Book Review

The Pleasures of Counting

Reviewed by Brian E. Blank

The Pleasures of Counting
T. W. Körner
Cambridge University Press
534 Pages
Hardcover (1996) $85.00
Paperback (1997) $34.95

For more than two thousand years some familiarity with mathematics has been regarded as an indispensable part of the intellectual equipment of every cultured person. Today the traditional place of mathematics in education is in grave danger. Unfortunately, professional representatives of mathematics share in the responsibility. The teaching of mathematics has sometimes degenerated into empty drill in problem solving. Applications and connections with other fields have been neglected. Teachers, students, and the educated public demand constructive reform. The goal is genuine comprehension of mathematics as a basis for scientific thinking and acting.

The reader has no doubt heard words to this effect many times in the last decade. These ideas, however, were not spawned by the current debate over the college mathematics curriculum. In fact, the first paragraph is a lightly edited extract from the preface of *What Is Mathematics?* [1], written by Richard Courant and Herbert Robbins in 1941. In writing their text, Courant and Robbins sought to acquaint their readers with the "content of living mathematics." Although they cautioned against "the dangerous tendency toward dodging all exertion," they considered their book to be popular in that it required only "a good high school course" by way of background.

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And it was a popular book. I remain very fond of it for reasons that are personal as well as objective. As a high school student I had access to numerous algebra and trigonometry books that those of a certain age and British Commonwealth lineage would associate with the style of *Hall and Knight*—unremitting drill, for those who have not shared the experience. But *What Is Mathematics?* was the only book in my high school library that suggested the *depth* of mathematics. Here was a book, after all, that discussed the Prime Number Theorem in the first chapter. I suspect that I am not the only one who learned from Courant and Robbins that the trees actually make up an interesting forest.

For better and for worse, times do change. Certainly the direction of mathematics is constantly shifting as new fields find favor while others fall by the wayside. One might expect Courant and Robbins to choose their topics differently were they writing their book today. Linkages and Mascheroni compass constructions might be dispensed with, continued fractions and projective geometry curtailed. But an even more fundamental dynamic would necessitate another approach: a good high school course is not what it used to be. Reading the precise definition of continuity and then using it to prove the Intermediate and Extreme Value Theorems requires an intellectual discipline that is no longer within the proximate reach of the high school graduate. It is very difficult nowadays to imagine sending a good college freshman to Courant and Robbins to find out what mathematics is about. Nor have many alternatives been written: one can envision the difficulties.

In the preface of his new book, *The Pleasures of Counting,* T. W. Körner writes "This book is meant, first of all, for able school children of 14 and over and first year undergraduates who are interested in mathematics and would like to learn something of what it looks like at a higher level.…the aim is so worthwhile and the number of such books so limited that I feel no hesitation in adding one more." There will be little hesitation among the readership of the *Notices* in welcoming a book that achieves the stated aim as engagingly as *The Pleasures of Counting* does. There remains little for the reviewer to do but to convey some of the topics that Körner has selected.

The first part of *The Pleasures of Counting* is concerned with the use of abstraction. Several historical problems have been selected as subjects of mathematical analysis by way of example. When, for instance, repeated epidemics of cholera ravaged Europe in the mid-nineteenth century, both the practice of medicine and the practice of statistics were in their infancy. Various theories were advanced to explain the spread of the disease, but medical science was not then able to validate any hypothesis. In 1849 Dr. John Snow proposed that the agent of the deadly illness was spread through contaminated drinking water. What distinguished Snow's conjecture from the others was not so much that it was the correct one, but that he undertook the collection of data to support it.

Conditioned as we now are to this simple act of counting, we will only with difficulty appreciate the underlying abstraction. Consider that statistical argument is only about two hundred years old, even though it might have originated at any time in the history of modern man. In the face of real monetary expenditure, Snow's argument must have seemed very abstract indeed. Incontrovertible statistical evidence was not enough, immediately, to justify the expense of water treatment. Response was predictably slow. When a virulent outbreak of cholera struck London in 1866, Snow was again able to trace the source to tainted water despite the denials of water company officials. Tightwads in Hamburg were even slower to react. That city was the site of the last major European cholera epidemic over a century ago.

This compelling case study gets Körner's book off to a fine start. Unfortunately it does little to illustrate the statistician's practice. So overwhelming are Snow's data that the need for statistical inference does not really arise. The next of Körner's case studies concerns the efficacy of aspirin, streptokinase, and the genetically engineered drug tPA as agents for the treatment of blood clots. It offers a greater opportunity to clarify the role of the statistician. One therefore regrets that the misconception of the statistician as a collector (as opposed to an analyzer) of data is not dispelled. That is my only criticism of the section. The more advanced reader might have already found in Körner's *Fourier Analysis* [3] an extended statistical exposure of Sir Cyril Burt's fraudulent IQ data. The reader who is disappointed with the much shallower discussion of statistics in the work under review should remember Körner's targeted audience. To his credit, Körner resists the temptation to dazzle. His goal is to coax the interested freshman into a deeper study of mathematics. Some restraint is properly brought to the task.

If this review is to serve as any kind of guide, then I am obliged to acknowledge that Körner had an ulterior motive when he chose to analyze thrombolytic agents. There are villains in his story—the profit motive of the drug company producing the expensive drug tPA and the biased coverage of the financial press. Here, as in many other parts of his book, Körner has an opinion. It is not his style to serve up a dispassionate analysis. Although his point of view often comes across unmistakably, it is not expressed in a heavy-handed fashion (excepting, perhaps, in his insistent use of "she" as the default third-person pronoun). Nonetheless, it is undeniable that the statement of an opinion is sure to provoke hostility among some readers. I can only counsel those who know themselves to be intolerant of opinion to seek out blander writing elsewhere.

The application of statistics to medicine behind him, Körner turns his attention to the uses of calculus in war. Consider, for example, the policy of convoying cargo ships as a defensive measure against submarine warfare. There are some obvious advantages to the strategy, but there are also several strong counterarguments: one has only to picture a large group of targets moving en masse no faster than its slowest component.

How does one decide if convoying is beneficial? The statistician, no doubt, would be able to devise some controlled experiments. The Admiralty, on the other hand, wanted an answer before all the data came in. The analysis of the convoy problem provides a wonderful introduction to mathematical modeling. It is at this early point in his book that Körner might begin to lose that fourteen-year-old mentioned in his preface. After a general discussion of considerable interest, the payoff, in the form of a differential equation, is shunted into an exercise. Körner plainly does not want his reader to dodge all exertion. The advantage of his approach is that he does not have to bring the level of his book down to that of the least-prepared reader. Instead, he uses his preface to caution the budding mathematician to expect some difficulties ahead. Were professional mathematicians to understand everything in a mathematics book, he explains, they would suspect that the material was too easy to be worthwhile. This is an important pedagogical point. If it were more forcefully made as part of the orientation of entering university students, then the responsibilities of mathematicians as educators would not be so misunderstood.

The second section of *The Pleasures of Counting* is called "Meditations on Measurements". The name is appropriate, both for its reference to the central theme of measurement and for the indication that more than hard science will be involved. Measurements, it must be admitted, often contradict fine theories. Biologists have long recognized that metabolic rate does not behave the way that it should. Specifically, as mammalian surface area and volume grow according to second and third power laws respectively, the relation between metabolic rate R and mass M should take the form $R \propto M^{2/3}$. How embarrassing, then, that a plot of $\log(R)$ as a function of $\log(M)$, the famous "mouse to elephant plot", yields a line of slope $3/4$. The $3/4$-power law remains a mystery. Although Körner convincingly argues that a $2/3$-power law might be too simple-minded, I do not see anywhere in his explanation why the no less simple-minded $3/4$-power law seems to be so effective a model. Moreover, when the author steps out from his role as mathematician, his expertise may be questioned. In discussing variation in design, Körner asserts that "there are no small mammals in arctic regions." One looks to the marmot, the tundra vole, and the lemming for counterexamples. Even the arctic fox is small, presumably as an adaptation to its environment. But never mind. The scaling arguments that Körner passes on are generally convincing. One learns, for example, why whales are so large, why squirrels climb with so little effort.

It is not a very great leap from scaling in biology to dimensional analysis in the physical sciences, and Körner does not disappoint. The power of dimensional analysis cannot fail to impress the student who encounters the method for the first time. Unfortunately, it is a method that is egregiously absent from standard courses in mathematics and physics. At the heart of the matter is the rarely explicated Buckingham π Theorem. Although a good treatment may be found in the excellent text of Logan [4], Körner's primary reference, few students will find their way to it there. Körner is therefore to be applauded for including a lengthy discussion that is supported by several examples. As is typical in this book, the examples have been chosen both for their importance and for their entertainment value. In one application after another, through dimensional arguments alone, we learn why helicopter blades are long, why fatty deposits on coronary arteries eventually become dangerous, and why the nationality of tanker crews influence the design of tanker engines.

The discussion of dimensional analysis is a handy gateway to an issue that the reviewer ought to at least feebly bring up. There are rather a lot of minor errors in the review copy. The dimensional analysis of the simple pendulum, for example, is marred by incorrect formulae. In fact, the thinking student will be able to use dimensional analysis to deduce that the formulae presented must be wrong. These errors, and an impressive list of others, are corrected on a Web page that Körner maintains. The URL is `www.dpmms.cam.ac.uk/home/emu/twk/.my-book-cor.html`. One supposes that later printings will correct the errors. Until then I suspect that very few readers will be seriously inconvenienced by these minor mistakes. Indeed, the list of errors speaks to the attentiveness of the book's readership.

Körner concludes his meditations on measurement with an extended account (two chapters totalling 68 pages) of the life and work of Lewis Fry Richardson, 1881–1953. This is one of the best parts of the book, amply justifying the lavish treatment of its subject. As a student Richardson studied mathematics, physics, chemistry, botany, and zoology. His eclectic student interests would be a harbinger of his diverse scientific output. In early positions Richardson was employed as both a chemist and a physicist. In 1912 he patented an early sonar device. The next year Richardson joined the Meteorological Office. There he became a pioneer in using mathematics to predict the weather, publishing *Weather Prediction by Numerical Process* in 1922. He later made important contributions to the study of atmospheric turbulence. The "Richardson number", a fundamental meteorological constant, is named after him. Richardson also applied mathematics to study the causes of war, publishing the *Statistics of Deadly Quarrels* in 1950. *The Pleasures of Counting* contains an introduction to all that I have described and more. Körner has done a great service in bringing the mathematical work of this exceptional scientist to wider notice.

The third part of *The Pleasures of Counting* is concerned with "The Pleasures of Computation". It is more conventional. The contents include the Euclidean algorithm, the Fibonacci numbers, Turing's Theorem, and an excellent discussion of the sorting problem (the Knock-Out Method, specifically). But there are less familiar things as well. The "Railroad Problem" or "Max-Flow Problem" asks how to get the maximum number of trains between two points on a network. The Ford-Fulkerson algorithm that solves the problem is given in detail. Nonspecialists who have not kept up with their graph theory might learn for the first time the relatively recent Braess Paradox: adding links in a congested network can be counterproductive.

The chapter that concludes with Turing's Theorem is called "Deeper Matters" and begins with the section "How safe?". It deserves special attention. Körner recounts the cautionary tale of the London Ambulance Service's computerized dispatch system. Designed to have ambulances reach 95%

of emergencies within fifteen minutes, the system did not reach the 20% mark on its first day of operation and only got worse thereafter. The situation improved after management switched to a semicomputerized system, but that soon crashed: errant code left over from a previous patch caused a memory leak of fatal proportions. The £1.5 million system had to be abandoned.

I have used the term "cautionary", as did Körner, but of course the tale of the London Ambulance Service is no such thing. If it were cautionary, we would not have had the Denver International Airport baggage fiasco only a few years later. As you might recall, a $200,000,000 computerized baggage-handling system was designed using 300 computers to guide 4,000 cars around 21 miles of track. In theory it was a marvel. In practice it not only destroyed the baggage it carried but also its own telecars and even its own tracks. As for errant code left over from an earlier software overhaul, we have a recent ghastly example. In August 1997 the veteran pilot of Korean Air Flight 801, in full control of his aircraft, flew into a hillside in Guam. Two hundred and twenty-six on board were killed. As of this writing no official cause of the accident has been released. But we do know of one reason why the crash was not prevented. The Radar Minimum Safe Altitude Warning system was intended to work when the distance r of the aircraft from Guam International Airport was less than or equal to 55 miles. Due to a software error it actually worked only for $54 \le r \le 55$.

Not every software problem is the programmer's fault. The Venus probe, Mariner 1, had to be destroyed a few minutes after launch. Mathematicians had designed its tracking system to employ readings of Mariner's *average* velocity $\overline{v}$. Someone along the chain of transmission left out the bar over the v. The programmer then coded v according to instructions. The use of actual velocity instead of smoothed velocity set in motion a series of "corrections" that resulted in a classic negative feedback loop. That feedback loop, and nothing else, caused the erratic path and eventual destruction of the rocket and probe.

The software problem is one of human fallibility, and Körner can give no solution. What he has to say, though, is worth reading. Since the one real exercise in the section, Exercise 12.1.1, is weak, let me add my own. A program is written to control a system of high-speed elevators in a skyscraper. Included in the system are sensors that detect the number of persons in each elevator and the number of persons that await the elevator at each of its potential stops. To improve efficiency, the elevator is instructed to bypass any floor with more persons waiting than the elevator can accommodate. The next available elevator with adequate capacity is sent instead. Such a system was designed and coded by a leading engineering company. The fatal operational flaw of the system was discovered only in simulation (but at least before the system was put into service). Exercise: (i) What was that flaw? (ii) Redesign the system to fix the problem. (iii) Identify all the new errors that you have introduced in doing so. Fix those and repeat the cycle.

Part IV of *The Pleasures of Counting* is titled "Enigma Variations" and is concerned with cryptography. The title stems not from music but from the German enciphering-deciphering machine used during World War II. By now you will have noticed that Körner's tastes are very wide-ranging—I must confess, more so than my own. I found the material on ciphers old-fashioned and somewhat tedious; the modern RSA algorithm, discussed at some length in [3], is consigned to an exercise here. At least the historical path Körner takes passes through some combinatorial probability theory. (I think it possible that the average mathematician, on seeing the word "counting" in the title, might have expected to find more discrete mathematics in the book.) Part IV gains momentum with the introduction of Shannon's Theorem and concludes well with a long exercise on Hamming error-correcting codes. If this material whets the appetite of the student, then it has done its job; Körner appropriately refers the interested reader to the books of Koblitz [2] and Thompson [6] for more thorough discussions.

The last part of *The Pleasures of Counting* is called "Pleasures of Thought". It is not especially apparent why this section (and not any one of the preceding four) is so named, perhaps because it contains the least amount of mathematics and concludes with a philosophical discussion patterned after Plato's dialogues. Here is a quick summary of the mathematical content. The disappearance of surnames—that is, the dying out of the male line—is the point of departure for a brief excursion that will prepare the student for a later course on Markov chains. It is followed by the logistic differential equation and another, more sophisticated model for the spread of disease. As the last mathematical topic this neatly brings us full circle to modern versions of the cholera epidemic. Quite a lot is left to the exercises in all of this.

There is not enough space in a review such as this even to hint at all the topics that *The Pleasures of Counting* touches. But there are also pleasures to be found that are not mathematical. This is a very entertaining, well-written book that is filled with good humor. Do not miss the note on the national traits of technological disasters; it follows in the best tradition of Flanders and Swann's *A Song of Patriotic Prejudice*. The professional mathematician will appreciate the frequent anecdotes, often found in the side notes. (Do you know why the so-called Professor So-Called Adams was so-called?)

When it comes to criticism, every savvy author is wise to make use of the natural head start he or she is afforded, anticipating possible criticism and deflecting it before the critic comes to the plate. To this end Körner quotes Montaigne: "Some may assert that I have merely gathered here a big bunch of other men's flowers, having furnished nothing of my own but the string to hold them together." Trumped from the outset in my role as critic, I have little to add but the observation that very few readers will have encountered all of these flowers on their own. We may be grateful to Körner for bringing them to our notice. We might not have picked all the same flowers, but surely the author is allowed his choice. It is true that other gardens might have been visited, but the bouquet is already full. Is there nothing else to criticize? Flitting about, from one topic to the next, one does get the impression that some discussions are too superficial. This complaint, however, is not too serious. *The Pleasures of Counting* serves as a mathematical anthology accompanied by a bibliography of 259 items. Detailed notes allow the interested reader ample opportunity for follow-up. Although the bibliography is a valuable resource, from time to time one feels that the references might be better judged. Körner, for example, is very fond of Knuth's *The Art of Computer Programming*. It is far and away the best reference for the experienced mathematician. We might do better, however, to send our students to the book written by Knuth's student (*Algorithms*, by Robert Sedgewick [5]).

It is up to the author to state for whom he is writing his book, and I have already let Körner have his say. As the reviewer I have the less humble task of stating who I think should read the book. I will get to that after allowing Körner one last word. Writing of books that *he* recommends, Körner states "Any book that you can learn from is a good book." By that criterion I feel sure that *The Pleasures of Counting* will be deemed a good book by nearly everyone who picks it up. Although I would not be so presumptuous as to state that all mathematicians should read it, I can at least recommend it to them unreservedly. I especially think that mathematicians with limited applied mathematics experience would benefit from its study. I count myself in that group. On those few occasions in which I have had a little involvement in an application of mathematics, I went blundering in with the misconceptions of a pure mathematician. One example will be enough to illustrate.

Any collision avoidance system might generate two kinds of error. These are usually called type I and type II errors in statistics texts, but let us refer to them by the more suggestive names of false positives and false negatives. A standard strategy in statistics is to consider only decision-making procedures that ensure one (specified) type of error is held to an acceptably low level. From within that class the statistician tries to find a procedure that best controls the remaining type of error. Given the catastrophic consequences of a false negative in an aircraft's collision avoidance system, the mathematician will naturally insist that the radar system be designed so as to have an infinitesimal chance of a false negative. But hold on! The system just described will generate a large number of false positives. The mathematician answers, "True, but so what? They are harmless." Not so: the mathematician is not the one flying the plane. Military aviation experience (and even some tragic commercial experience) reveals that pilots simply turn off devices that produce too many false alarms. (The radar failure in Guam that I mentioned earlier was apparently introduced in an effort to reduce the number of false alarms.) A particular strength of Körner's book is that it reveals not only the rich vein of problems encountered in applied mathematics but also a human side of mathematics that is often absent from pure research.

I find myself at the end of my review, and I have not yet addressed Körner's targeted readership. That must be *your* assignment, for that readership will not see this review. If you anticipate being in the position of influencing a talented student considering mathematics as a career, then I urge you to read this book so that you can refer it to the student with authority. Make sure to recommend it to your local high school library. If they are slow to purchase it, then donate a copy. The mathematics shelves at bookstores are brimming with books titled *Fractal This* and *Chaos That*. Körner has given us a popular text that lacks a glitzy title but which is rich in substance. Let us do our best to make sure that it is read. I was not alive when Courant and Robbins wrote the words with which I opened this review, but I cannot believe that they were truer then than now. There has been a pressing need for a lively, accessible book that will answer the modern student who asks, "What is mathematics?" *The Pleasures of Counting* fits that bill.

References

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[2] N. Koblitz, *A course in number theory and cryptography*, Springer-Verlag, Berlin and New York, 1987.

[3] T. W. Körner, *Fourier analysis*, Cambridge University Press, 1988.

[4] J. David Logan, *Applied mathematics*, Second Ed., Wiley, New York, 1997.

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[6] T. M. Thompson, *From error correcting codes through sphere packings to simple groups*, Math. Assoc. Amer., Washington, DC, 1983.