Nature's Numbers

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I. Stewart
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Mathematicians are frequently called upon to write and talk about their work to a general audience in order to convince those who pay our salaries and (if we are lucky) support our research activity that the research we do is worthwhile and interesting. Indeed, the editorial column in these *Notices* has itself urged just this on more than one occasion. Most of us find this task such a daunting challenge that we put it aside for a rainy day, only to find that there are always more pressing things to be done, whether it is writing up a new piece of research or preparing a new course. There is a small but significant number of mathematicians, however, who succeed in this task and produce a (seemingly) constant stream of entertaining (and informative) books about mathematics for a "general audience". Ian Stewart is one of these, and one of the most successful.

Despite being a professional mathematician, and therefore by definition not one of the intended audience, I enjoy reading these "popular" mathematical books just as I enjoy reading many popularizations of science. In the case of mathematics books, however, I always find myself wondering who my fellow readers actually are. Are they, like me, other mathematicians wanting a relaxed good read? Are they the same people who read Stewart's "Mathematical Recreations" column in *Scientific American* each month? Are they mathematics graduates in nonmathematical careers who like to keep their mathematical hand in? Or people who always wanted to be able to understand mathematics better but left it behind at an earlier age?

Nature's Numbers is the sixth in a series of fairly short books called The Science Masters, in which leading scientists communicate their ideas to general readers. As far as I know (from the list of current and planned future contributors on the back cover) Stewart is the only mathematician currently on the list. Having read two or three of the books in the series, I would say that the format is very successful. The books are short enough to be read in one or two sittings and are very accessible to their intended audience. At least that is true of the other books in the series; it is rather dangerous for a mathematician to judge what is or is not accessible to nonmathematicians.

The starting point for *Nature's Numbers* is that there are many patterns to be found in nature, including numerical patterns (such as Fibonacci numbers in sunflowers) and shape patterns (such as in snowflakes). The first chapter already seeks to define what we actually mean by "a pattern", coming to the conclusion that we see a pattern in something that has symmetry, but not too much symmetry — that is, broken symmetry. We do not see a pattern in the still surface of a pond, as there is too much symmetry. If we then toss the traditional pebble into the pond, however, the perfect symmetry is broken: we lose all but the symmetries which fix the point where the pebble strikes the surface in the resulting pattern of concentric circles. This theme of broken symmetry is a recurring one in the book, particularly in Chapter 6. The

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idea that symmetries are operations that may be combined together, rather than static properties of a shape, is brought out. Nature has symmetries at every scale, from elementary particles and atoms right up to galaxies. The role of mathematics is to describe symmetry-breaking processes in order to explain in a unified way the fact that the patterns we see in sand dunes and zebras' stripes are caused by processes which, while physically different, are mathematically very similar.

In the early chapters of the book there is an excellent attempt to answer the questions What is mathematics for? and What is mathematics about?. For the former, the answers given include solving puzzles in nature (such as why planets move in the way that they do), describing changing quantities via calculus, modeling change (such as the evolution of the eye), and the prediction and control of physical systems. The useful point is made that there is often a very long time lag between the original, "pure" mathematical work and the eventual application, so that purely goal-directed research is often inappropriate in mathematics. I hope that the grant-awarding bodies read as far as this (page 29)!

Under the heading "What Mathematics Is About" Stewart emphasizes that it is not just about numbers, but also about operations (also known as functions or transformations), about the logical relationships between facts, and about proof. He gives a good example of the process of finding a proof. There is also an interesting section on the "thingification of processes" as a basic mathematical process. I have previously read about this (using the more standard word "reification", which Stewart says sounds pretentious) only in articles on mathematics education; here it is made clear what a universal abstraction process this is, not just in mathematics.

There is not space here to describe the contents of each of the book's nine chapters in detail: the pace is fast, and hence a lot of ground is covered quickly. I must confess that when I started to read it, I was expecting more of a bias towards nonlinear dynamics and chaos, but this does not appear (except briefly) until the last two or three chapters. Prospective readers who have not been prepared to tackle a whole book on these topics (such as Stewart's own *Does God Play Dice?*) could do a lot worse than to read Chapters 8 and 9 of this book for an entertaining and readable account. In the last chapter, entitled "Drops, Dynamics and Daisies", there are three examples of "simplicity emerging from complexity": the formation of water droplets, population dynamics, and Fibonacci numbers cropping up in the formation of daisy petals. I found it particularly surprising to learn that it was not until 1993 that the latter phenomenon was given a satisfactory dynamical explanation. In each of the three examples the case is made that nonlinear dynamics provides a better insight and explanation into what happens than earlier attempts, which were either purely descriptive or overly formula-bound.

The epilogue to the book is entitled "Morphomatics". It appears to be a kind of manifesto for a new mathematical theory of that name, or even for a new kind of mathematics that will complement rather than replace current scientific thinking. This new theory does not currently exist, but Stewart sees its creation as a necessary way forward in the use of mathematics to understand the natural world. Not all mathematicians will necessarily agree with this manifesto, but it should certainly stimulate an interesting debate. It is a little puzzling, however, for such a manifesto to appear in the epilogue of a book that is not aimed at mathematicians!

Finally, there are several dozen references for further reading, divided up by chapter. These are mostly articles and books published within the last ten years, but include a few earlier items such as the classic *On Growth and Form* by D'Arcy Thompson (originally published in 1917, and still in print). I would like to add one title: *Patterns in Nature* by Peter S. Stevens (Little, Brown & Co., 1974) is a beautifully illustrated account covering some of the same ground.

This book claims it will equip its readers with a mathematician's eyes and hence change the way they see the world. In this aim it stands a good chance of succeeding—provided that they are not already mathematicians, of course.