Chih-Han Sah (1934–1997)

Johan Dupont, Anthony Phillips, Vladimir Retakh, Judith Roitman, and Mark Saul

A Sketch of an Intellectual Biography

Anthony Phillips

Chih-Han Sah, who died in Stony Brook on July 22, was born in Beijing in 1934 into an old and distinguished family. An ancestor of his was Genghis Khan's commander in Foochow. Han's father, Adam Pen-Tung Sah, earned a Ph.D. in physics from Worcester Polytechnic University in the 1920s, wrote the general physics text that was the standard in China from 1930 until 1950, rose to become president of Xiamen University, and served as secretary general of the Academica Sinica from 1945 until his death in 1949. Han's mother, Shu-Shen Huang, was an exceptional athlete: she represented China in the Olympic Games, competing in javelin and discus. Later she took a master's degree in mathematics from the University of Illinois and taught mathematics at Slippery Rock State College in Pennsylvania until she retired.

Han's father spent the 1935–36 academic year on sabbatical in the United States. During a visit to Ohio State University the Sahs made the acquaintance of William and Dorothy Everitt, who turned out to play a crucial role in the education of Han and of his older brother, Chih-Tang (Tom): when Han's father died in 1949, the Everitts took on the two boys as unofficial foster children. The boys were able to move from the chaos of postwar China to the calm and stability of Urbana, Illinois, where William Everitt was now dean of engineering.

Han had just turned fourteen. Before leaving China he had been hustled from one school to another, with several years during the war of no school at all, but he had managed to complete the tenth grade. In Urbana he came into the university's experimental high school as a sophomore Though starting with only "a 200-word [English] vocabulary at perhaps the kindergarten level" as he described it, he was able to skip the eleventh grade and to finish as the class valedictorian. He recounted this scene from his first year:

"I had been sitting in class mute (the custom in China) for two weeks when the teacher decided to give a quiz on factorization of polynomials. I finished the quiz about thirty seconds after she had finished writing the questions on the blackboard, including…one about $X^4 + X^2Y^2 + Y^4$, and turned it in. Some ten minutes later, after she finished some other paperwork, and while the rest of the class was still struggling with the quiz, she looked at what she obviously expected to be a blank sheet. The change of expression on her face was a marvel to watch…."

Han started the University of Illinois as an engineering physics major. Here is how he described his transition to mathematics:

"When I was an undergraduate lab assistant in the physics accelerator lab, I found that I could not understand the purpose of the nuclear physics experiment we were running. I decided to read up on quantum mechanics in the library. To my frustration, I discovered that I could not understand the mathematics used in the texts. Not long after, I asked my best friend in college what he was studying in the way of mathematics. I was shocked to find that I could not read past the first few pages

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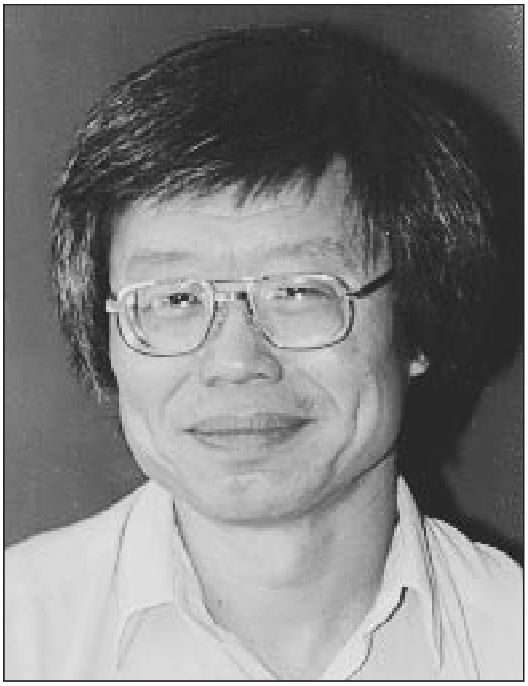

Chi-Han Sah

of his book with the esoteric title *Theory of Groups*. Since my physics teachers all told me that I had already overdosed on mathematics, I decided to ask my math professor about the propriety of beginning to study some pure math. (I had in mind the vague idea of spending most of my fourth college year to that end.) His answer was: 'It is too late to begin studying pure math at the age of 19.' Two weeks later, I pigheadedly decided to graduate early and applied to the Mathematics Department to study math full-time.…Soon I was, despite warnings from my science teachers, firmly seduced by 'useless' mathematics. At the same time, I never lost my interest in science, engineering, and the much more difficult and fascinating endeavors in the humanities."

Kenneth Mount, who had been a fellow student and friend of Han's at "Uni-Hi", went on with him to the university. He reports: "We were never in the same classes together, since I became a math major before he did. We did work on problems together. Indeed, that was a daily and nightly occurrence. We spent many hours playing 'pick a group', which was a version of twenty questions." This is an early and typical example of Han's enthusiasm for discussing mathematics and of his often playful approach to the subject.

Han earned a master's degree in mathematics from Illinois. He then entered the graduate program at Princeton, where he wrote a thesis under the direction of Timothy O'Meara. O'Meara told me how he would run into Han in the Common Room, ask him how he was doing, and suggest one or two further problems—Han would turn up the next day with the answers. "He glided through his entire thesis like that."

Han's ebullient mathematical personality flourished at Princeton. Here are some of Barry Mazur's reminiscences:

"Han Sah was the real energy behind a small band of us graduate students, the hours of our days studded with the 'grad-student-run seminars' that you mentioned. It was to those seminars, and not the formal Princeton classes, that our passions were truly directed. Han's enthusiasm shaped many of those seminars, and, happily, he had many enthusiasms:

"'We must learn about quantum mechanics!' We soon found ourselves taking turns covering blackboards with the disembodied 'brackets' coming out of the pages of Dirac's *Principles of Quantum Mechanics*.

"We heard, or made up, the gossip that Chevalley adamantly REFUSED to draw pictures when he did his algebraic geometry. The idealism, the wild-eyed rigor and asceticism of this gesture fired our imagination: we also decided 'WE DRAW NO PICTURES!' and in another graduate student seminar we macheted our way through Chevalley's *Introduction to the Theory of Algebraic Functions of One Variable*.

"But there were other seminars replete with pictures. Chevalley, with pictures or without, was a great favorite for seminars, with others of his books (e.g., *Theory of Lie Groups*) being text and pretext.

"Besides the seminars, there was (along with John Stallings and Jim Stasheff) the constant quest for examples and (much more delicious!) counterexamples in topology and algebra. Here pictures were often THE driving force, and here sometimes I think we relished our failures as much as our successes. If some property about functions on, or mappings from, the unit interval eluded us, our revelling carol would be: 'WE DON'T EVEN KNOW THE UNIT INTERVAL!' No sooner, however, did we find a topological space, or ring, or group with some weird property but the cry went out for one with a yet weirder property. Han was our resident finite group theorist and adroit at finding finite groups obeying whatever prescription was called for. Both our inspiration and our tutor.

"The force of Han's enthusiasm carried you along; it was catching. And when it was Han's turn to lecture to us, he would do so with a cheerful, staccato, relatively fast delivery, with a singing voice, and a cloud of chalk."

Han graduated from Princeton with a Ph.D. in 1959 but stayed on one more year as an instructor. From 1960 to 1963 he served as a Benjamin Peirce Instructor at Harvard. He continued in the research direction of his thesis, mainly on finite groups and on quadratic forms over fields of characteristic 2. While he was at Harvard he developed an introductory course in Abstract Algebra; his notes became a textbook with the same title that was published in 1967 by Academic Press and that is still a standard, if formidable, reference.

In 1963 Han joined the University of Pennsylvania faculty. There he collaborated with Oscar Goldman on locally compact rings. Also, through his work with Leonard Charlap on the classification of flat Riemannian manifolds, he began working on the homology and cohomology of groups, topics that would preoccupy him for the rest of his career.

In 1970 Han came to Stony Brook, where he stayed. He continued his research in finite groups and group cohomology but also began work on applications of group cohomology, to Hilbert's Third Problem, about "scissors congruences". He published a volume on the topic in the Pitman Research Notes Series. This book came to the attention of Johan Dupont, who told the participants in Han's memorial gathering how surprised he had been that someone else was thinking about scissors congruences. Thus began a transatlantic collaboration and a close friendship that lasted until Han's death.

The mathematics department at Stony Brook is fortunate in its proximity to the Institute for Theoretical Physics. This serendipidity has been commented on elsewhere, in connection with the "dictionary" between gauge field theory and differential geometry. But Han soon became the reference of choice for physicists working on exactly solvable models, and their discussions led to joint work: with Barry McCoy, Jacques Perk, and Shuang Tang, and with Eduardo Ramos and Robert Shrock. In recent years his various interests came together: he and Dupont published a paper in *Communications in Mathematical Physics* on "Dilogarithm identities in conformal field theory and in group homology".

As Han said himself, "I never lost my interest in science...." Although he loved our "useless" mathematics, he was very happy when it could be used in elucidating problems in physics and chemistry. One of his main concerns about the education of mathematics majors was that they were not getting the solid exposure to science courses that would allow them the satisfaction of applying their mathematical knowledge to real-life situations and also allow them access to the vast source of mathematical problems and phenomena encoded in the physical world.

His Role in Education

Judith Roitman and Mark Saul

In recent years Chih-Han Sah became increasingly interested in mathematics education. His work in education is difficult to reconstruct, since so much of it took place behind the scenes: in private conversation or as an advisor on other people's projects (often with no official recognition). When asked by Al Cuoco for an autobiographical statement, Han wrote, "I [am] a confirmed believer in the Avis philosophy. Let somebody else be #1 and take the brunt. I am for #2 and hide in the background."

In the early 1990s Han became involved with the Gelfand Outreach Program in Mathematics (GOPM), an American adaptation of I. M. Gelfand's Soviet Union-wide correspondence school. Han helped to develop new material for this program and to pilot special applications for teacher training and in-school instruction. A pilot program in Puerto Rico continues as one of the most productive parts of this project.

Han was one of the leaders of an innovative proposal to the National Science Foundation that would have established a national program for postdoctoral fellows with responsibilities in both mathematics and K–12 education. A pilot version of this program exists and is quite successful at Case Western Reserve, but despite strong support from many mathematicians a national program has not yet been funded.

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Han was a thoughtful critic of reform in both calculus and K–12 education. His specific contributions here are again difficult to document, taking the form of memos, e-mail, and phone conversations rather than position papers or public talks. But he was the best kind of critic, the kind who, for example, would look up student records to see what actually happened rather than relying on his own notion of what he had expected. He was deeply concerned about the fate of individual students and took the "long view" on assessing educational programs.

The most public of his involvements in education was the e-mail list `mathed` he established in August 1993. This grew out of his concern with education reform, most specifically out of a series of e-mail conversations with Hung-Hsi Wu and Dick Askey, who shared his concern. It was Han who

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took the conversation to a broader audience by adding first a few people, then more and more until he had a full-fledged list to manage; and, typically for Han, he managed the whole time-consuming e-list structure himself—with all of its open participants and auditors. Most remarkably, Han was bigger than his own opinions. He took pains to include people whose views were bound to differ from his own. He sought out researchers in mathematics education as well as in pure and applied mathematics. He included classroom teachers as well as university professors. And he took great pains to keep the discussion civil as well as passionate. As Hung-Hsi Wu writes, "Ultimately time may prove that the way Han handled the `mathed` group was his greatest contribution to education."

But for many of us, the most treasured contributions that Chih-Han Sah made to mathematics education were his tireless private and semipublic discussions, ranging, as Al Cuoco writes, "from mathematics to education to household repairs," and memorably on the history of Chinese mathematics. In private conversation or on one of the public lists of which Han was a member, he would respond to questions related to school mathematics in enthusiastic and profound ways. Here is a description of one such conversation, from Al Cuoco: "About 8 p.m. one evening I sent him a message asking if he knew how a planimeter (a mechanical device for finding areas of closed curves) worked. He responded immediately that he didn't but would find out. The next morning at 5 a.m. I found a three-page message in which Han proposed two devices that would do the integration (complete with proofs that they worked). In other words, he found out about planimeters by inventing two of them. Later he looked at the devices that are actually used, videotaped a couple, found references in nineteenth-century calculus books, and wrote a long post for `mathed`, calling it his 'planimeter adventure'. He took me along on many similar adventures, always looking at new ideas with a passionate combination of seriousness and delight, with a grin that I could picture long before I saw it firsthand."

His Mathematical Works

Johan Dupont and Vladimir Retakh

Han's interests in mathematics have varied over a broad range of subjects: group theory, quadratic forms, rings, Riemann surfaces, algebraic topology, scissors congruences, algebraic K-theory, polylogarithms, combinatorial geometry, applications to electrical engineering, conformal quantum field theory and statistical quantum mechanics, and structures of fullerenes in chemistry.

He started his work in mathematics with finite group theory. Together with R. Brauer, Han edited the proceedings of one of the most influential meetings in the 1960s, which helped set the classification program of finite simple groups on the road.

His famous works with O. Goldman in the theory of locally compact rings led to a number of interesting results and new tools, such as the Goldman-Sah product. He also studied questions of the existence of normal complements and automorphisms in group theory and finite quotient groups of discontinuous groups connected with Riemann surfaces.

In view of the strong geometric environment at Stony Brook, it is not surprising that he became increasingly involved in geometric problems. Perhaps his best-known contributions are in connection with the subject of "scissors congruences" of polytopes in Euclidean, spherical, or hyperbolic n-space, a field in which he was to become a world authority.

Two polytopes are called *scissors congruent* (s.c.) if they can be cut into the same finite number of subpolytopes such that the pieces of the two polytopes are pairwise congruent by means of isometries of the geometry in question. This notion occurs in connection with an elementary definition of the concept of "area" in the Euclidean plane and has a long history. The first explicit proof that polygons in the plane have the same area if and only if they are s.c. seems to have been given by W. Wallace (1807). But already Gauss (1844) noticed that a similar elementary approach to the concept of "volume" of polyhedra in 3-space is lacking, and, referring to this, Hilbert (1900) stated as the third problem on his famous list the challenge of finding two polyhedra of the same volume that are not s.c. This problem was immediately solved in this form by M. Dehn (1900), who introduced additional necessary conditions for two polyhedra to be s.c. and showed that these are not satisfied for the regular cube and regular tetrahedron. (An earlier proof by R. Bricard (1896) used a more restricted notion of s.c.) The subject of s.c. was subsequently forgotten except among a few geometers.

Han was introduced to the subject in the mid-1970s by D. Sullivan, and in 1979 his book *Hilbert's Third Problem: Scissors Congruence* was published in the Pitman Research Notes Series. The main result was the beautiful theorem that in all dimensions the so-called Hadwiger invariants determine Euclidean polytopes up to translational s.c. (i.e., the

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only congruences allowed are translations). It turned out that this theorem had already been proved (but not published) a few years before by two Danish mathematicians, B. Jessen and A. Thorup. A more important contribution of the Pitman Notes is that they pointed to the close relation between the notion of scissors congruences and homological algebra, in particular the cohomology theory of groups (in this case, of the group of isometries involved).

For a long time the subject of s.c. was considered somewhat exotic and removed from mainstream mathematics, even by the workers in the field. But according to Han this was unjustified for two reasons. One is the historical fact that the subject of s.c. is related to such fundamental mathematical concepts as "area" and "volume". The other is that more than once an important mathematical idea has appeared as a special case in the context of s.c. long before it was formally introduced. Thus Dehn's conditions for s.c. involved an example of the tensor product of two abelian groups thirty years before such a tensor product was defined by Whitney. Also the *group of polytopes* (nowadays called the *s.c. group*), defined by B. Jessen in 1941 in a paper written in Danish, is really the algebraic K-group in the sense of Quillen (1971) for the category of polyhedra with conguences as the morphisms.

Today, thanks to the efforts of Han and his collaborators, this area of research is fully integrated into modern mathematics, having close connections to well-established fields such as homological algebra and algebraic K-theory, characteristic classes for flat bundles and foliations, hyperbolic 3-manifolds, and even (though more speculatively) subjects like motivic cohomology and conformal field theory.

For instance, by a classical geometric construction (going back to Gerling, a contemporary of Gauss), the s.c. group is 2-divisible also in non-Euclidean geometry, and the generalization to p-divisibility for any prime p for hyperbolic 3-space led to the proof of the first nonsolvable case (for $SL(2, \mathbb{C})$) of the so-called Friedlander-Milnor Conjecture on the homology of the discrete underlying group of a Lie group.

In the opposite direction, applications of well-known theorems in algebraic K-theory by Borel and Suslin have greatly clarified the structure of the s.c. group in spherical and hyperbolic 3-space, and the remaining problem is equivalent to the so-called "rigidity question" in algebraic K-theory. In particular, explicit necessary and sufficient conditions are now known for s.c. in these geometries provided the vertices of the polyhedra are defined over the field of algebraic numbers.

Han made several other beautiful contributions to s.c., but he also worked on a wide range of problems in all kinds of mathematics extending into physics and chemistry ("buckyballs"). He was one of the "unofficial" conduits of mathematical information to physics colleagues.

As he once wrote: "My 'engineering upbringing' is such that I am not selective in terms of areas—they are all interesting to me (I only understand very small parts of what I read and hear but usually can find someone that is an expert to explain to me the details)." He was happy when he could help other people solve their problems, especially when the problems involved elementary algebra, geometry, and number theory. Very early on he arranged seminars with physicists and had numerous discussions with them, trying to overcome their traditional preoccupation with analysis and believing that they needed more algebra and geometry. He was always very open minded and generous, both in scientific matters and in personal relations, and he will be greatly missed.

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