

Garrett Birkhoff and the *Survey of Modern Algebra*

Saunders Mac Lane

Garrett Birkhoff became fascinated with finite groups when he was an undergraduate major in mathematics at Harvard. Upon graduation in 1932, at the age of nineteen, he traveled in Europe with a Henry fellowship. There he studied Speiser's book on group theory and van der Waerden on modern algebra. At Cambridge University he was impressed by the elegant group theoretic ideas of Philip Hall. Then Garrett also discovered the idea of a lattice—a poset with both lower and upper bounds (only later did it turn out that this structure had been found by Dedekind, in a little noted study of what he called a “dual group”). While in Cambridge, Garrett also conceived the idea that there could be a real “universal algebra” and realized this idea by proving what is now known as Birkhoff's theorem, characterizing varieties of algebras.

He considered algebras given by a set with specified operations (unary, binary, etc.) which satisfy a given list of identities. All such constitute a “variety”. Birkhoff's theorem states that a class of algebras is such a variety if and only if it is closed under the formation of subalgebras, direct products, and homomorphic images. This result became the starting point for the subsequent active development of universal algebra.

After holding a Junior Fellowship at Harvard 1933–36, Garrett became an instructor in 1936. At

that time Harvard provided a full year undergraduate course in geometry; Garrett advocated the establishment of a corresponding full year course in algebra, to be called Mathematics 6. He taught the first version of this course in 1937–38, emphasizing Boolean algebra, set theory, vectors and group theory; he prepared notes of his course. I taught a somewhat different version of Mathematics 6 in the next year, 1939–40, after I joined the Harvard faculty in 1938; I also provided typed notes of my version of the course. In the subsequent years Garrett and I combined our preliminary notes to publish with MacMillan in 1941 our joint book, *Survey of Modern Algebra*. It provided a clear and enthusiastic emphasis on the then new modern and axiomatic view of algebra, as advocated by Emmy Noether, Emil Artin, van der Waerden, and Philip Hall. We aimed to combine the abstract ideas with suitable emphasis on examples and illustrations. Groups were started by examples such as the group of symmetries of the square. Vector spaces were introduced by axioms, but with n -tuples of numbers as illustrations. The chapter on matrices began with linear transformations and explained matrix multiplication in terms of the composition of the corresponding linear transformations. The Galois theory was presented with the conceptual ideas of Emil Artin, which made the Galois correspondence (subgroups to subfields) vivid. In brief, the emphasis was axiomatic and abstract, but built on examples.

At that time one of my good midwestern friends told me that our survey “would not fly beyond the Charles River.” For a year or two this was perhaps so. But American mathematics, spurred by the in-

Editor's Note: Garrett Birkhoff passed away on November 11, 1996, at the age of 85.

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fluence of refugees from Europe and the urgent needs of war research, was rapidly developing. Throughout mathematics, ideas in their abstract form mattered. Our *Survey* was at hand and provided these ideas with examples. It soon became, and was for many years, the text of choice for an undergraduate course in algebra. We were fortunate to be there, young and enthusiastic, at the time when new views of algebra came to fruition. And these ideas are still there; Garrett and I were both pleased with the recent publication of the fifth edition of *Survey* (A. K. Peters, 1996).

We enjoyed teaching and writing algebra because it was clear, exciting, and fun to present. The book was prepared at a time when both of us were assistant professors, so without tenure. Yes, we did know then that research mattered for tenure, but our joy in teaching was somehow connected with our respective research. Also, the mathematics department at Harvard both emphasized research and expected all faculty members to be steadily active in teaching undergraduates. These responsibilities were in effect combined in our activity. Then and later we took part in the flow of new ideas from discovery to use and to present to students.

Garrett's own research was involved. It was then primarily in lattice theory and universal algebra. His original slim colloquium volume on lattice theory was later expanded to a much more comprehensive version, reflecting the growth in this field. During the war Garrett's interests grew to include hydrodynamics and other applied mathematics—with an occasional pause to prepare revised editions of *Survey*. On the fiftieth anniversary of its publication, the *Mathematical Intelligencer*, in its column "Years Ago", edited by Karen V. H. Parshall, gave a description of *Survey*, complete with pictures of the authors (vol. 14, no. 1, 1992, 26–31 pp.).

References

- [1] GARRETT BIRKHOFF, *On the combination of subalgebras*, Proc. Cambridge Philos. Soc. **29** (1933), 441–464.
- [2] ———, *Lattice theory*, Colloquium Publications, vol. 25, Amer. Math. Soc., Providence, RI, first edition, 1940, 155 pp.; third edition, 1967, 418 pp.