foundation. What is the meaning of proof with today's use of the computer? they are asked. Why should I believe a computer? These are questions that concern mathematicians, and they should concern our students as well. I believe that mathematical, life, and physical science majors and engineering majors will have a far deeper and more useful understanding of mathematics and its role in their own areas of interest if they have had a first-year college course based upon a text such as *The Mathematical Experience*. They will approach other mathematics courses quite differently. The interlocking nature of mathematics as a single intellectual enterprise will become more visible. The traditional artificial barriers between analysis, geometry, algebra, statistics, and so on will shrink in contrast to the strong unifying principles that run through the mathematical sciences.

To be sure, some mathematicians may find *The Mathematical Experience* lacking. Choices have been made. "Where is the discussion of the quadratic formula?" some will ask. "The treatment of the Pythagorean Theorem and geometry, in general, is not sufficient," others might assert. "The book is inconsistent in viewpoint and contains errors that detract from it," they may say. Indeed, such concerns are legitimate. But the fact is that, while not perfect, this book does provide what is required to initiate students into a wider, more meaningful exploration of the world of mathematics. Most mathematicians seem to agree that to understand mathematics you must do mathematics, and indeed there are many opportunities to do just that in connection with each of the chapters.

This is a book about the human experience of mathematics, connecting with each person's own experience doing mathematics. However, as a collection of essays about mathematics from these different perspectives it is not entirely consistent. Asking for a definition or description of "mathematics" is comparable to asking physicists for a definition of "particle" or seeking the meaning of "love" from your neighbors. In the latter case, for example, you may hear biblical references, or the perspectives in Shakespeare, a quotation from "Peanuts" or "Calvin and Hobbes", or a verse from the Spice Girls' latest song. What we understand seems to depend on our individual experience and the experiences of others with whom we interact. Often what we understand is altered by how we say it and by how it is heard. Is not mathematics much the same? The authors state, "Most writers on the subject seem to agree that the typical working mathematician is a Platonist on weekdays and a formalist on Sundays." The substance of the mathematics appears to change with experience and depends on the person recounting the story. But it has an objective reality that is independent of the person. Alas, when precision is required, it is common to retreat to the formalist position that mathematics is only a created structure of axioms, definitions, and their consequences. For students of mathematics the lack of attention to and acknowledgment of these and other fundamental issues and questions leads to their anemic understanding of the science of mathematics.

In a chapter entitled "Mathematical Reality" there is a discussion of the human aspect of mathematical proof, the impact of computers (the four-color problem, the distribution of primes, the Riemann hypothesis), and the robustness of mathematical programs encompassing thousands of lines of code. What about extensive collaborative programs involving the work of many researchers (such as the classification of finite simple groups)? Examples such as these challenge mathematicians to provide more robust arguments supporting the notion that mathematical truth is eternal and immutable and omnipotent. Too often explanations are one dimensional. *The Mathematical Experience* describes mathematics as a complex multidimensional system whose nature and direction is no longer possible to confine or to predict. The challenges are enormous for an instructor faced with discussing the nature of computer results whose origin is a program or system that never stops and while still running is correcting, updating, or extending itself. Described to me by Louise Moser, a colleague in computer science, this concept of "eternal" programs or systems demands the consideration of calculations or computer studies whose structure changes over time. If the object of study does not conform to a simple fixed set of rules, how do we describe its correctness to our students? Such questions are a natural constituent of the discussions growing out of this chapter.

The authors' treatment of questions about teaching and learning is similarly challenging. They note that the first edition was published in 1981 prior to the publication of the "Curriculum and Evaluation Standards" by the National Council of Teachers of Mathematics in 1989. *The Mathematical Experience*'s attention to the "thinking" and "problem-solving" dimensions of mathematics is one example of how this book tried to raise some critical aspects of mathematics teaching before they were more widely popular. They feel that they were a bit out of sync with their time. But, referring to the NCTM Standards, in the present edition the authors note that, "To a large extent, they validated our enterprise."

In the mathematics education arena the authors try to expand the range of discussion, to provoke questions of educational goals, to stimulate reflection upon and expansion of the range of teaching strategies, and to encourage teachers to develop honest measures of the degree to which educational goals have been achieved in their classes. They challenge us to consider whether we have only to "transmit" mathematical information in a clear and complete manner or whether we have some responsibility to ensure that learning is actually taking place; what is our standard against which we measure student accomplishment or performance? Consider the question of "learning for whom?" Is there a responsibility to all students or only to "the mathematically able"? If the former, how might this influence interactions with students? Does "dumbing down the course" or "grade inflation" inevitably follow? In this book it is impossible to devote much space to these questions. This is a place where one might choose to supplement the book in a course. More likely, as in my own situation, it gives an opportunity to establish links to other courses devoted to these questions. I try to focus on the foundational paradigms that define an area and provide a solid bridge to further study. For students interested in our mathematics education program, a discussion of how students learn (in the context of a course) helps students raise their personal standards for mathematical performance and establishes a link to our introductory course on problem solving and mathematics teaching.

This is a people-centered book about mathematics, and as such it provides an opportunity to explore fundamental issues that are typically absent from the experience of most college and university students (as well as their teachers). This new edition provides an excellent initiation of students into some of the more challenging aspects of mathematics. It can help bring them from a vision of mathematics as arithmetic and memorization to an understanding of mathematics as an intellectually challenging and creative experience—one in which there are surprises at every turn, one in which today's understanding is never sufficient but more like a foundation upon which to build. The more one learns, the more one knows how little is known. An appreciation for the accomplishments of the past is important if one is to understand the potential for the future. But mathematicians must also work to develop a public appreciation for the challenges and the opportunities provided by new mathematics, for explorations beyond our current knowledge. Mathematics defines and lights the road ahead. Mathematics will move us from today to beyond tomorrow. The study edition of *The Mathematical Experience* will help its readers acquire a real understanding of mathematics.