

Lars Valerian Ahlfors

(1907–1996)

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Lars Valerian Ahlfors was born in Helsingfors, Finland, on April 18, 1907. He studied mathematics with Ernst Lindelöf at Helsingfors University and earned the doctorate in 1928. Ahlfors accepted a position at Harvard University and moved to Cambridge in 1935. He spent most of his professional life at Harvard, with a period in Europe (primarily at the University of Helsinki) from 1938 to 1946.

Lars Ahlfors retired from his position at Harvard in 1977. He continued to study mathematics until the end of his life on October 11, 1996.

Lars Valerian Ahlfors was arguably the preeminent complex function theorist of the twentieth century. With a career spanning more than sixty years, Ahlfors made decisive contributions to areas ranging from meromorphic curves to value distribution theory, Riemann surfaces, conformal geometry, extremal length, quasiconformal mappings, and Kleinian groups ([7] serves as a map of Ahlfors's contributions to the subject). Ahlfors was both role model and mentor to his graduate students and to the many mathematicians around the world who learned from his example. He is remembered warmly, both as a mathematician and as a man.

Ahlfors's life contained many firsts. In 1936 he received, along with Jesse Douglas, one of the first two Fields Medals. Ahlfors's remarks concerning that occasion, taken from his *Collected Papers* [4], typify his honesty and humility:

I was in for the surprise of my life when in 1936, at the International Congress in Oslo, I was told only hours before the ceremony that I was to receive one of

the first two Fields Medals ever awarded. The prestige was perhaps not yet the same as it is now, but in any case I felt singled out and greatly honored. The citation by Carathéodory mentions explicitly my paper "Zur Theorie der Überlagerungsflächen," which threw some new light on Nevanlinna's theory of meromorphic functions. The award contributed in great measure to the confidence I felt in my work.

During World War II Ahlfors was forced to pawn his Fields Medal in order to obtain the money to secure safe passage from Finland to Zurich at the end of the war. Ahlfors's remarks on the incident were:

I can give one very definite benefit [of winning a Fields Medal]. When I was able to leave Finland to go to Sweden, I was not allowed to take more than 10 crowns with me, and I wanted to take a train to where my wife was waiting for me. So what did I do? I smuggled out my Fields Medal, and I pawned it in the pawn shop and got enough money. I had no other way, no other way at all. And I'm sure that it's the only Fields Medal that has been in a pawn shop... As soon as I got a little money, some people in Switzerland helped me retrieve it.

Another remarkable achievement of Lars Ahlfors is that he was invited to be a plenary speaker at the International Congress of Mathematicians on three different occasions. No other mathematician in the past seventy years has achieved such a distinction. His plenary addresses were about three completely distinct areas in which he had made seminal contributions.

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Ahlfors's text *Complex Analysis* [1], which was first published in 1953 and has gone through three editions, has been the definitive text in complex function theory for the past forty-five years; there are few other texts in modern mathematics that have played such a dominant role.

At a conference held in Storrs, Connecticut, to commemorate Ahlfors's seventy-fifth birthday, Ahlfors remarked that "Retirement is wonderful. I can't perish anymore, so I don't have to publish." The upshot of his remarks was that he could devote himself full time to understanding the new developments in his field without the pressure of writing them up. He seemed to think of this as a mathematician's paradise. The NSF program officer in complex analysis told me about fifteen years ago that Lars Ahlfors's entire NSF proposal consisted of a single sentence: "I will continue to study the work of Thurston." Ahlfors was seventy-five years old at the time, and he was devoting himself to learning the newest ideas in the subject!

In what follows, seven mathematicians relate their personal reminiscences of Lars Ahlfors.

—Steven G. Krantz, Editor

James Jenkins

Lars Ahlfors returned to Harvard in the fall of 1946, the same time that I arrived to begin my graduate work there. David Widder was at that time the chairman of the mathematics department, and he was recruiting people to work with Ahlfors. Since I had some background in function theory from my work at the University of Toronto, he approached me in this context (not that I was reluctant).

In those days there was no mathematics building at Harvard; indeed, the faculty members did not have regular offices. Some of the senior people had faculty studies in Widener Library, and Ahlfors shared one with Julian Coolidge, professor emeritus and a geometer of the old school. I had my first few meetings with Ahlfors there. On one occasion he had the duty of testing my proficiency in foreign languages with Coolidge present. In a passage in French making a mildly pejorative statement about "géomètres", I translated the word as "geometers", and Ahlfors was quick to point out that it was used in the more general sense of "mathematicians".

Every faculty member at Harvard had an affiliation with one of the residential houses (which had a certain analogy with colleges at the English universities). Ahlfors's affiliation was with Dunster House, and he had a sort of office there. We met there a few times, but afterwards always at his home. Ahlfors was very conscientious in meeting with his graduate students, but his role was largely

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Early photograph of Lars Ahlfors, who is reputed to have worn a beret until the day he died.

reactive rather than presenting suggestions or ideas (at least in my case).

In the spring of 1947 Ahlfors gave a course on the method of the extremal metric, which was then in its infancy. In the year 1947-48 he gave a course on the calculus of variations. In the year 1948-49 he gave a course on Riemann surfaces, directed largely to a generalization of his work on Schwarz's lemma to finite-bordered Riemann surfaces. Ahlfors did not conduct a regular seminar at any time during my stay. One year he gave a series of talks on Teichmüller's papers, probably with a view to preparing himself for his work on quasi-conformal mappings.

Ahlfors had a remarkable command of English for one for whom it was not the native language. However, English is tricky. One semester, when he was teaching a course in elementary calculus, he asked students on the examination to find the three first derivatives of an explicit function of one real variable (creating considerable havoc). Also, under stress one tends to revert: at a tense moment in a lecture "j" came out as "yot".

There were occasionally people who came to work with Ahlfors, for example, a Swiss by the name of Haefeli. In the year 1948-49 Arne Beurling came to Harvard as visiting professor. He gave two rather well-known one-semester courses on his brands of functional analysis and harmonic analysis. He and Ahlfors were working on a book on extremal length and in fact had a rough draft of some of the material. No book ever appeared, but some of the contents showed up in Ahlfors's book *Conformal Invariants* [2].

Robert Osserman

When I reached the point in my graduate studies where it was time to select a Ph.D. advisor, I naturally consulted fellow students who were more advanced. One of those was a young student from Finland named Vidar Wolontis. He may not have been totally objective in his recommendation that I study with his countryman Ahlfors, but he did mention one reason that I still remember. He said, “Look at Ahlfors’s bibliography. It is really short. Only a relatively small number of papers, but every one significant.” I later learned of Gauss’s famed motto: “*Pauca sed matura*” or “Few, but ripe.” Perhaps Ahlfors’s unstated motto was “*Pauca sed egregia*”—“Few, but outstanding.” That was an idea that appealed to me greatly, and I did become Ahlfors’s student, a decision I have always felt lucky to have made.

Ahlfors’s style as mentor suited me perfectly. He was always available but never intrusive. He provided me with several opportunities that proved important to my work. One was a 1951 summer program in Stillwater, Oklahoma, where he was invited to give a series of lectures. He asked if I would like to be the official note taker. It was actually a paid position, or at least what passed for pay in those days—\$150, if I remember right. (Perhaps I should add that when I started graduate school at Harvard, the tuition was \$600 per year, so that my summer salary was one quarter of a year’s tuition. Also, I paid \$10 per month for a room that came with a piano in it.) I worked closely with Ahlfors on reworking his lectures and learned an enormous amount in the process. Murray Gerstenhaber was also there for the program, and he too was a great help to me in working through the lectures. When the program ended, Murray and Ahlfors and I drove in Murray’s newly acquired car from Stillwater to Chicago. A highlight of the trip was dinner the first night in a fancy restaurant in Missouri on what Murray reminded me recently was Bastille Day. I remember Ahlfors’s great delight in celebrating the end of a month of living under the restrictions imposed by the dry state of Oklahoma.

Later in 1951 Ahlfors suggested that I accompany him to Princeton for a special conference for the one-hundredth anniversary of the birth of the Riemann surface in Riemann’s doctoral dissertation, presented in public in 1851. The keynote lecture was given by Ahlfors, who presented a masterful overview of the evolution of Riemann’s ideas during the preceding century.

This conference was particularly significant for me; right after Ahlfors’s talk, Lipman Bers stood up and said he had two problems concerning Riemann surfaces that he thought the group in at-

tendance would be ideally suited to work on. After we returned to Harvard Ahlfors suggested that I work on one of the two problems, and he had some ideas about how to proceed. It turned out that the particular approach he suggested did not work, but I was able to solve the problem by a different method, and it became part of my Ph.D. thesis.

A high point of the Princeton conference was a cameo appearance by Albert Einstein, who expressed his own indebtedness to Riemann and the importance to him in particular of Riemann’s introduction of the idea of a Riemann surface. What Einstein was referring to was of course the idea of a Riemannian manifold, and it was a legitimate confusion to think that a Riemann surface might be just a two-(real-like)-dimensional Riemannian manifold. As the terminology evolved, by the middle of this century the term “Riemann surface” meant a one-dimensional complex manifold or, equivalently, a two-(real)-dimensional surface with a conformal structure. A Riemannian manifold could have any dimension and was endowed with a Riemannian metric, so that lengths, for example, make sense on a Riemannian manifold but not on a Riemann surface. The two subjects had diverged so much that there were many practitioners of each who had nothing to do with the other. On the other hand, there was, and is, a considerable area of overlap; it was precisely in exploiting that area that Ahlfors was most ingenious and adept, especially in the use of conformal metrics to study analytic mappings and Riemann surfaces. My own later work also made significant use of the connection, but chiefly in the opposite direction: applying the theory of Riemann surfaces to problems in differential geometry.

In the years following my doctorate Ahlfors continued in his supportive role. He made a number of suggested changes in a paper I sent him for submission to the *Proceedings of the National Academy of Sciences*, including the deletion of the word “unfortunately” that I used at one point, which he described as “too passionate.”

In later years I had less contact with Ahlfors as I drifted away from the areas in which he worked and more toward differential geometry. However, the mathematical style and professional ideals that I had imprinted during my graduate work persisted throughout my career. On the mathematical side I was always drawn to areas that involved analysis with a geometric flavor. And professionally I have tried to live up to Ahlfors’s high standards. I might add that one of his characteristics that I have always greatly admired was his scrupulous honesty and generosity in giving credit to others. Ahlfors’s influence and legacy will permeate mathematical life for a long time to come, and I am very grateful to have been able to be a part of it.

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Clifford Earle

Toward the end of the “Author’s Preface” to his *Collected Papers* [4] Ahlfors wrote, “I have enjoyed the many excellent students Harvard has had to offer, many of whom I have watched become leaders in my own or some other field.” Many of them, both undergraduates and graduate students, attended Math 213, the two-semester graduate course in complex function theory that Ahlfors taught many times at Harvard. His well-known book *Complex Analysis* [1] grew in part from his experience in Math 213 and has been a standard textbook in such courses for many years.

The final paragraph of the preface of the first edition of *Complex Analysis* [1] begins as follows:

One more point: the author makes abundant and unblushing use of the words “clearly”, “obviously”, “evidently”, etc. They are not used to blur the picture. On the contrary, they test the reader’s understanding, for if he does not agree that the omitted reasoning is clear, obvious, and evident, he had better turn back a few pages and make a fresh start.

Ahlfors’s classroom manner belied these blunt statements, which disappeared in the later editions of the book. When he used a word like “obviously” in his Math 213 lectures, the “obvious” statement was always followed by a friendly glance around the classroom and then, more often than not, by the polysyllable “Well” and an extended explanation or example.

Higher-level graduate courses on topics in geometric function theory and Riemann surface theory played an important role in Ahlfors’s teaching of graduate students and postgraduate visitors to Harvard. These courses were often given before books on their subjects had been published. Ahlfors made his detailed handwritten lecture notes available for reading in the library after class. Such notebooks were the basis of his books *Lectures on Quasiconformal Mappings* [3] and *Conformal Invariants* [2].

Besides his classroom notes, Ahlfors, of course, shared his latest research papers with his graduate students and encouraged the reading of classic texts. Carathéodory’s two volumes on function theory [6] and the books of Rolf Nevanlinna were high on his reading list.

But from 1959 through the 1960s an Ahlfors graduate student’s most memorable experience was participation in the Ahlfors seminar. In this

weekly seminar graduate students gave expository talks about papers in the literature and Ahlfors lectured about his recent research, but most of the talks were given by short- or long-term visitors who lectured about topics of their choice, usually related to their latest work. Regular attendance at the seminar exposed the participants to a broad range of topics in complex analysis. Among the visitors who gave a lecture or series of lectures in the seminar during those years are Accola, Beardon, Belinskii, Gehring, Hayman, Kodaira, Osserman, Pommerenke, Rickman, Rodin, Sario, Väisälä, Weill, and Wermer. This stream of visitors was made possible by the dramatic increase in federal support of mathematics after the Sputnik flight in 1957 and represents a highly effective use of such funding to benefit both mathematics research and the education of young mathematicians (if these can be separated) simultaneously.

Dennis Hejhal

During the fall of 1964 I was a fifteen-year-old high school student at Lane Technical High School in Chicago. I was reading some analytic function theory on my own at the time and discussing it with one of my teachers. She informed me that Ahlfors’s book was a classic, that she had used it during her grad school days, and that maybe I should try to work through her copy. I did so and encountered a variety of questions in connection with the exercises. As I recall today, there was an error in one that prompted me to write a letter to Ahlfors to ask about the matter.

As a lowly high school student I was pleasantly shocked when I promptly got a reply from the author, and even more so when it was addressed to someone called “Dr. D. A. Hejhal”. I immediately wrote back with a fuller letter stating that I was still in high school and was just beginning my study of function theory.

Thus began my correspondence and long-distance “tutelage” with Lars Ahlfors.

I was profoundly influenced by the guidance he offered me, as I sent him my various reports telling him what I was up to. Very early on I developed a sense of the quality Ahlfors represented. I also felt encouraged that a Harvard mathematician of his caliber would have the time and interest to correspond with a high school student.

Our early exchanges covered a variety of topics: what type of books I had access to, whether I knew any university faculty, what I wanted to study next, my project for the National Science Fair, etc.

He did not really try to influence my choice of study topics, but rather encouraged me to go where

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Ahlfors (standing in top photograph and in foreground of bottom photo) at Harvard math picnics, circa 1950–1952.

my interests took me. About the only thing he did strongly suggest, after two or three letters, was that I take a copy of one of his letters and “report to” the chairman’s office at the University of Chicago mathematics department with it, basically seeking someone to help “look after me.”

Herstein was the acting chairman, and the first thing he did was give me a note saying that it was okay for Hejhal to check out library books in his name. The Eckhart Hall Library is a first-rate mathematics library, and this was like opening the door

to infinity for me. This high school student had never seen anything like it!

In this way, through Ahlfors, I came to appreciate that mathematics includes components of scholarship and history concomitant to research. In a 1985 paper Ahlfors credits me with providing him with some decisive historical references thanks to my “gargantuan appetite for reading.” I can only respond by saying that it was he who both whetted and precipitated that appetite—and that he exaggerated.

During my undergraduate days at the University of Chicago, I continued to keep Ahlfors posted with progress reports and copies of my papers. Paul Sally turned out to be the ideal local person to keep me headed in the right direction. One paper that I wrote while in my second year, on kernel functions (answering a question posed to me by R. R. Coifman), was thought possibly to be suitable for a Ph.D. thesis. I was not sure what course was best and received a mixture of views. Building on the sense of trust that had evolved earlier, I decided to listen to Ahlfors; he wrote that rather than “racing,” he felt it better that someone in my shoes substantially deepen their knowledge first and work on more things before taking a Ph.D.

This ultimately led to my going to Stanford in 1970 for my graduate work (a place that has rightly been described as a “hotbed” of complex analysis in those days, particularly for someone enthusiastic about kernel functions and conformal mappings). Since Ahlfors was planning to be away from Harvard during part of that time, he concurred that Stanford was the natural place for my graduate work.

Though new influences abounded there for me (Pólya, Bergman, Schiffer, Royden, Cohen, Zalcman), it has to be said that it was Ahlfors who, in effect, started me in serious motion down the wonderful path of complex analysis.

It was only in the spring of 1972—just before I applied for a job—that Lars Ahlfors and I first met in person. He invited me to come to Harvard for a month.

There is an interesting story behind this invitation. Whenever I play “tour guide” for my visitors in Sweden, we always go to the Mittag-Leffler Institute near Stockholm. The library there contains many old books, spread out over the third and fourth floors. On the third floor in the corner, looking out over “his books”, is an enormous bronze statue of Mittag-Leffler (commissioned by same!) holding his reading glasses. In 1972 my work with Max Schiffer on the Szegő kernel function led me to discover the wondrous gold mine of a book by H. F. Baker [5]. Ahlfors was visiting the Mittag-Leffler Institute when I wrote him describing my results, citing the reference to Baker’s book. I offered to come to the Mittag-Leffler Institute if he wanted to hear more. I soon received

a reply. Ahlfors said it would be preferable to wait until he returned to Harvard; let's set something up, etc. Then, with obvious delight, he went on to say that he found Baker's book, "but that in order to get the book, I first had to get a ladder, prop it up against the statue, and quite literally stand *on top of* Mittag-Leffler to grab it from the (nearly inaccessible) shelf above his head."

That scene with the sixty-five-year-old Ahlfors would have been priceless to catch in a photo. Isaac Newton's famous quotation¹ could have served as its title.

One presumes the same story was told to Lennart Carleson, then director of the Mittag-Leffler Institute, because shortly afterward Baker migrated to a more easily accessible spot on the fourth-floor balcony level.

With Ahlfors safely back at Harvard, my one-month visit took place as planned. During my stay there I was offered a Benjamin Peirce Assistant Professorship, thus leading to the next stage in our relationship, viz., working as a junior colleague with my former long-distance mentor.

I remember my Harvard days with great warmth. The atmosphere that Ahlfors created there—e.g., in our weekly animated discussion sessions and in the people to whom he introduced me (Weil, Selberg, Bers, Carleson, etc.)—is one that I am deeply grateful for and will always remember.

During my days as an impressionable student/apprentice I would have to say that, in his old-fashioned way, Ahlfors taught me much about being a mathematician and about constantly seeking to achieve something of quality.

One hears today various things about "the dumbing down of society". In my dealings with Lars Ahlfors I always got the impression that here was somebody who truly believed in excellence, someone who not only would *not* lower the bar but who would instead *raise it* to facilitate bringing out talent that he intuited was there.

I feel myself exceedingly fortunate to have encountered someone like this in my formative days as a mathematician. One could not have asked for a better start.

Troels Jorgensen

It was my good fortune to know Lars Ahlfors for a quarter of a century. He was an inspiring mathematician and a very special person. His intuition for and joy in mathematics, combined with a great capacity for work, made him a harmonious craftsman through a career that spanned over sixty years. He savored the recognition and awards he

¹*"If I have seen further than others, it is by standing on the shoulders of giants."*

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Ahlfors's Students

Lars Ahlfors had a large number of Ph.D. students, many of whom are leading mathematicians today. The most complete list that could be assembled includes these names:

- Robert D. M. Accola, Brown University
- S. Thomas Bagby, Indiana University
- Harvey Cohn, City University, New York
- Clifford J. Earle Jr., Cornell University
- John Fay, St. Joseph's College
- Paul R. Garabedian, Courant Institute
- Dale Husemoller, Haverford College
- James A. Jenkins, Washington University
- Roger B. Kirchner, Carleton College
- Saul Kravetz (deceased)
- Henry Landau, AT&T Bell Labs
- Albert Marden, University of Minnesota
- Peter Evans Martin, Dickinson College
- Audrey Wishad McMillan (retired)
- Robert Osserman, Stanford University, now at MSRI
- Henry O. Pollak, AT&T Bell Labs
- J. Ian Richards, University of Minnesota (deceased)
- Halsey Royden, Stanford University (deceased)
- Ernest C. Schlesinger, Connecticut College
- George Sethares, Bridgewater State College
- George Springer, Indiana University
- F. Ulrich (deceased)
- Eoin Whitney (deceased)
- Vidar Wolontis

There were so many others who found inspiration in his work. A partial list includes Lipman Bers, Arne Beurling, Christopher Bishop, Frederick Gardiner, Frederick Gehring, David Hamilton, Dennis Hejhal, Peter Jones, Troels Jorgenson, Linda Keen, Irwin Kra, Bernard Maskit, Howard Masur, and Seppo Rickman.

received yet was very modest about his accomplishments. At a banquet celebrating his seventieth birthday Ahlfors said that "He liked to go fishing where the fish are, rather than trying exclusively for the big one."

Lars was grateful for the opportunity to serve at Harvard and took much pride in his colleagues, enjoying their professional successes as well as their good company. A strong constitution allowed him to stay fit without spending much time on exercise and to share many a happy evening with his family and friends from around the world. Yet he was always ready for work the next morning. As with his mathematical interests, his interest in other human affairs was often strong and addressed forcefully: "That is just it! That is just it!" would resound when something met with his approval. One could have a wonderful time discussing almost any matter with Lars.

Lars often recalled his delight when, as a young man, he found out that one could become a mathematician. The ultimate blessing, however, was his lifelong happy marriage to Erna. The example set by Lars and Erna of affection for each other and



Ahlfors with wife Erna around 1987.

for the whole world should be remembered, along with his mathematics, with deep admiration.

Albert Marden

Lars Ahlfors has been a role model of course for his taste and style in mathematical research, but also for his conduct of professional life.

As an undergraduate I took 2.5 courses from Lars: Math 105 (Advanced Calculus) as a sophomore in 1953, Math 213 (Complex Analysis) as a junior, and Math 240 (Conformal Invariants) as a senior. It was characteristic of his sensitivity that he gave me an *A* in the latter course. As I look back over my meticulously taken course notes, it is painful to recognize how immature I was, too immature to understand the material. Instead, what penetrated was the clarity, elegance, and utter authority with which Ahlfors wove together analysis and geometry to resolve a seemingly intractable problem. I remember following him into the Sever Hall faculty room as he lit up a cigarette and considered how to answer my questions. After getting medical school out of my system, I began thesis work under Ahlfors about 1960. I generalized some of Accola's results connecting extremal length of homology classes to various canonical harmonic differentials and finished in 1962.

In 1981 I arranged for Lars to be invited for a quarter as the first Ordway Professor at Minnesota. That was at the beginning of his search for higher-dimensional analogs of the theory of Kleinian groups, in which he was the pioneer. Later, Burt

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Rodin brought Lars and Erna to La Jolla for a quarter, and, thanks to Burt, I had the good fortune to be there as well. We watched the sun set from Lars and Erna's apartment on the shore as we enjoyed their incomparable hospitality.

By the early 1980s Thurston's work had transformed the subject of Kleinian groups beyond all recognition. Lars had great respect for Thurston but railed against the lack of detailed written proofs, as this was a great frustration to his effort to understand the new theory. Lars was a genius at computing and finding the right perspective and formula for a situation, but he did not have the insight and turn of mind of a three-dimensional topologist.

In our time perhaps only Nevanlinna himself had anything approaching the universal esteem with which Lars was regarded in complex analysis. One reason was that Lars made germinal contributions to most of the active areas in the field. One could argue that Ahlfors defined the field by the scope of his work. Another reason was the objectivity and evenhandedness with which Ahlfors regarded and supported other mathematicians. One did not gain his preference simply by living in a particular political territory. Many appreciated his formal, refined, and reserved personal style; at the same time he was tolerant and understanding.

Raoul Bott

Ahlfors was a concept for me long before we met when my family and I moved here (to Cambridge) in 1959. Nevanlinna had visited our former home in Michigan, and in any case Lars's name came up often there in a department much concerned with the theory of complex variables. Much later, in the 1970s and in a collaboration with Chern, I was finally introduced to Lars's work firsthand and even had the foolish hope of trying to improve on it. This was his virtuoso achievement on equidistribution, to which Herman Weyl alluded as "a vineyard planted by that grand gardener from the North."

And so now—miracle of miracles—in 1959 it appeared that we would be colleagues henceforth! At that time the department fell into several age groups. Walsh and Widder were the most senior, in their sixties. Then came the trio of "foreigners": Ahlfors, Brauer, and Zariski. These were followed by Garrett Birkhoff, Lynn Loomis, George Mackey, and Andy Gleason. John Tate and I were

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the new and youngest additions to the professorial stable at ages thirty-five and thirty-six. Actually, in 1959 Barry Mazur, Shlomo Sternberg, and David Mumford were also already on board as Junior Fellows and assistant professors.

I was of course scared to death to join a department that included such names as Ahlfors, Brauer, Zariski, and I tried hard to be on my best behavior. The department was extremely hospitable; we were wined and dined, and soon learned that nobody would bite us.

Still, it was only after a delightful evening at the Ahlforses' that Phyl and I truly felt at ease here. Somehow, in their house, matters of protocol or hierarchy—as well as being on one's good behavior—were completely missing. We were there to have a good time and to bare our souls to each other, helped along with the obligatory Skol in vodka, often followed by champagne.

In the department Lars was a quiet, reserved, and benevolent presence. He was always perfectly attired, attended the meetings punctually, and only now and then entered the discussions, often decisively. For me it was always a pleasure to run into him in the corridors. It reassured me: this was indeed a good place to be.

At home Lars was the warmest of hosts, a delightful drinking companion, a devotee of beauty and class in every form—from literature, music, and painting to the admiration of the opposite sex. Yet there was no impropriety in Lars's gallantry. All his life he was enthralled with his wonderful wife, Erna, who had such a mercurial way of dealing with his natural Finnish reserve and his shyness.

I now remember with special affection a summer month we spent together with the Ahlfors family on the coast of Rhode Island, with the many minor misadventures that invariably occur in a joint rental venture. For instance, each family had a dog, both male, who unfortunately would not tolerate each other. So I remember playing bridge with Erna and Lars and Phyl after our young brood had been put to bed; Lars and I would each hold a growling dog at our feet, with some sort of makeshift barrier between them. Amazingly, this did the job; "out of sight, out of mind" really works for dogs.

In other ways we were not so evenly matched: our four youngsters, two of them under five years old, against one spirited teenager of the Ahlforses'. Actually, on second thought and on observing my grandchildren, maybe four kids under five years of age is an even match for one child over fourteen. In any case, it was a memorable month, with Erna and Phyl outdoing each other with their consummate culinary skills.

Lars was a great Harvard patriot. On many occasions he delivered himself of the opinion that Harvard was the best of all universities and the

Boston/Cambridge area the most desirable place to live. Of course, he came here at a time when Harvard had a rather grander way about it. At dinner parties Lars would reminisce: it was understood that men would turn up in black tie and the ladies in evening gowns. Although he relished these memories, he was ever on the side of us youngsters and did not seem to mind our much more slovenly ways.

Lars was also a man of action and had an instinct for achieving his ends in the simplest way, be it in the exposition of his mathematics or in real life. When, already in retirement, Lars found himself accosted by a knife-wielding assailant on the doorstep of his Beacon Street apartment in downtown Boston, he did not hesitate. It happened that he was returning from a shopping expedition with a bottle of whiskey (first class, of course) under his arm. So he instinctively hit the chap over the head with the bottle and managed to open his door to safety before the assailant recovered his wits. Thereafter, it was mainly the loss of that fine whiskey that Lars lamented.

Let me end by giving thanks that we have had Lars among us for so many years. We must spend our time not in mourning but in celebration of a wonderful life—a life of brilliant achievement, great professional success, lived amidst much human warmth and mirth. Yes, there was great tragedy also. But who, in a life span of nearly ninety years, can avoid that, especially in this century of ours?

And so we must be consoled by the thought that mortal man cannot wish for much more than to be remembered, not only as a pioneer in the annals of the culture of his day, but also with love, affection, and gratitude in the hearts of his family, friends, and colleagues.

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