Commentary

In My Opinion

What Do Engineers Really Want?

The revision in ABET standards, dramatic decline in calculus enrollments, and proposals to shift some mathematics instruction to engineering faculty have focused attention on the design of mathematics courses for students in engineering. Many mathematicians believe that this is a call for cookbook courses that emphasize formulas and computation. Some may even feel that they would rather abandon the courses than the mathematics they hold dear. But I claim that a better understanding of what engineers and physical scientists really need to know can make applied mathematics *more*, not less, satisfying to teach. Let me illustrate with a few examples from the traditional calculus curriculum.

Example 1: Some years back a well-known mathematician publicly stated that asking students to differentiate something like $x^{\sin x}$ was silly because no one in his physics department would know how to work such a problem or would care. But he was only half right. *Every* physicist I asked found the problem unfamiliar but immediately rewrote the function as $e^{(\sin x)(\ln x)}$ and realized that differentiation was then straightforward. Although engineers and physicists may not use functions like $x^{\sin x}$, or even $5^{\sin x}$, they *do* need to know that $a^b = e^{b \ln a}$. Thus, the real issue is not "Will an engineer encounter this problem?" but "Is asking students to differentiate $x^{\sin x}$ an effective way to teach the meaning and use of exponential functions?" Is there a more effective way? Is memorizing a formula for differentiating a^x counterproductive?

Example 2: Too often traditional topics are taught year after year as if they were ends in themselves, with little memory of the motives for introducing them into the curriculum. I can see two important reasons for teaching the standard "solid of revolution" volume problems. One is to provide students with experience relating a practical quantity that is easily approximated by a sum to a definite integral. The other is that converting a problem such as "compute the volume remaining when a hole of radius r is drilled through the center of a sphere of radius R" to an integral involving specific functions may help the student develop the skill needed to analyze more realistic applied problems. However, if we confine ourselves to asking them to use a standard formula to find the volume when $f(x) = \ldots$ is rotated around the x-axis between $x = 1$ and $x = 5$, then *neither* goal will be met.

Example 3: A chemist once complained that he taught the chain rule in physical chemistry because the students did not learn the multivariable form in calculus. How can this be true? Chemists, physicists, and engineers will never need to compute anything like $\frac{\partial}{\partial x} y \sin(x^3 y)$, much less subsequently find $\frac{\partial}{\partial u}$ under some bizarre change of variables. Even transformations to standard, e.g., spherical, coordinates rarely require the explicit use of the chain rule, because the formulas for divergence, Laplacian, etc., are readily available. But the thermodynamic relationship

$$\left(\frac{\partial u}{\partial T}\right)_P = c_V + \left(\frac{\partial u}{\partial V}\right)_T \left(\frac{\partial V}{\partial T}\right)_P$$

is nothing more than a rewriting of the chain rule and the definition $c_V = \left(\frac{\partial u}{\partial T}\right)_V$. Chemical engineers and physical scientists use the chain rule to study the relationship between various quantities that arise in thermodynamics. Much mathematical meat is buried in the (possibly unfamiliar) notation $\left(\frac{\partial V}{\partial T}\right)_P$. First, it is assumed that P, V, T satisfy an "equation of state" of the form $f(P, V, T) = 1$ and that this implicitly defines three functions $P = p(V, T)$, etc. Then the expression above means $\frac{\partial V}{\partial T}$ in the usual sense, with the subscript P to indicate that V is regarded as a function of P and T. Developing relationships such as

$$\left(\frac{\partial P}{\partial V}\right)_T \left(\frac{\partial V}{\partial T}\right)_P \left(\frac{\partial T}{\partial P}\right)_V = -1$$

requires a basic understanding of the meaning of the implicit function theorem as well as the chain rule. Why should we leave this instruction to physical chemists and engineers? True, neither they nor their students want rigorous proofs. But not all theory is irrelevant. The implicit function theorem will fail at precisely those points where a phase transition occurs. We counter the attitude that counterexamples are mathematical pathologies by explaining how the failure of certain hypotheses is related to real physical phenomena.

Some may object that we are beginning to cross the line and teach things best left to engineers. But the boundary is fuzzy precisely because mathematical concepts, not just formulas, are important in applied fields. We should not strive for zero overlap, but for better communications between mathematicians and those in other fields.

—Mary Beth Ruskai
Associate Editor