The AMS and Mathematics Education: The Revision of the "NCTM Standards"

Roger Howe

Readers of the *Notices* will be aware, through recent articles by Allyn Jackson [1] and maybe otherwise also, that there is currently great ferment in mathematics education, from kindergarten through graduate school. Today's mathematics education reform (=: mathedreform) movement advocates sweeping changes in the philosophy, pedagogy, and content of mathematics education in the U.S. If implemented thoroughly, it will have widespread effects on American society; in particular it would affect future mathematical research. Thus, it behooves the AMS to work to ensure that the changes are beneficial to American children and to American mathematics.

The NCTM Standards

The central documents in the current debates on K–12 mathematics education are the "NCTM Standards" [2], a collection of three volumes containing ideas about curriculum, pedagogy, and assessment published by the National Council of Teachers of Mathematics (NCTM). These have been enormously influential, especially among bodies which set broad educational policy. For example, many state "frameworks" for the mathematics curriculum have borrowed heavily from the "Standards", sometimes adopting them almost wholesale. They have also inspired the creation of numerous novel curricula. In particular, the Division of Education and Human Resources of the National Science Foundation (NSF) has sponsored several projects for the development of "Standards"-based curricula [3]. The enormous public relations success of the "Standards" has placed NCTM firmly in the driver's seat of the school mathedreform bus.

NCTM is now planning to issue a revised version of its "Standards", tentatively in the year 2000, and it has assembled a writing team for the purpose. There have been calls for revision from within the mathedreform community for several years [4], and some have always considered a revision to be part of the plan. Revision will allow NCTM to take into account extensive comments from some university mathematicians and to eliminate ambiguities which have allowed, most visibly in California, what some regard as excesses in the direction of changes suggested in the "Standards" [1, 5].

To further the revision process, NCTM has solicited input from most of the other mathematics professional organizations (AMS, MAA, AWM, ASA, ASL, SIAM,…). These have responded by creating committees, known collectively as ARGs (Association Resource Groups) to provide comment as solicited by the NCTM writing group. The AMSARG was appointed as a subcommittee of the Committee on Education (CoE) by Hyman Bass, chair of CoE. AMSARG consists of Richard Askey, Wayne Bishop, Roger Howe (chair), Alfred Manaster, David Moore, Judy Roitman, Mark Saul, and William Thurston. (Manaster and Roitman are also on the NCTM revision writing team.)

So far NCTM has solicited two reports from AMSARG. NCTM's questions and our reports are reproduced elsewhere in this issue of the *Notices*. To provide some background for the reports, this article summarizes some of the most basic issues in the mathedreform debate, then gives a brief sum-

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mary of the reports. This should be sufficient background for understanding most of the remarks in the reports, although some of the comments will not be thoroughly intelligible to someone not familiar with the "Standards".

Though AMSARG is not large, we have found that we are far from being of one mind on many issues, and the debates we conducted to frame our reports were both intense and extensive. Nevertheless, after the discussions there were significant points of agreement, as formulated in the reports. Very probably there are aspects of the issues that were not reflected in our thinking, and AMS members with ideas they think would illuminate NCTM's questions in useful ways are invited to communicate with us by writing to: Ms. Monica Foulkes, American Mathematical Society, 1527 18th St., NW, Washington, DC 20036-1358, e-mail `mxf@ams.org`, with a cover note indicating a message for AMSARG.

Issues

The debate on mathematics education is complicated. It involves many overlapping, sometimes contradictory, concerns. Here are some of the issues that figure in discussions of K–12 education.

1) Relative performance.

International comparisons of mathematics competence have regularly put the U.S. far behind the leaders in aggregate performance. The latest and largest such comparison is TIMSS (Third International Mathematics and Science Study). In TIMSS, U.S. eighth-graders come out below the median among the forty participating nations [6], and fourth-graders somewhat above, but still in the middle of the pack [7]. In studies with fewer participating countries, the U.S. has tended to place near the bottom. Asian countries tend to do well. Harold Stevenson and colleagues have presented a detailed picture of contrasts between U.S. and Asian mathematics classrooms (see, e.g., [8]) in practice and achievement.

2) Equity.

It is no secret that in the U.S. achievement varies widely from state to state and within any region tends to increase with community wealth. Some states put in performances which bear international comparison; others would come in near the bottom of any heap [9]. Similarly, suburbs outscore inner cities. This geographical variation tends to get correlated with ethnic data. Though not related to geography, gender variation gets attention too as an equity issue.

3) Technology.

Technology affects mathematics education in at least three distinct ways:

(i) The increased role of information processing in society creates new job opportunities for mathematically apt people, and it creates demand for higher levels of mathematical training for the less apt.

(ii) Computer technology can be used in the classroom. Calculators and computer graphics software offer alternatives to chalk for presenting mathematics and invite consideration of how they can best be used.

(iii) On the most fundamental level, technology requires rethinking not only of the "how" but of the "what" of teaching mathematics. It is pretty clear that in the future no one is going to get a job based on the ability to add long columns of numbers accurately. Recently we have seen the appearance of calculators and computer software that can perform much of the repertoire of undergraduate mathematics and beyond. Even if everything had been fine with U.S. math education, we would have to pay attention now to how the availability of sophisticated calculational tools changes what is important to teach. The automation of computation challenges the notion that mastery of computational technique should be the main criterion of mathematical success. The relation between computational expertise and conceptual understanding, and how each supports the other, is complex and requires careful study and thought.

4) Demography.

More students are taking more advanced degrees, and vastly more mathematics, than was the case when today's senior academics were getting their degrees. Over 750,000 students a year take calculus [10, 11], which has become a prerequisite for pursuit of a large variety of desirable careers. A larger percentage of a given age cohort now does graduate study of some kind than matriculated in colleges before World War II. A useful mantra here is "College is high school; graduate school is college", but even that does not account for the extent of the change. At the same time that mathematics course enrollments were swelling, the average SAT scores of college matriculants were decreasing. (Both these trends seem to have reversed to some extent in the 1990s.) The remarks above refer to the demography of academia and of mathematics courses. There is also the society-wide trend toward a more diverse population.

5) Subject matter.

Much of the school curriculum has been in place for a long time. One can, and probably should, ask if there are parts which are no longer germane. Also, research has created new fields of inquiry and application. Statistics figures prominently here. Some pieces of operations research and discrete mathematics also vie for a place in the curriculum. Although it is not much discussed, it should be relevant to AMS members that research in pure mathematics has shed new light on topics of school mathematics, and this insight has only very partially and imperfectly trickled down to the K–12 classroom.

6) Pedagogy.

a) Classroom procedure. The traditional lecture is questioned as the sole means of conveying subject matter. Lectures are admitted to be an efficient way to present information, but absorption rates are questioned. More varied practice is advocated. Doctrines of "constructivism", which emphasizes the role of the internal mental processes and installed database of the individual student in his or her learning, enjoy wide currency among educators. Small group work is often put forward, sometimes as a corollary of constructivism, as a way to ensure "active, hands-on" learning. Problem solving is put forth as a major method and goal. Contextualism, the view that school mathematics should be rooted in the problems of the real world, asks more specifically for "real-world problems" with "real data". There are calls for deemphasis of memorization and of repetitive practice.

b) Rigor and technique. AMS members should find it easy to approve of calls for "mathematical power", "increased understanding", and "mathematical reasoning". However, implementation of teaching consistent with these slogans may involve tradeoffs that some will find troublesome. In this area, there are two issues especially relevant to the AMS. One is how far mathematical reasoning should be developed, in particular, whether it includes an appreciation of and ability to deal with proof. A second concern is the extent to which technical skills are necessary for conceptual understanding, or, put more starkly, how far technical skills can be neglected before understanding and mathematical power also flag. The technique debate has been of concern to the general public also. Deemphasis of "basic skills" has been one of the red flags that have aroused parental ire in some communities.

7) Teacher preparation and certification.

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Dear Roger,

This follows up on an earlier letter in which Gail Burrill asked that you set up a Review Group within the American Mathematical Society to help us as we update the NCTM Standards for the coming century. We are very pleased that you have appointed such a group, and we look forward to fruitful collaboration as we proceed with this project.

Our first request is to get your reactions to the statements of the content standards as they are now phrased. To that end I have enclosed copies of the "barebones" curriculum standards from the 1989 *Curriculum and Evaluation Standards for School Mathematics*, without any of the explanatory or illustrative material that followed each of them. (This set of standards is on green paper.) With regard just to these curriculum standards, here are three questions to which we would like your Review Group to respond:

1. Your view of Mathematics. Consider the nature of Mathematics—its content, processes, and procedures—that you feel is important for students from pre-Kindergarten through grade 12. Do the current statements of the Standards adequately communicate your view of the discipline?
2. Consistency and growth. Do the statements of the current curriculum Standards convey a sense of consistency and growth in content themes as the student moves across the grade levels? (Content themes would include, for example, ideas of measurement, number sense, and algebraic and geometric thinking.)
3. Expected understanding of content. Do the statements of the content Standards adequately reflect the mathematical understanding expected of a student graduating in the 21st century? Do they reflect the needs of students who are planning post-secondary study in a Mathematics-related discipline?

As you know, one of the goals of the updated document is to merge the classroom-related aspects of curriculum, teaching, and assessment so that the interrelationships among them are clear. (We have also enclosed standards from the *Professional Standards for Teaching Mathematics* and from the *Assessment Standards for School Mathematics*, on blue and gray paper, respectively.) Exactly how this is to be done is the subject of the fourth question:

4. Blending the three sets of Standards. The NCTM project to reissue a Standards document in the year 2000 intends to meld together the dimensions of content, teaching, and assessment. What suggestions could you make as to the most effective ways of blending these ideas?

The Commission on the Future of the Standards and the Writing Groups will be reconvening again on February 1, 1997, and we would very much like to have your input by mid-January so that we can compile responses from several associations in time for that meeting. Of course there will be other opportunities for you to provide advice later. We also welcome your group's suggestions of other ways they can be involved. Some groups are thinking about holding forums or discussions, although we are not specifically requesting this now.

We appreciate your willingness to help NCTM with this important project. Please let me know if there is anything I can do to facilitate your response.

Sincerely yours,

Mary M. Lindquist
Chair

Enclosures

There is substantial evidence that teachers in other countries, especially high-scoring countries like Japan, know more mathematics than do U.S. teachers [8, 12, 13]. Assuming that it would be desirable for U.S. teachers to have better mastery of their subject matter, it is not obvious that the simple step of requiring more mathematics courses for future teachers will achieve this goal. Standard undergraduate courses do not dwell heavily on the relevance of their subject matter to the high school curriculum, and teacher candidates may not be able to make the connections on their own.

8) Assessment.
Along with calls for changes in subject matter and pedagogy, there is a strong chorus urging changes in the way mathematics achievement is evaluated. Multiple-choice tests are decried as inadequate. There are calls for tests which require writing and human evaluation. There are calls for evaluation by means of "portfolios" of accumulated work and for complex profiles of progress to replace simple letter grades. Debates about measurement complicate all other aspects of mathedreform. They cloud the criteria for success, and they make agreement on what constitutes progress harder to reach.

9) High performers.
This issue is relatively neglected in K–12 discussions but is one that should concern AMS members. There is plenty of evidence of problems here. In the TIMSS only 5 percent of U.S. students performed in the top 10 percent internationally [6]. In graduate programs a declining and now minority portion of students are U.S. born [14].

10) New curricula.
There has been extensive development of new mathematics programs which attempt to implement the reform ideas. NSF has sponsored several programs at the high school and junior high school level, and there are also commercially developed new programs. These programs have been a rich mine of controversy. It is remarkably difficult to get a clear picture of their effects [1]. There is heartfelt anecdotal testimony both positive and negative. When statistical evidence, favorable or unfavorable, becomes available, its significance can be disputed on the basis of the issues sketched above, as well as on technical statistical grounds.

The list of ten issues above attempts to present the main ingredients of mathedreform as distinct issues, although some interactions are noted. In the give-and-take of debate, the issues of course interact in complex ways, and sometimes the interactions are comparable or even greater in importance than the separate ingredients.

The ARG Reports

Here is a brief summary of the reports of AMSARG. The full reports appear on pages 270–276 in this issue of the *Notices*.

First Report: This was in response to general questions about the "Standards". The main point of the AMSARG report is that the revised "Standards" need to be more specific and less ambiguous. Particular areas in which we sought greater emphasis and clarity are: (i) in the role of the teacher as a leader of the class and as guarantor of mathematical closure, (ii) in the time development of the curriculum—specification of what should be learned when, (iii) in clarifying the notion of algorithm as a key feature in the automatization of mathematics, (iv) in developing the notion that proof and logical argument are the appropriate continuation of successful nurturing of mathematical reasoning ability, and (v) in articulating the connections between algebra and geometry. We also made a suggestion about the organization of the "Standards" revision.

Second Report: The second round of questions focussed on the ideas of proof and algorithm. Our report articulated some of the important features of algorithms and of the concept of algorithm. It also addressed the role of the algorithms of arithmetic. In this connection it noted that decimal notation is itself a highly sophisticated algorithm, whose structure strongly constrains the form of algorithms for performing arithmetic operations. Concerning proof, we stated our sense that mathematics should be taught as a subject which makes sense in itself and which helps to make sense of the world. We also argued that proof is the ultimate formulation of this sense-making practice and that ability to understand and construct proofs should be the end result of the successful development of reasoning skills. We did not advocate strict axiomatic developments of whole areas, but rather a practice of "semilocal" proof, in which substantial numbers of results are shown to follow from a much smaller set of accepted principles. We stated a preference for developing specific examples of abstract structures rather than a formal treatment of them in high school.

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